EXAM 1

Thursday, Feb 12 8:00-9:30 PM

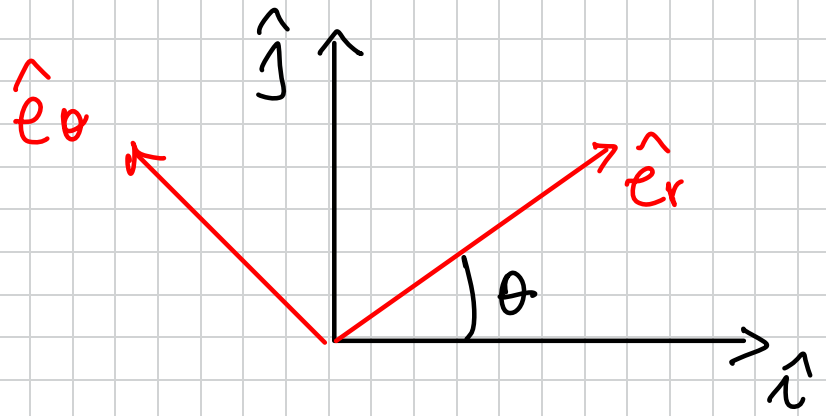
WTHR 200

Projection of Motion

$$\vec{a} = a_r \hat{e}_r + a_\theta \hat{e}_\theta$$

a_r: known; a_θ: known

Find the horizontal and vertical components of $\vec{a}$



Need $\vec{a} = a_x \hat{\imath} + a_y \hat{j}$

a_x: unknown; a_y: unknown

$$a_x = \vec{a} \cdot \hat{\imath}$$

Horiz. component of $\vec{a}$

$$a_y = \vec{a} \cdot \hat{j}$$

$$\hat{\imath} = \cos\theta \hat{e}_r - \sin\theta \hat{e}_\theta$$

$$\hat{j} = \sin\theta \hat{e}_r + \cos\theta \hat{e}_\theta$$

$$\Rightarrow a_x = \underbrace{(a_r \hat{e}_r + a_\theta \hat{e}_\theta)}_{\vec{a}} \cdot \underbrace{(\cos\theta \hat{e}_r - \sin\theta \hat{e}_\theta)}_{\hat{\imath}}$$

$a_x = a_r \cos\theta - a_\theta \sin\theta$ ⟵ Horizontal comp. $\vec{a}$

$a_y = a_r \sin\theta + a_\theta \cos\theta$ ⟵ Vertical

$$\vec{a} = a_x \hat{\imath} + a_y \hat{\jmath}$$

Question C1.3

Consider the path description for the motion of a point P. Circle the item below that most accurately describes the acceleration of P, $\vec{a}_P$:

(a) The rate of change of speed for P, $\dot{v}_P$, is ALWAYS the same as the magnitude of its acceleration, $|\vec{a}_P|$.

(b) The rate of change of speed for P, $\dot{v}_P$, is the same as the magnitude of its acceleration, $|\vec{a}_P|$ only if the path of P is STRAIGHT.

(c) The acceleration of point P is always PERPENDICULAR to the path of P.

(d) None of the above.

$$\vec{a}_p = \dot{v}_p \hat{e}_t + \frac{v_p^2}{\rho} \hat{e}_n$$

$$|\vec{a}_p| = \sqrt{\dot{v}^2 + \left(\frac{v_p^2}{\rho}\right)^2}$$

$|\vec{a}_p| = \dot{v}$ is true only if

$$\left(\frac{v_p^2}{\rho}\right) = 0 \Rightarrow \rho = \infty$$

$\therefore$ straight path

Question C1.4

Point P represents a passenger traveling in a automobile. The velocity and acceleration of P, $\vec{v}_P$ and $\vec{a}_P$, respectively, are shown below at a given instant in time. Circle the item below that most accurately describes the motion of P:

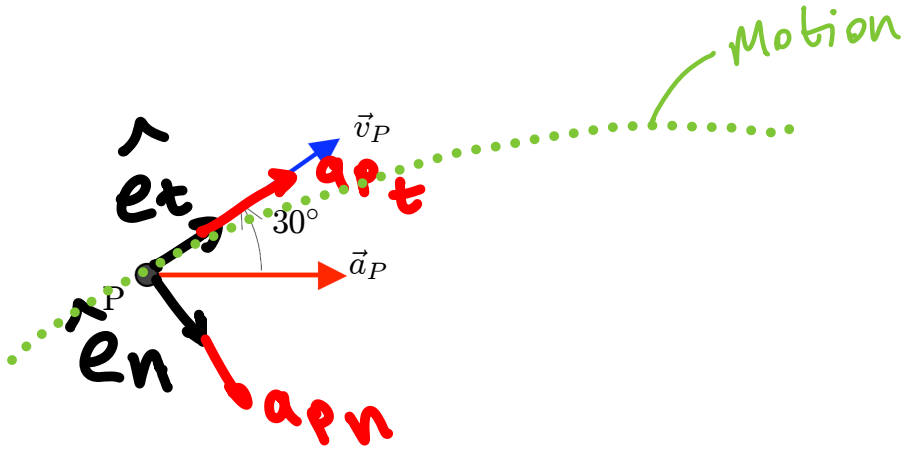
(a) The speed of P is decreasing and P is turning left.

(b) The speed of P is increasing and P is turning left.

(c) The speed of P is decreasing and P is turning right.

(d) The speed of P is increasing and P is turning right.

