ME 274 Lecture 9

Planar kinematics: Rigid Bodies (Instant Centers)

Eugenio “Henny” Frias-Miranda

2/2/26

Housekeeping/Announcements

1. **HW 8 due tonight!!**
2. **Exam Next Thursday at 8PM…**
    1. 90 minutes long
    2. 2 questions will be like homework questions
    3. 1 question will be a multipart series of conceptual questions
    4. Equation sheet will be posted on ME274 website
    5. Pi Tau Review session on Friday.
    6. Prof. Krousgrill will host a review session on Zoom Tuesday before exam
3. Remote Office hours today! (https://purdue-edu.zoom.us/j/96722067110)

***Reminder for Henny to wear a mic during the lecture.

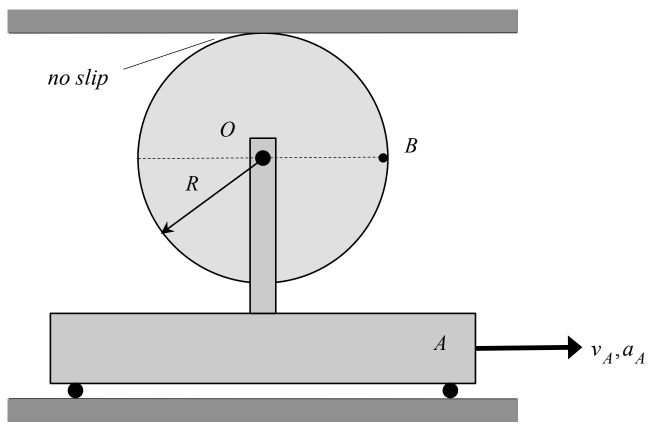
Homework H2.E

Given: Cart A moves to the right with a speed of v_A and an acceleration a_A. A disk of radius R is attached to a shaft on the cart at the center O of the disk. The disk is in contact with a horizontal surface at its top surface; as the cart moves, the disk rolls without slipping on this horizontal surface.

Find: For this problem:

(a) Determine the angular velocity and angular acceleration of the disk. Write your answers as vectors.

(b) Determine the acceleration of point B on the circumference of the disk and with B being immediately to right of O at the instant shown.

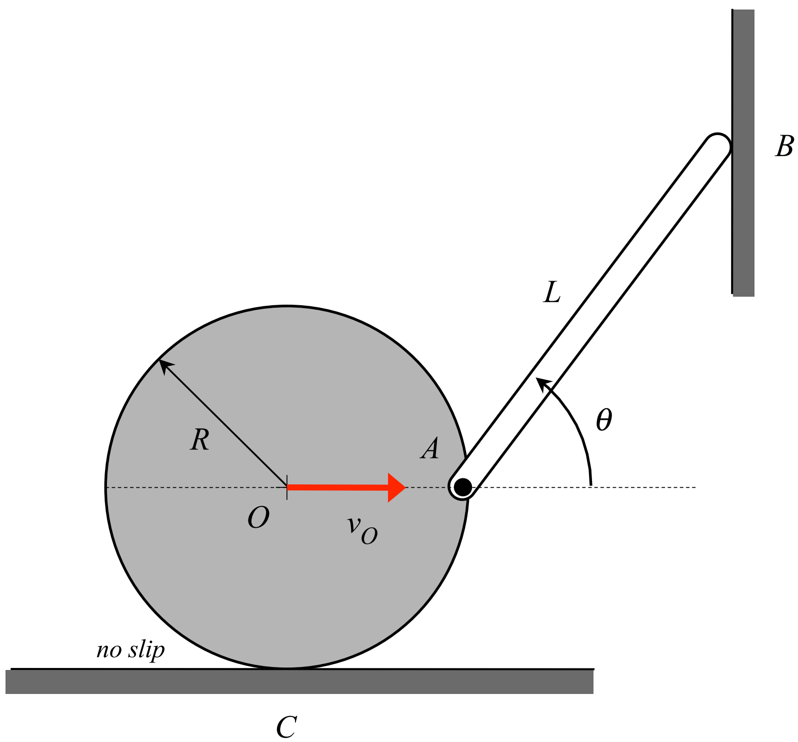


Use the following parameters in your analysis: $R = 2$ ft, $v_A = 4$ ft/s and $a_A = 5$ ft/s^2.

