2/9/2024

Fixed Reference Frame Equns:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

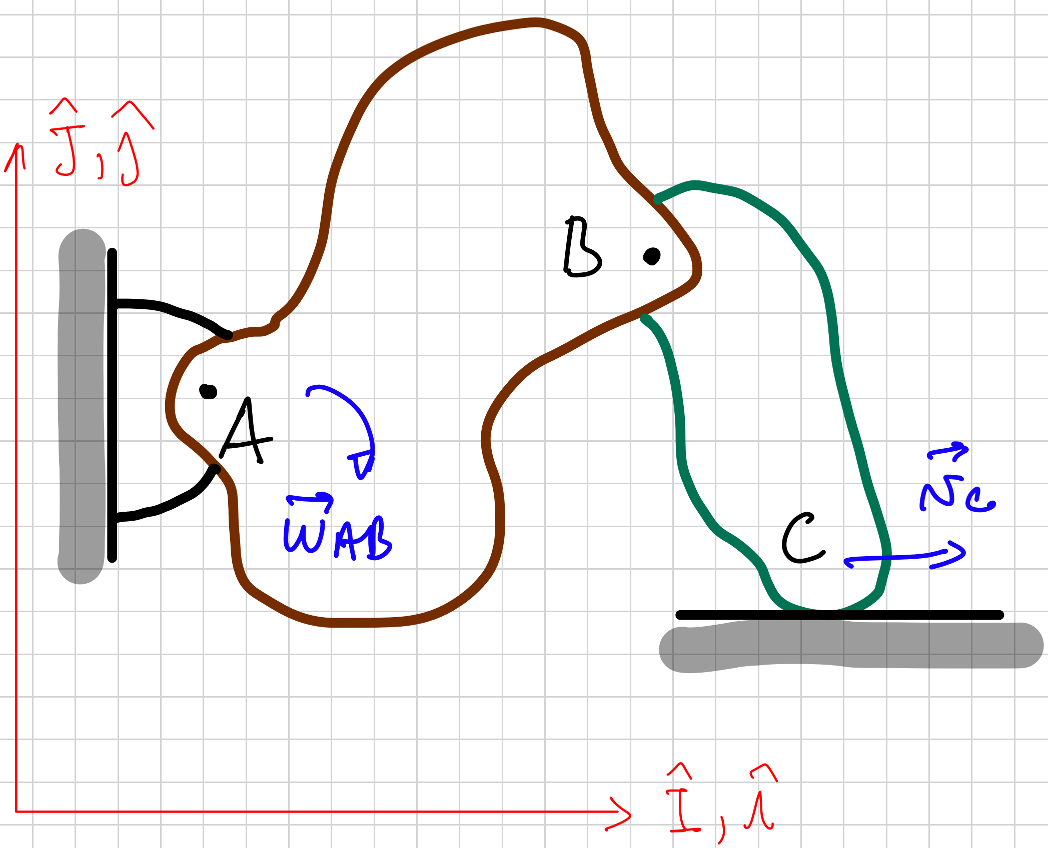
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}]$$