(e) There is insufficient information given for answering this question.



a_{Pt} in same direction of $\vec{v}_P$

$\Rightarrow$ speeding up

a_{Pn} indicates direction of rotation

$\Rightarrow$ turning right from perspective of person riding in vehicle

Question C1.5

Point P represents a passenger traveling in a automobile. The velocity and acceleration of P, $\vec{v}_P$ and $\vec{a}_P$, respectively, are shown below at a given instant in time. Circle the item below that most accurately describes the motion of P:

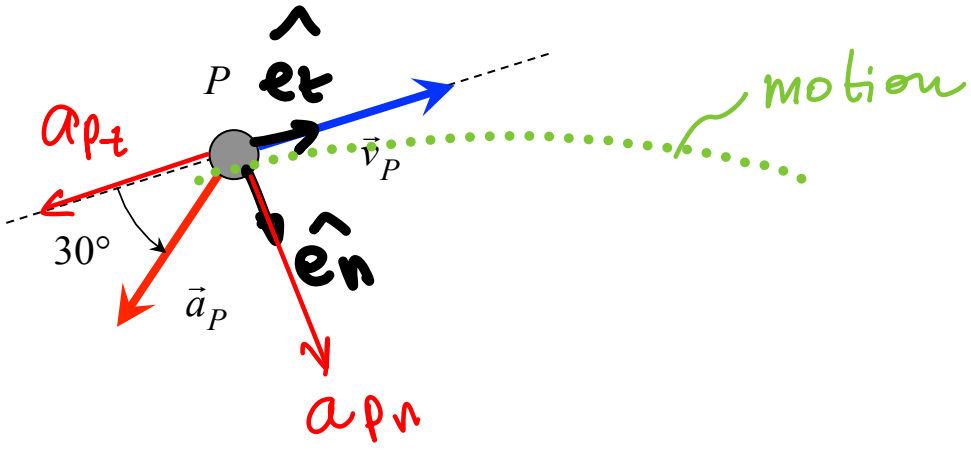
(a) The speed of P is decreasing and P is turning left.

(b) The speed of P is increasing and P is turning left.

(c) The speed of P is decreasing and P is turning right.

(d) The speed of P is increasing and P is turning right.

(e) There is insufficient information given for answering this question.



a_{pt} in opposite direction of $\vec{v}_P$

$\Rightarrow$ slowing down

a_{pn} indicates direction of rotation

$\Rightarrow$ turning right from the perspective of a person riding in the vehicle

Question C1.7

A polar description with variables r and θ is used to describe the kinematics of point P. For a position with $r = 0.5$ m and $\theta = 2$ radians, the velocity and acceleration vectors for P are known to be:

$$\vec{v}_P = (-6\hat{e}_r + 2\hat{e}_\theta) \text{ m/s}$$

$$\vec{a}_P = (10\hat{e}_r) \text{ m/s}^2$$

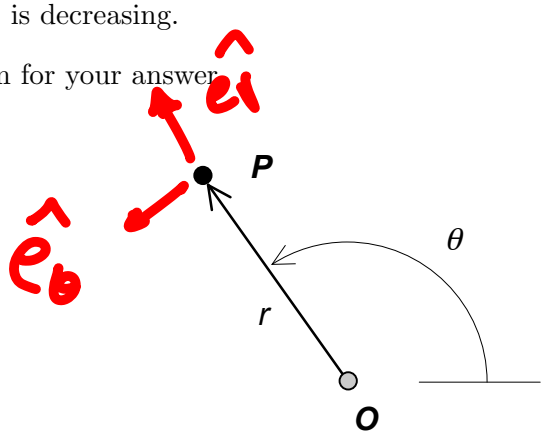
respectively. Circle the item below that most accurately describes the speed of P:

(a) The speed of P is increasing.

(b) The speed of P is not changing.

(c) The speed of P is decreasing.

Provide a justification for your answer.



$\vec{v}_P = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$= -6\hat{e}_r + 2\hat{e}_\theta$

$\Rightarrow \dot{r} = -6$ m/s

$\Rightarrow r\dot{\theta} = 2$

$\dot{\theta} = 4$ rad/s

$\vec{a}_P = \_\_\_\hat{e}_r + \_\_\_\hat{e}_\theta = 10\hat{e}_r$

$\ddot{r} - r\dot{\theta}^2 = 10$

$\ddot{r} = 10 + (0.5)(4)$

$\ddot{r} = 12$ m/s^2 speed increasing

Question C1.11

Blocks A and B are connected by an inextensible cable, with this cable being wrapped around pulleys at A and E. Assume that the radii of these two pulleys are small compared with the other dimensions of the problem. Block A moves downward with a speed of v_A. Let v_B be the speed of block B when $s_A > 0$. Circle the answer below that most accurately describes the speed of B as compared to the speed of A:

(a) $0 < v_B < 2v_A$ ✓✓

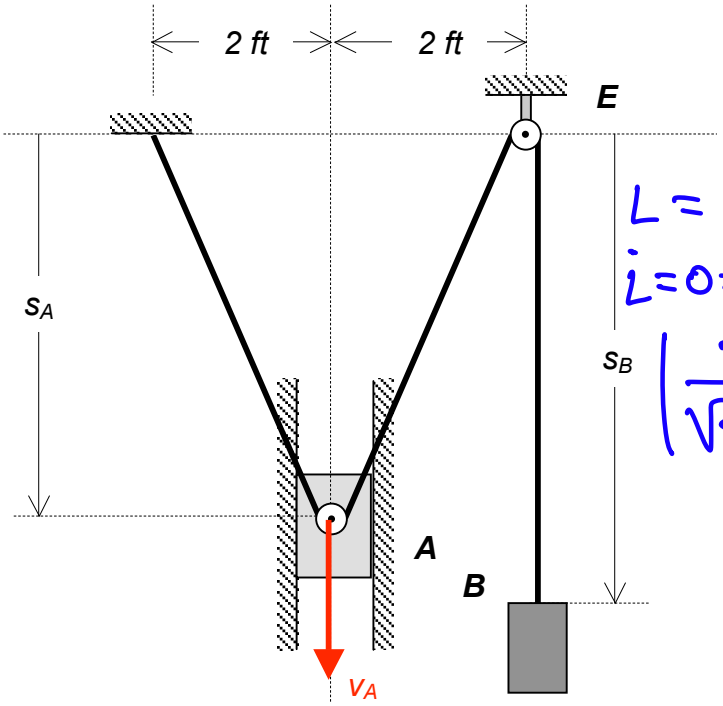
(b) $v_B = v_A$ ← no because $N_B = \frac{2s_A}{\sqrt{s_A^2+4}} N_A$

(c) $v_B = 2v_A$ ← only if s_A is large

(d) $v_B > 2v_A$ ← no, because $N_B < 2N_A$

(e) More information is needed about the problem in order to answer this question.

Provide a mathematical justification for your answer.



$L = 2\sqrt{s_A^2+4} + s_B$

$\dot{L} = 0 = 2\left(\frac{1}{2}\right)(s_A^2+4)^{-1/2}(2s_A)\dot{s}_A + \dot{s}_B$

$\left|\frac{2s_A}{\sqrt{s_A^2+4}}\dot{s}_A\right| = |\dot{s}_B|$

$\frac{2s_A}{\sqrt{s_A^2+4}} N_A = N_B$

if $s_A \cong 0 \Rightarrow N_B = 0$

if s_A is $\gg 0 \Rightarrow$

$\frac{s_A}{\sqrt{s_A^2+4}} \cong 1 \cdot 2N_A = N_B$

$\Rightarrow N_B \cong 2N_A$

CHAPTER 2 REVIEW

Topics covered in Ch. 2

- Kinematics of rigid bodies
  - ↳ velocity
  - ↳ acceleration
- Instant centers of rotation
- Rolling without slipping

EXAM 1 COVERS MATERIAL UP TO THIS REVIEW

FUNDAMENTAL EQUATIONS:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}]$$

$$= \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

$$\vec{v}_{IC} = 0$$

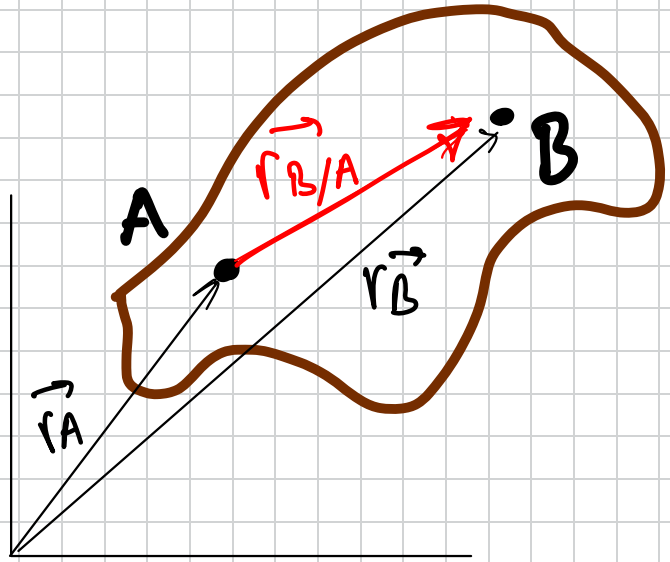
$$|\vec{v}_P| = |\vec{\omega}| \cdot |\vec{r}_{P/IC}|$$