Homework H2.F

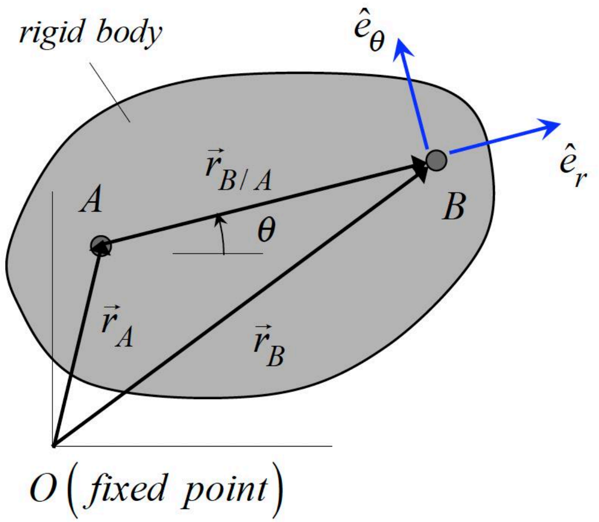
Given: A wheel of radius R rolls without slipping on a horizontal surface with its center O traveling with a constant speed of v_O. Bar AB is pinned to the outer perimeter of the wheel at end A, and end B of the bar is constrained to slide along a vertical wall. At the position shown, pin A is directly to the right of O.

Find: For the position show, determine the angular velocity of link AB and the angular acceleration of link AB.



Use the following parameters in your analysis: $\theta = 53.13°$, $L = 1.5$ ft, $R = 0.5$ ft and $v_O = 10$ ft/s.

Rigid Body Kinematics



1. A rigid body is an object where the **distance between any two points on the object remains fixed,** regardless of the motion of the object.

2. Velocity and acceleration equations for ***planar motion of a rigid body:***

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right] + \vec{\alpha} \times \vec{r}_{B/A}$$

When in the same plane… :

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}.$$

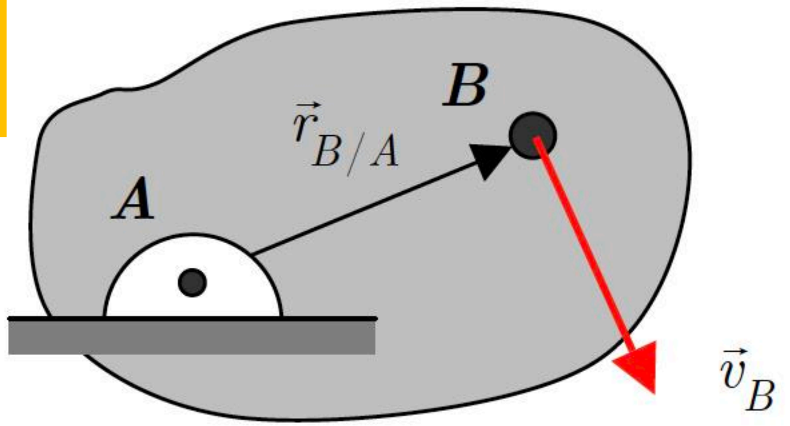
[pg. 85 content]

Laying the Groundwork for Instantaneous Centers of Rotation
(***Instant Centers*** – *for short*)

- The Concept of Instant Centers is similar to having a Rigid Body that is Pinned to the ground at point A (v_A = 0)
- We can say that the body is *rotating about point A* or A is the center of rotation.
- The velocity of B is perpendicular to the line connecting B with the center of rotation A

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} =$$

- Speed of B is given by:



- We generally use the concept of Instant Centers for:
  1. Solving for **angular velocity (w) or a velocity** at a point
  2. At the end of a Planar Rigid Body (Ch 2) Problem to **determine the direction of** angular velocity (w) or velocity

[pg. 103 content]