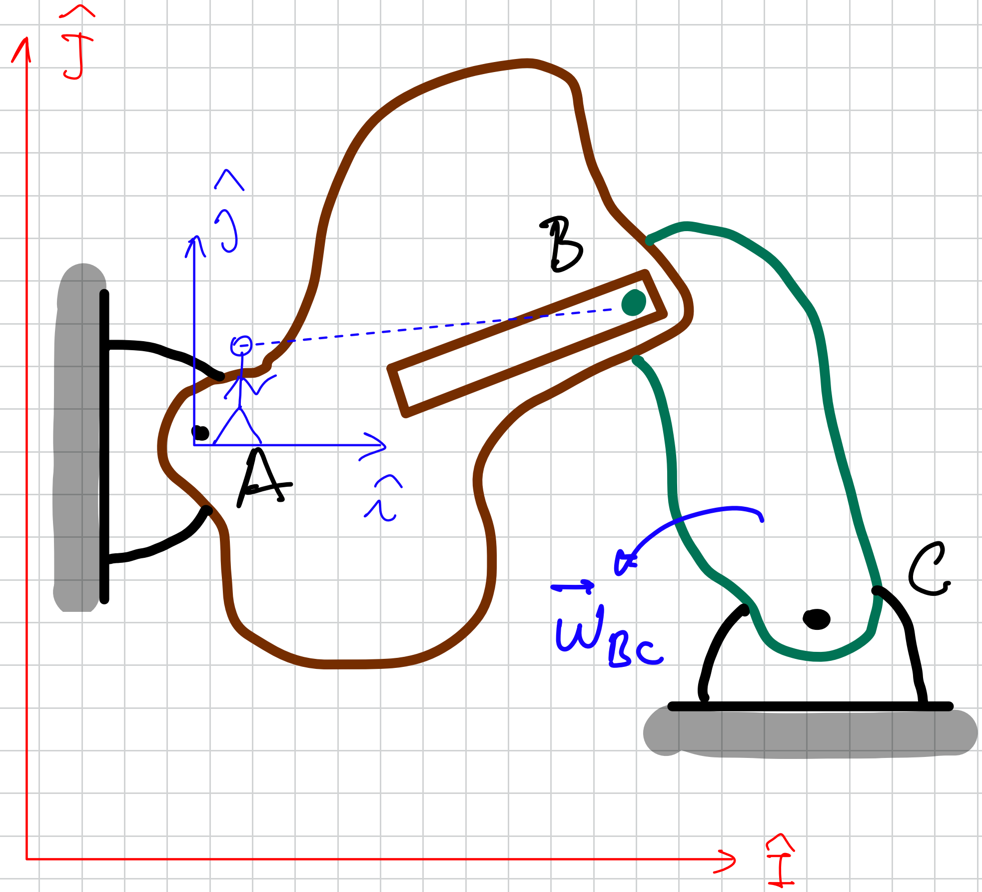
Rotating reference frame equns:

$$\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times [\omega \times \vec{r}_{B/A}]$$