$$\vec{v}_{C/G} = 0$$

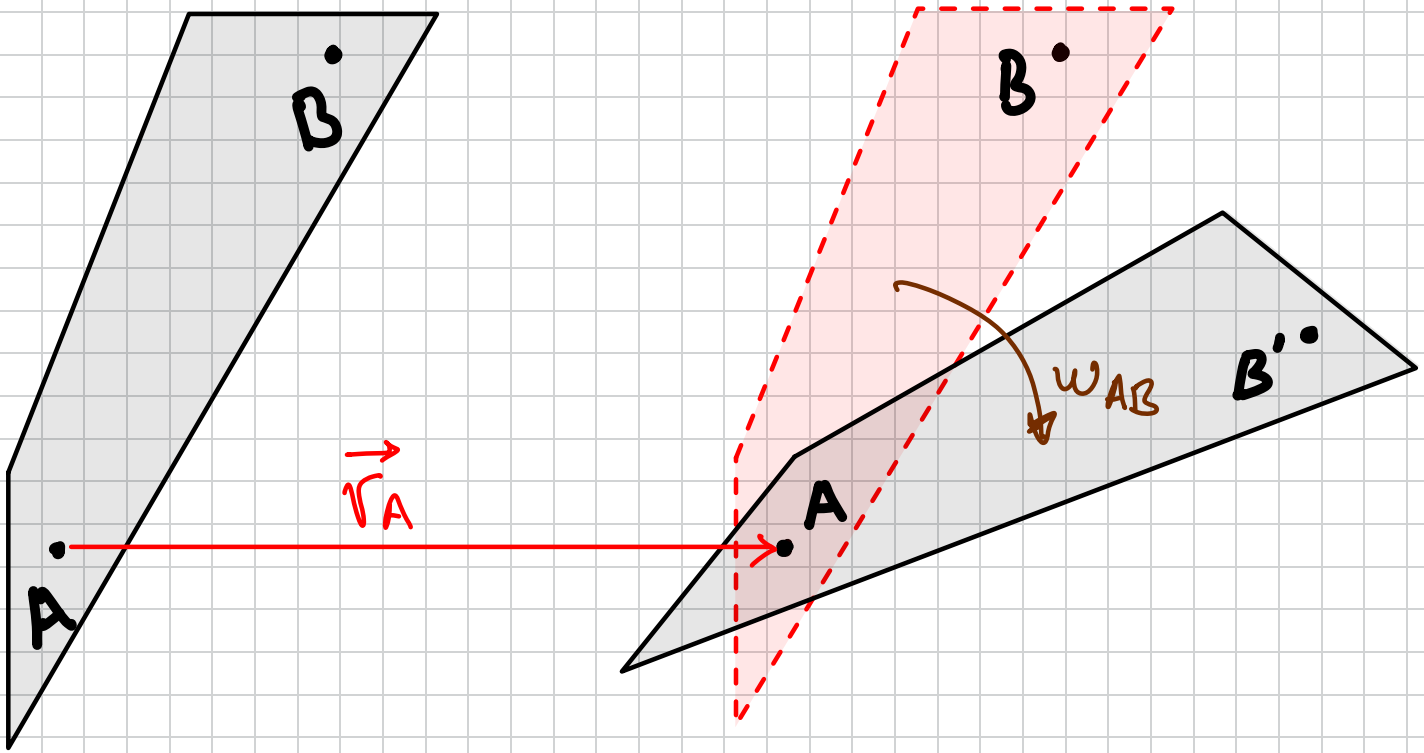
$$a_{C/G} = \omega^2 R$$

$$v_O = \omega R$$

$$a_O = \alpha R$$



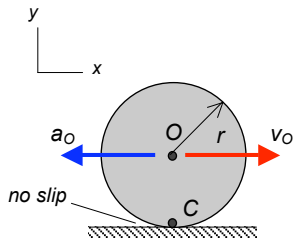
$$\vec{v}_B = \vec{v}_A + \omega \times \vec{r}_{B/A}$$



ERRATUM

Question C2.2

A sphere of radius r rolls without slipping to the right on a rough, horizontal surface. The center of the sphere, O, has a speed of v_O, with this speed decreasing at a rate of a_O.



Circle the figure below that most accurately represents the direction of the acceleration of the contact point C.

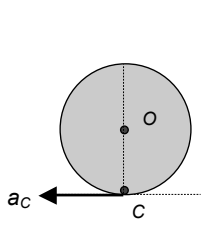


Figure A

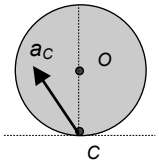


Figure B

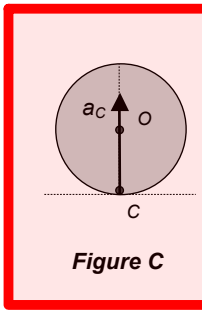


Figure C

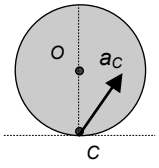


Figure D

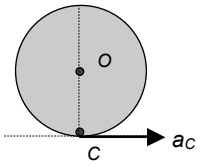


Figure E

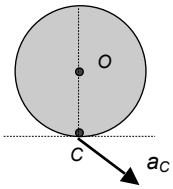


Figure F

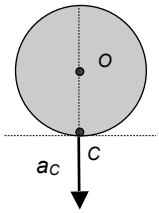


Figure G

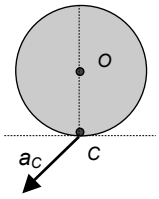


Figure H

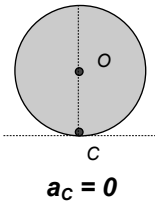


Figure I

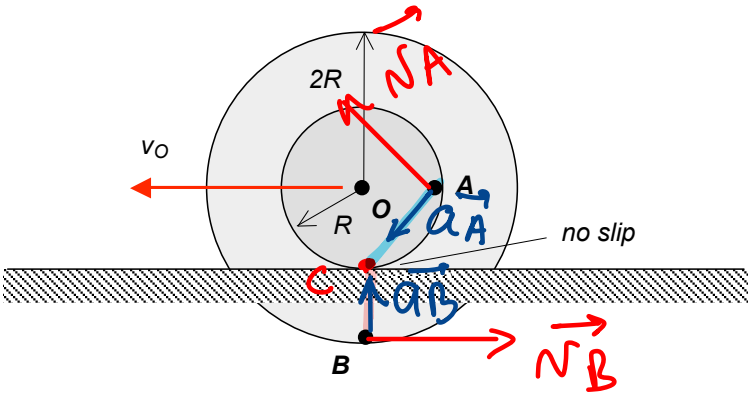
The contact point is an instant center with $v_C = 0$ $a_{Cx} = 0$. At all other instants a_{Cx} depends on a_O and α

Question C2.3

A stepped drum has inner and outer radii of R and $2R$, respectively. The drum rolls to the left with its center O having a constant speed of v_O, as shown below. Point A and B lie on the inner and outer radii, respectively, of the drum. At the instant shown, A is directly to the right of O, and B is directly below O. For this position:

(a) Make a sketch of the velocity vectors for A and B.