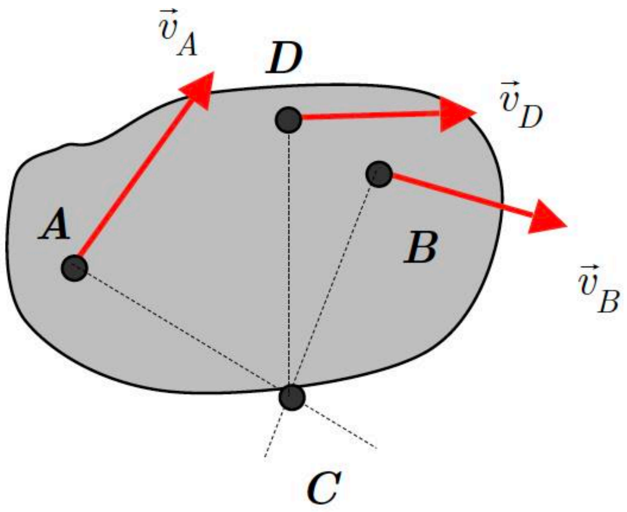
Where is the location of the instant center?

1. Locate two points (A and B) on the rigid body, where you know the
   Draw them out (red arrows on figure below).

2. The instant center is at

3. is known as the "Instantaneous Center of Rotation" (or simply "Instant Center").

4. The (e.g. Point D) must be perpendicular to the line connecting C and D



[pg. 103-104 content]

Things to Know About Instant Centers

1. The speed of any point is proportional to the distance from that point to C.
   - We can write this as :

2. An Instant Center necessarily need to lie within the rigid body it describes

3. What does it say about w if we find that ?
   -

4. With the concept of Instant Centers, you are able to look at a problem and get a sense of what direction it is rotating in.

5. When the rigid body moves to a new position, the location of C (instant center)
Ie. This is why we call it an *Instant* Center)

6. As a general rule, you cannot use instant centers when determining a .
We just use them for velocity and angular velocity.

[pg. 104 content]

Methods: Instant Centers

1. Locate two points A and B on the rigid Body
    - Let A be a point for which you know:
        - **Magnitude**
        - **Direction**
    - Let B be a point for which you know:
        - **Direction**

2. Draw the directions of the velocity vectors

3. Draw the lines that are perpendicular to the velocity vectors

4. The intersection of the two perpendiculars is the instant center, C, of the body. For this we know that v_C = 0.

5. From this we can find:
    - **Angular velocity, w:**

$$\omega = \frac{|\vec{v}_A|}{|\vec{r}_{A/C}|}$$

    - **Direction of angular velocity from looking at the velocity vectors (ie CW or CCW)**

6. Velocity of any point D on the body is perpendicular to the line connecting C and D:

$$|\vec{v}_D| = |\vec{\omega}|\ |\vec{r}_{D/C}|$$

[pg. 106 content]