Essential differences between Fixed & Moving frames



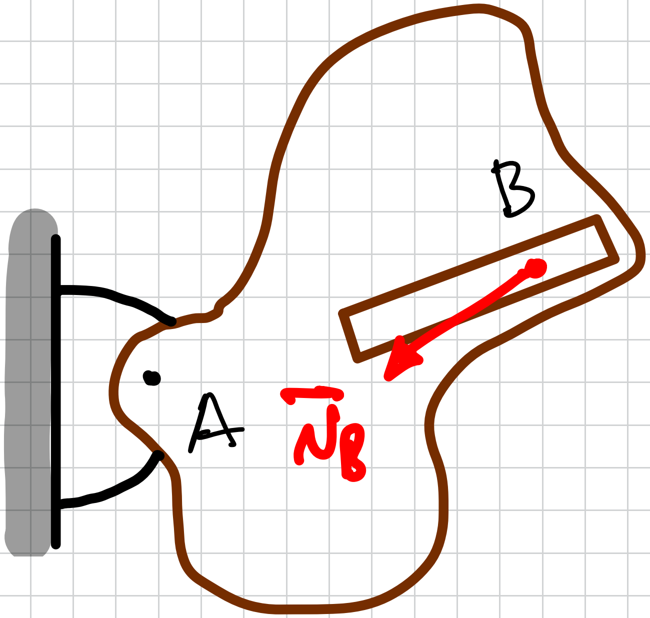


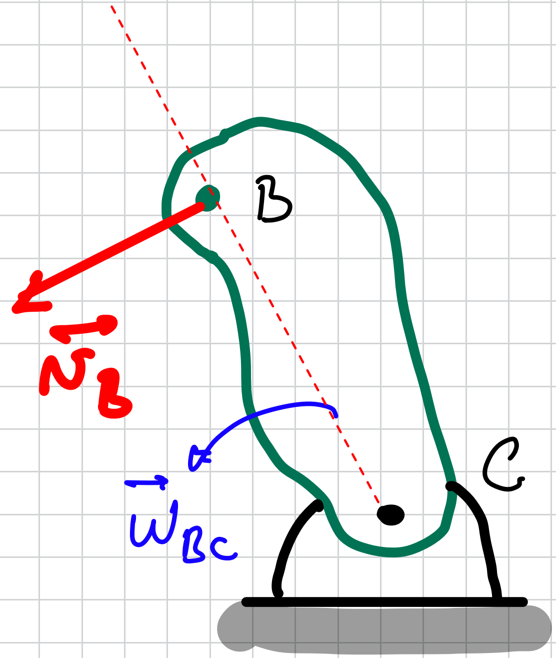
$\overline{AB}$ Constant

$\overline{BC}$ Constant

$\overline{AB}$ not constant

$\overline{BC}$ constant





$$\vec{v}_B = \vec{v}_B$$

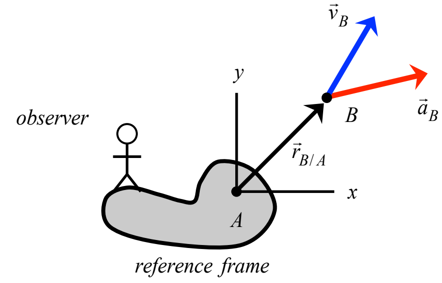
$$\vec{a}_B = \vec{a}_B$$

Summary: 2D Moving Reference Frame Kinematics 2

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point B.

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$



APPLICATION: Using 2D MRF equations in solving problems in the kinematics of mechanisms.

AP (rigid body):

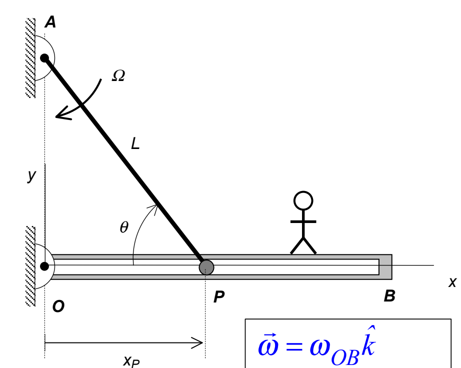
$$\vec{v}_P = \left(-\Omega\hat{k}\right) \times \vec{r}_{P/A}$$

$$\vec{a}_P = \left(-\dot{\Omega}\hat{k}\right) \times \vec{r}_{P/A} + \left(-\Omega\hat{k}\right) \times \left[\left(-\Omega\hat{k}\right) \times \vec{r}_{P/A}\right]$$

OP (*<u>not</u>* a rigid body):

$$\vec{v}_P = \dot{x}_P\hat{i} + \left(\omega_{OB}\hat{k}\right) \times \vec{r}_{P/A}$$

$$\vec{a}_P = \ddot{x}_P\hat{i} + \left(\alpha_{OB}\hat{k}\right) \times \vec{r}_{P/A} + 2\left(\omega_{OB}\hat{k}\right) \times \left(\dot{x}_P\hat{i}\right) + \left(\omega_{OB}\hat{k}\right) \times \left[\left(\omega_{OB}\hat{k}\right) \times \vec{r}_{P/A}\right]$$



With observer on OB:

$$\vec{\omega} = \omega_{OB}\hat{k}$$
$$\vec{\alpha} = \alpha_{OB}\hat{k}$$
$$\left(\vec{v}_{P/A}\right)_{rel} = \dot{x}_P\hat{i}$$
$$\left(\vec{a}_{P/A}\right)_{rel} = \ddot{x}_P\hat{i}$$

Example 3.A.4

Given: The disk shown below is rotating counterclockwise at a constant rate of ω. Link AP is vertical. Pin P slides within a straight slot cut into the disk. Let the xyz axes be attached to the disk. The slot is oriented at an angle of θ as measured from the x-axis. At the instant shown, P is on the x-axis, and the x-axis is horizontal.

Find: Determine:

(a) The angular velocity of link AP at this instant; and $\vec{\omega}_{AP}$

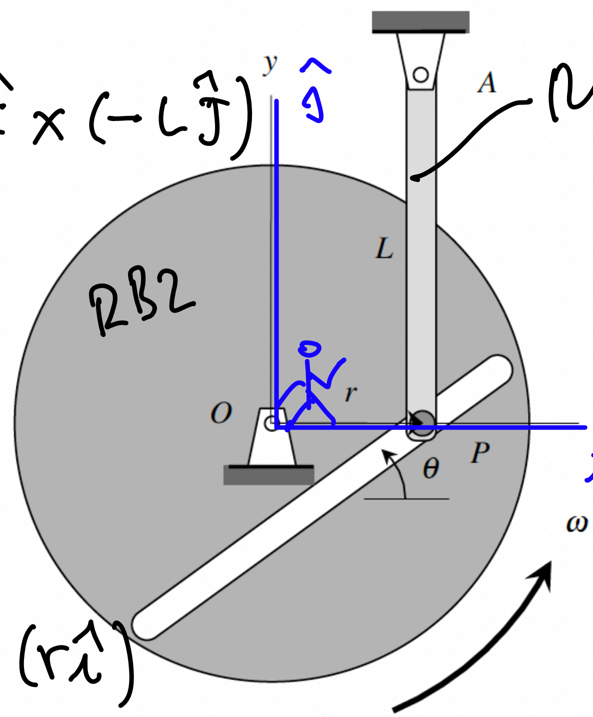
(b) The angular acceleration of link AP at this instant. $\vec{\alpha}_{AP}$

Use the following parameters in your analysis: $\omega = 8$ rad/s, $r = 0.2$ m, $L = 0.3$ m and $\theta = 36.87°$.