(b) Make a sketch of the acceleration vectors for A and B.



$a_O = 0$

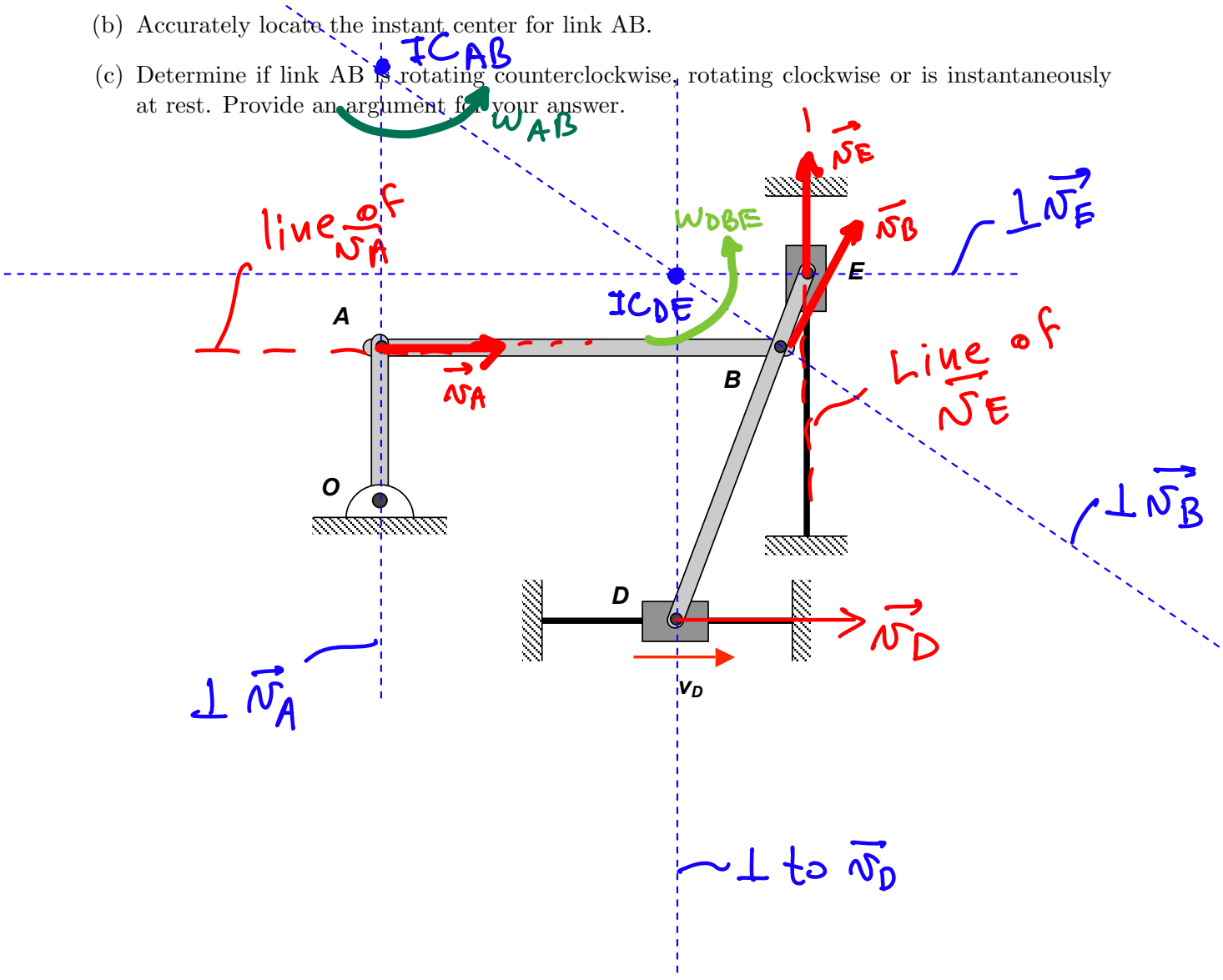
Question C2.5

A mechanism is made up of links OA, AB and DE. Pins D and E on link DE are constrained to move along straight guides. Link OA is pinned to ground at O and pinned to link AB at A. Link AB is also pinned to link DE at point B. Pin D moves to the right with a speed of v_D. For the position shown:

(a) Accurately locate the instant center for link DE.

(b) Accurately locate the instant center for link AB.

(c) Determine if link AB is rotating counterclockwise, rotating clockwise or is instantaneously at rest. Provide an argument for your answer.