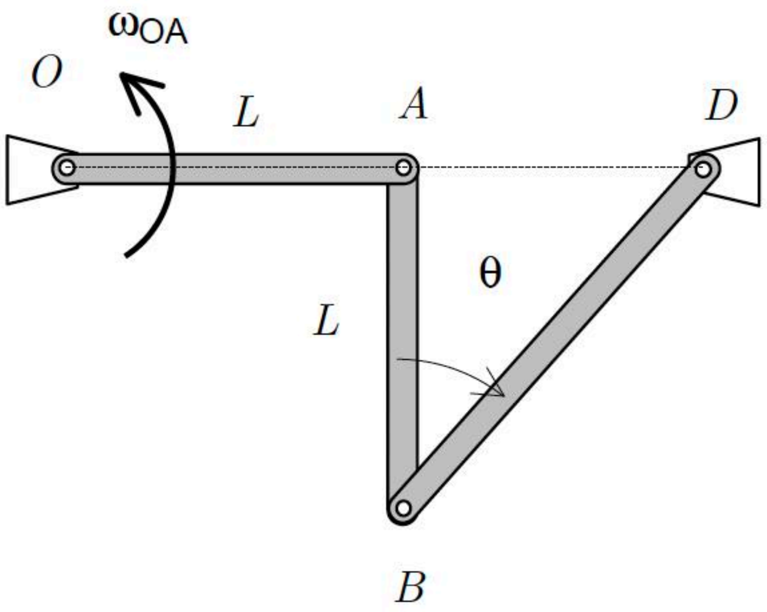
Example 2.B.1

Given: Link OA rotates with an angular speed of $\omega_{OA} = 3$ rad/s with a counterclockwise sense about pin O. At the instant shown, link OA is horizontal, AB is vertical and $\theta = 36.87°$.

Find:

(a) Locate the instant center IC_{AB} for link AB.

(b) Using the location of IC_{AB}, determine the angular velocities of links AB and DB.

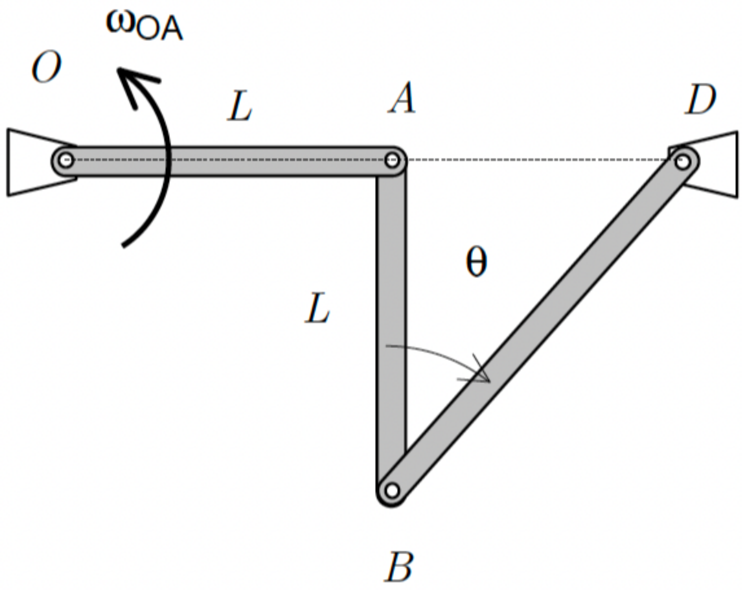


[pg. 109]

Example 2.B.1 p.109

Given: Link OA rotates with an angular speed of $\omega_{OA} = 3$ rad/s with a counterclockwise sense about pin O. At the instant shown, link OA is horizontal, AB is vertical and $\theta = 36.87°$.

Find:
(a) Locate the instant center IC_{AB} for link AB.
(b) Using the location of IC_{AB}, determine the angular velocities of links AB and DB.



Example 2.B.2

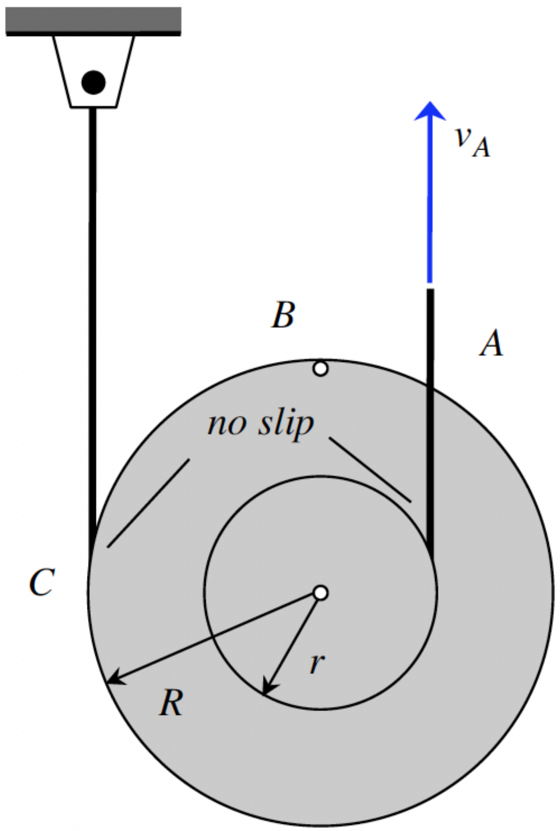
Given: A cable, wrapped around the inner radius of the pulley shown, is being raised at a rate of v_A. A second cable is wrapped around the outer radius of the same pulley with the upper end of this cable attached to ground. Assume that the pulley does not slip on either cable.