Bar AP (Fixed R.F)

$$\vec{v}_P = \cancel{\vec{v}_A}^{\,0} + \vec{\omega}_{AP} \times \vec{r}_{P/A} = \omega_{AP}\hat{K} \times (-L\hat{J})$$

$$\vec{v}_P = \omega_{AP} L \hat{I} \quad (1)$$



Disk (Moving R.F.)

$$\vec{v}_P = \cancel{\vec{v}_O} + (\vec{v}_{P/O})_{REL} + \vec{\omega} \times \vec{r}_{P/O}$$

$$\vec{v}_P = v_{REL}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + \omega\hat{k} \times (r\hat{i})$$

$$= v_{REL}\cos\theta\,\hat{i} + v_{REL}\sin\theta\,\hat{j} + \omega r\,\hat{j} \quad (2)$$

$$\vec{v}_{P/O} = v_{REL}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

At Instant shown, $\hat{i} = \hat{I}$, $\hat{j} = \hat{J}$, $\hat{k} = \hat{K}$

$$\vec{v}_P^{\,①} = v_P^{\,②}$$

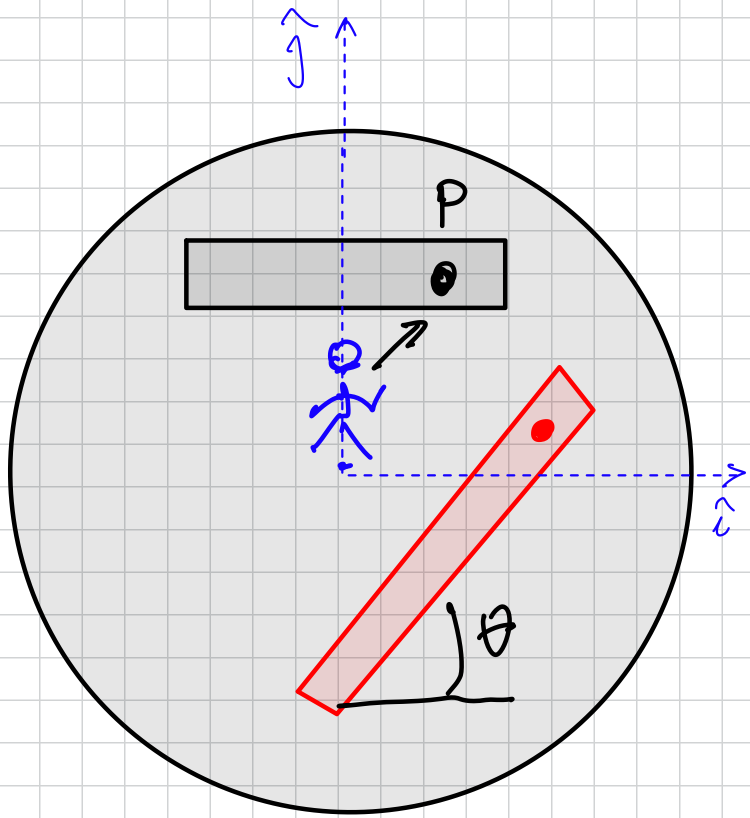
$$\omega_{AP} L \hat{i} = v_{REL} \cos\theta \, \hat{i} + v_{REL} \sin\theta \, \hat{j} + \omega r \, \hat{j}$$

$\hat{i}$: $\omega_{AP} L = v_{REL} \cos\theta$ $\longrightarrow$ $v_{REL} = -\dfrac{\omega r}{\sin\theta}$