AB CCW

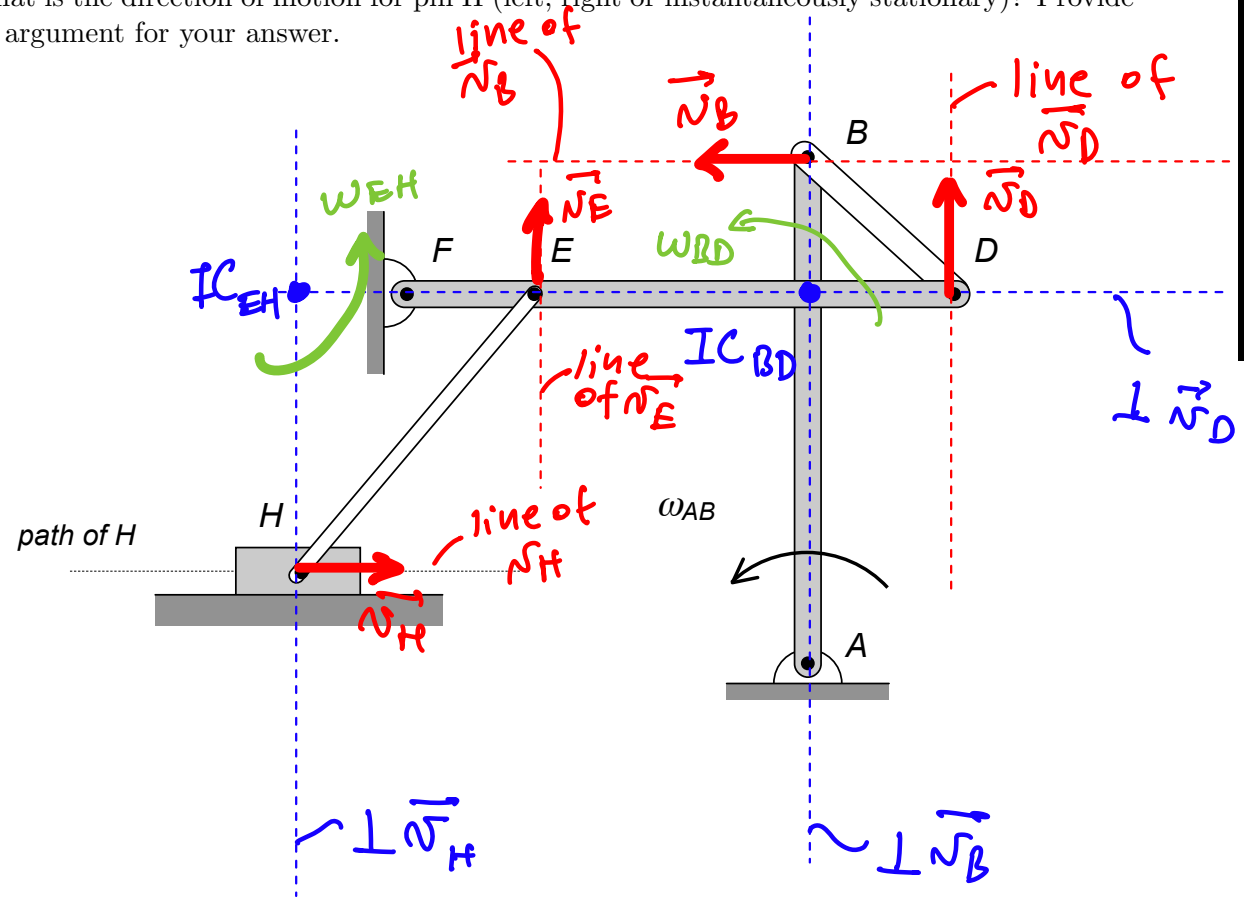
Question C2.6

The mechanism shown below is made up of links AB, BD, DF and EH. Links AB and DF are pinned to ground at pins A and F, respectively. Link EH is pinned to link DF at E. Pin H is constrained to move along a straight, horizontal path. Link AB is rotating counterclockwise, as shown.

(a) Accurately locate the instant center for link BD.

(b) Accurately locate the instant center for link EH.

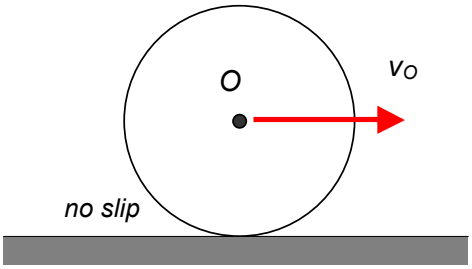
(c) What is the direction of motion for pin H (left, right or instantaneously stationary)? Provide an argument for your answer.

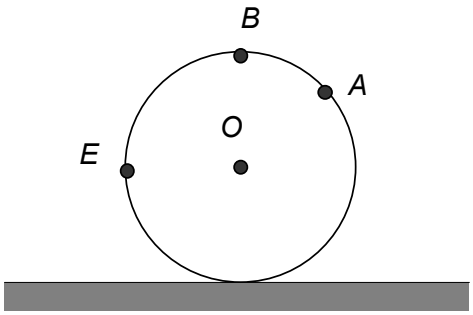


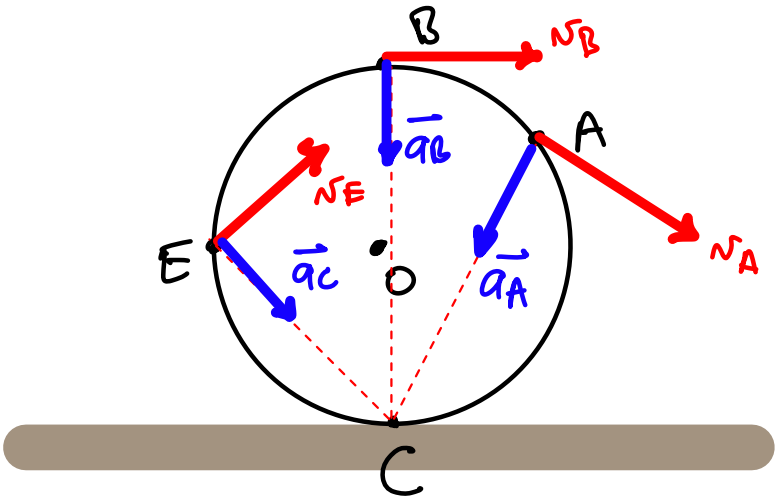
c) H moves to the right as ω_{EH} is counter clockwise and $\vec{v}_E$ goes up

Question C2.11

A wheel rolls without slipping with its center O having a constant speed of v_O. In the figure below, sketch the directions for the acceleration vectors for points A, B and E on the wheel.