Find: Determine:

(a) The location of the instant center for the pulley; and

(b) The velocity of point B on the outer radius of the pulley when B is directly above the center O of the pulley. Sketch this velocity vector.

Use the following parameters in your analysis: $v_A = 3$ m/s, $r = 0.5$ ft and $R = 1$ ft.



[pg. 110]

Example 2.B.2 p. 110

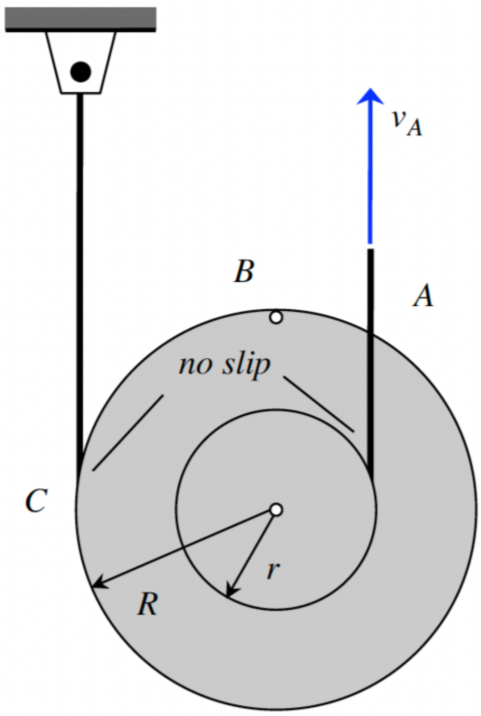
Given: A cable, wrapped around the inner radius of the pulley shown, is being raised at a rate of v_A. A second cable is wrapped around the outer radius of the same pulley with the upper end of this cable attached to ground. Assume that the pulley does not slip on either cable.

Find: Determine:

(a) The location of the instant center for the pulley; and

(b) The velocity of point B on the outer radius of the pulley when B is directly above the center O of the pulley. Sketch this velocity vector.

Use the following parameters in your analysis: $v_A = 3$ m/s, $r = 0.5$ ft and $R = 1$ ft.

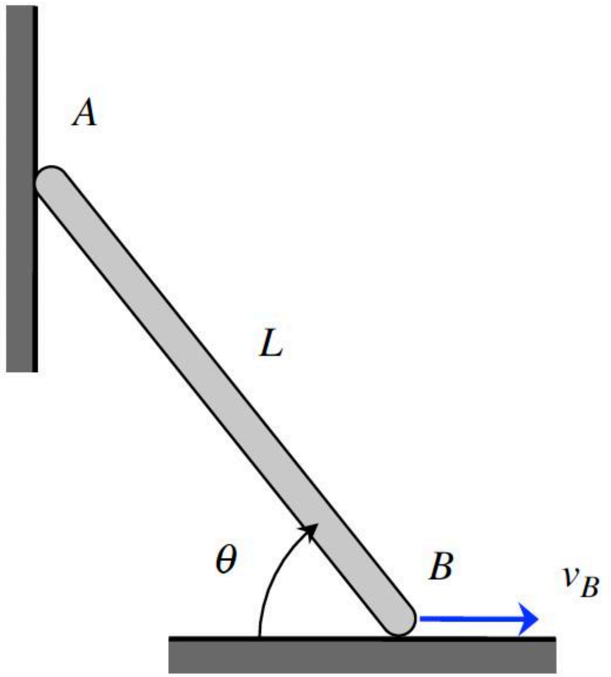


Example 2.B.4

Given: End B of the link moves to the right with a speed of v_B.

Find: For the position where $\theta = 30°$, using the method of instant centers, find the angular velocity of AB.