$\hat{j}$: $0 = v_{REL} \sin\theta + \omega r$

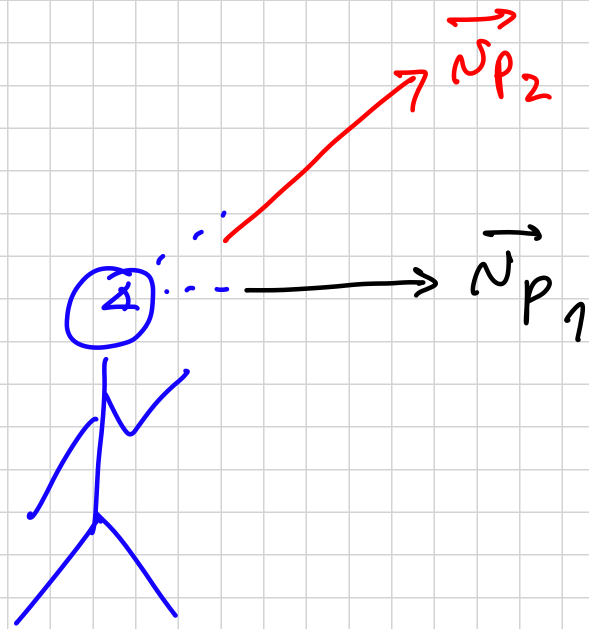
$$\boxed{\omega_{AP} = -\frac{\omega r}{L} \frac{\cos\theta}{\sin\theta}}$$

ANSWER (c)

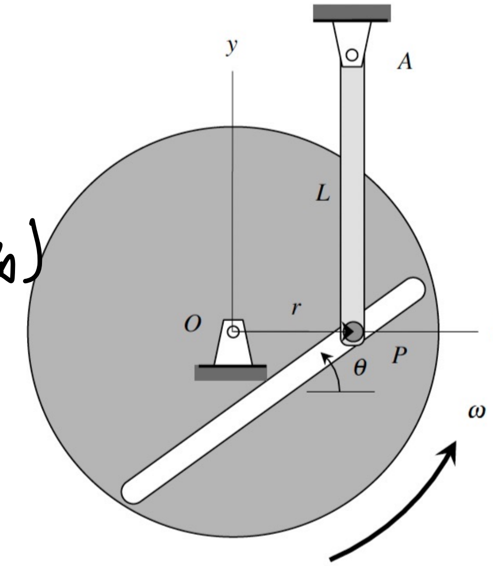


$$\vec{N}_{P_1} = N_{REL}\,\hat{i}$$

$$\vec{N}_{P_2} = N_{REL}\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$



Now, For acceleration



$$\vec{a}_P = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha}_{AP} \times \vec{r}_{P/A} + \vec{\omega}_{AP} \times (\vec{\omega}_{AP} \times \vec{r}_{P/A})$$

$$= \alpha_{AP}\,\hat{k} \times (-L\hat{j}) - \omega_{AP}^2\,(-L\hat{j})$$

$$= +\alpha_{AP} L\,\hat{i} + \omega_{AP}^2 L\,\hat{j} \quad 3$$

Moving RF:

$$\vec{a}_P = \cancel{\vec{a}_O}^{\,0} + (\vec{a}_{P/O})_{REL} + \cancel{\vec{\alpha}_O}^{\,0} \times \vec{r}_{P/O} + 2\omega_O \times (\vec{v}_{P/O})_{REL} - \omega^2 \vec{r}_{P/O}$$

$$= a_{REL}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + 2\omega\,\hat{k} \times (v_{REL}\cos\theta\,\hat{i} + v_{REL}\sin\theta\,\hat{j})$$

$$- \omega^2 (r\hat{i})$$

$$\vec{a}_P = a_{REL}\cos\theta\,\hat{i} + a_{REL}\sin\theta\,\hat{j} + 2\omega v_{REL}\cos\theta\,\hat{j} - 2\omega v_{REL}\sin\theta\,\hat{i}$$

$$- \omega^2 r\,\hat{i} \quad 4$$

$$\vec{a}_p \; ③ = \vec{a}_p \; ④$$

$$\hat{i}: \quad \alpha_{AP} L = a_{REL} \cos\theta - 2\omega v_{REL} \sin\theta - \omega^2 r$$

$$\hat{j}: \quad \omega_{AP}^2 L = a_{REL} \sin\theta + 2\omega v_{REL} \cos\theta$$

$$\boxed{\vec{\alpha}_{AP} = \left[\frac{\omega^2 r^2 \cos^3\theta}{L^2 \sin^3\theta} + \frac{2\omega^2 r \cos^2\theta}{L^2 \sin\theta} + \frac{\omega^2 r}{L} \right] \hat{k}}$$

ANSWER (b)