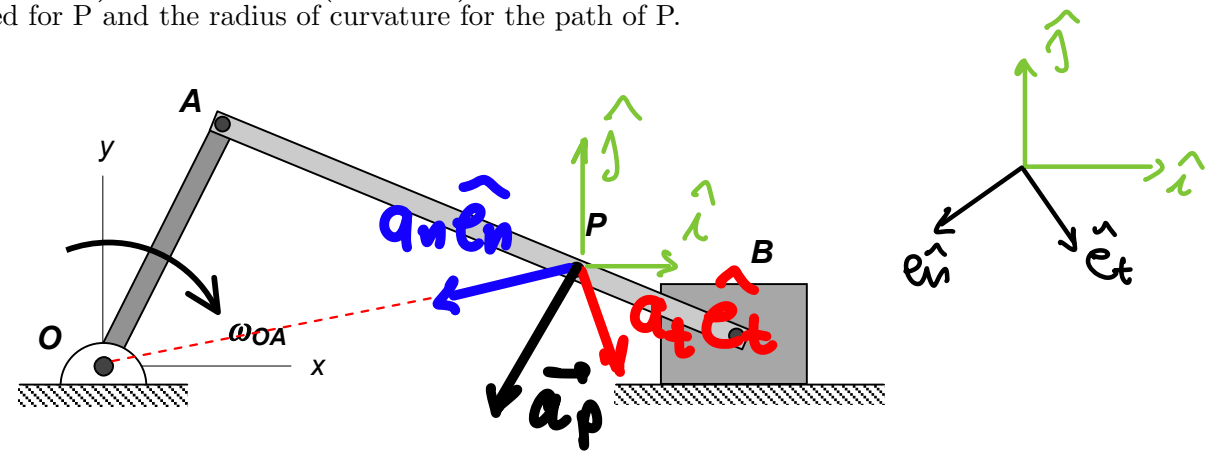






Question C2.15

The mechanism shown below is made up of links OA and AB. At the instant shown, link OA is rotating in the clockwise direction with the velocity and acceleration of point P on link AB known to be: $\vec{v}_P = \left(20\hat{i} - 4\hat{j}\right)$ m/s and $\vec{a}_P = \left(-10\hat{i} - 5\hat{j}\right)$ m/s^2. For this position, determine the rate of change of speed for P and the radius of curvature for the path of P.



$$\vec{a}_P = \dot{v}_P\,\hat{e}_t + \frac{v_P^2}{\rho}\,e_n = -10\hat{i} - 5\hat{j}$$

Need $\quad a_{P_t} = \vec{a}_P \cdot \hat{e}_t$

$$\vec{v} = v\hat{e}_t \Rightarrow \hat{e}_t = \frac{\vec{v}}{|\vec{v}|} = \frac{20\hat{i} - 4\hat{j}}{\sqrt{20^2+4^2}}$$

$$\hat{e}_t = 0.98\hat{i} - 0.19\hat{j}$$

$$\dot{v} = a_{P_t} = \vec{a}_P \cdot \hat{e}_t = \left(-10\hat{i} - 5\hat{j}\right)\cdot\left(0.98\hat{i} - 0.19\hat{j}\right)$$

$$\dot{v} = a_{P_t} = -9.8 + 0.95 = -10.75 \text{ m/s}^2$$

$$|\vec{a}| = \sqrt{\dot{v}^2 + \left(\frac{v^2}{\rho}\right)^2} = \sqrt{10^2+5^2} = 11.18$$

$$(-10.75)^2 - (11.18)^2 = -\left(\frac{\sqrt{20^2+4^2}}{\rho}\right)^2$$

$$\rho = 6.64 \text{ m}$$

Quiz Question

Question C2.14

A wheel rolls without slipping as its center O moves to the right with a constant speed of v_0. Point A is on the circumference of the wheel. At the instant shown, A is located directly to the left of O. At this instant (circle the correct answer):

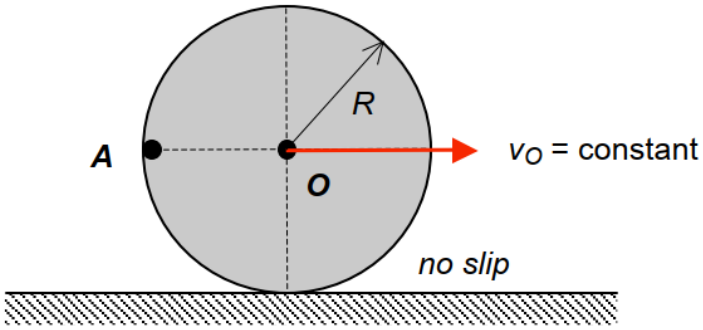
(a) The speed of A is decreasing.

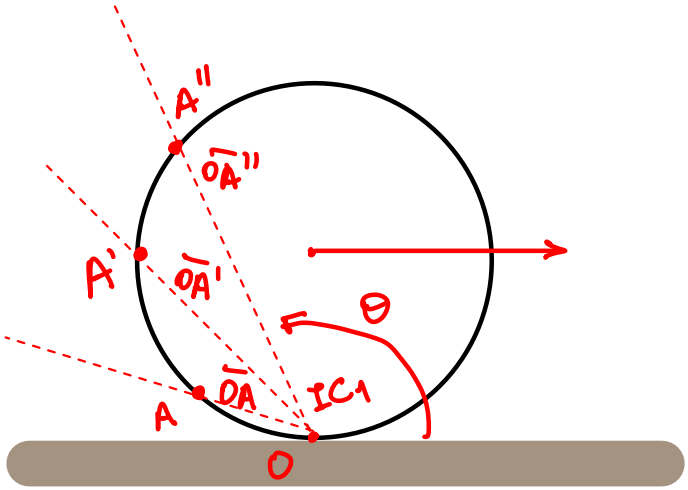
(b) The speed of A is constant.

(c) The speed of A is increasing.

(d) Numerical values for v_O and the radius R of the wheel are needed to answer this question.

Using IC ! $v = \bar{OA}\,\omega$





distance $\bar{OA}$ keeps growing until $\theta = 90°$ and

v_O = constant $= \omega$ = constant

$\Rightarrow$

$v_A = \bar{OA}\,\omega$]<

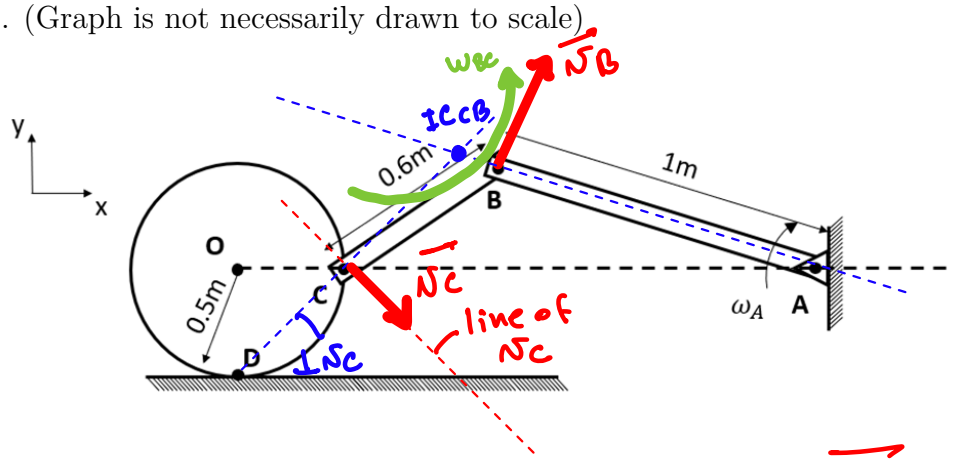
$v_{A'} = \bar{OA}'\,\omega$]<

$v_{A''} = \bar{OA}''\,\omega$

$\Rightarrow v_A$ is increasing

Quiz Question

Given the system set up as Figure. Bar AB is rotating clockwise with constant angular speed. Disk O is rolling without slipping on ground. At this instance, point C is located directly to the right of center O, which also aligns horizontally with point A. Point D is at the bottom of the disk where it is in contact with ground. Please answer the following questions. (Graph is not necessarily drawn to scale)



- Which direction is Bar BC rotating?
  - (A) Clockwise
  - (B) Counterclockwise
  - (C) No Rotation
  - (D) Not enough information is given
- Which direction is Disk rotating?
  - (A) Clockwise
  - (B) Counterclockwise
  - (C) No Rotation
  - (D) Not enough information is given
- Which point has a higher speed between B and C?
  - (A) $v_B > v_C$
  - (B) $v_B = v_C$
  - (C) $v_B < v_C$
  - (D) Not enough information is given

$\vec{v}_B = \vec{\omega}_{AB} \cdot \vec{r}_{AB}$

$v_B = \omega_{AB}$

$\checkmark \quad ? \quad ?$

$v_B = \omega_{CB} \cdot IC_{CB}B$

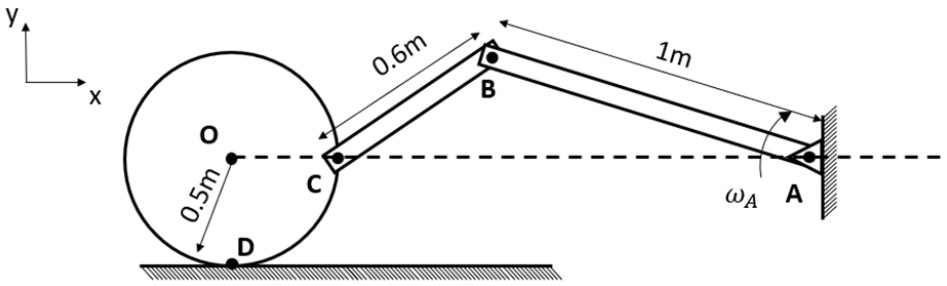
$? \quad ? \quad ?$

$v_C = \omega_{CB} \cdot IC_{CB}C$

2 eqns 3 unknowns

Quiz Question

Given the system set up as Figure. Bar AB is rotating clockwise with constant angular speed. Disk O is rolling without slipping on ground. At this instance, point C is located directly to the right of center O, which also aligns horizontally with point A. Point D is at the bottom of the disk where it is in contact with ground. Please answer the following questions. (Graph is not necessarily drawn to scale)



- ONLY USING Instant center method, which direction is point O going?

  (A) O moving to the left

  (B) O moving to the right

  (C) O not moving

  (D) Instant center method is not useful in this case

- ONLY USING Instant center method, is point O speeding up or slowing down?

  (A) O is speeding up

  (B) O is slowing down

  (C) O is not changing in speed

  (D) Instant center method is not useful in this case