Use the following parameters in your analysis: $v_B = 3$ m/s, $L = 0.5$ m and $\theta = 36.87°$.



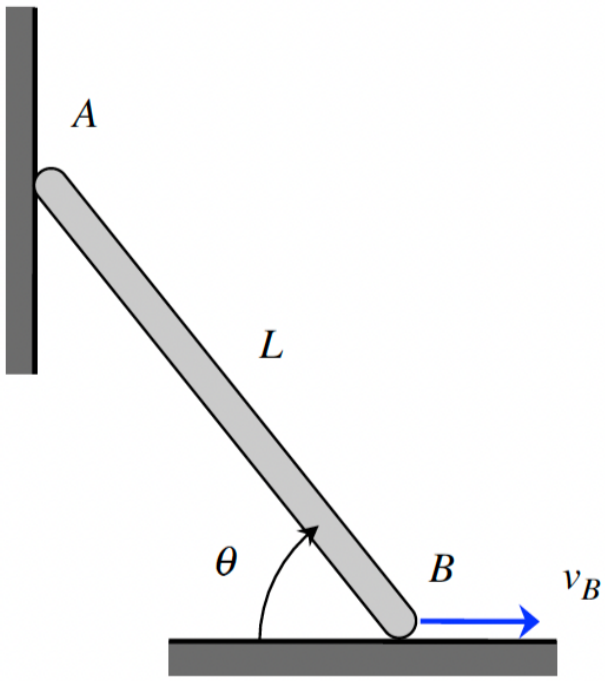
[pg. 112]

Example 2.B.4 p.112

Given: End B of the link moves to the right with a speed of v_B.

Find: For the position where $\theta = 30°$, using the method of instant centers, find the angular velocity of AB.

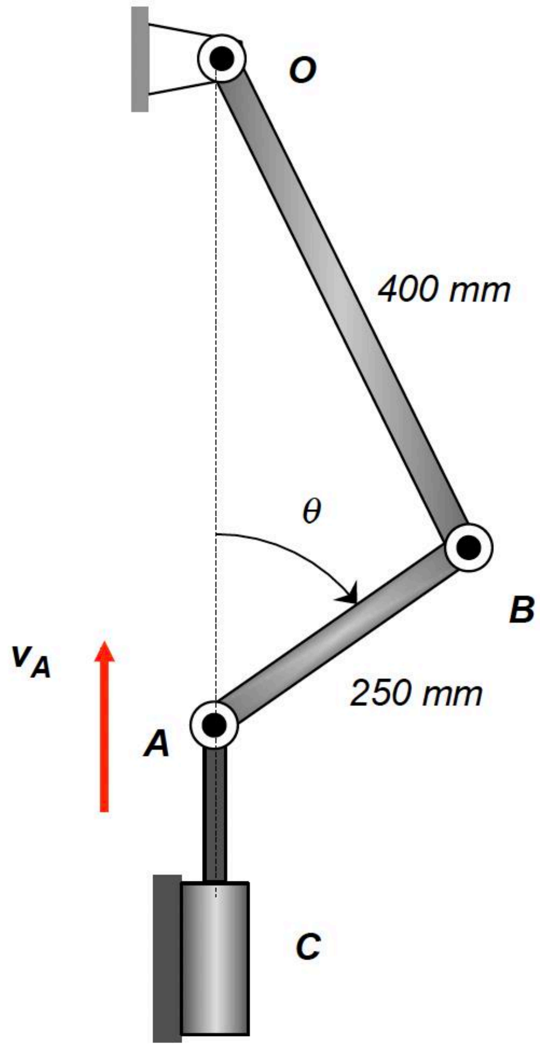
Use the following parameters in your analysis: $v_B = 3$ m/s, $L = 0.5$ m and $\theta = 36.87°$.



Example 2.B.5

Given: The piston rod of the hydraulic cylinder C imparts a speed to pin A of $v_A = 4$ m/s (upward).

Find: For the instant when $\theta = 45°$, using the method of instant centers, determine the angular velocity of link OB.

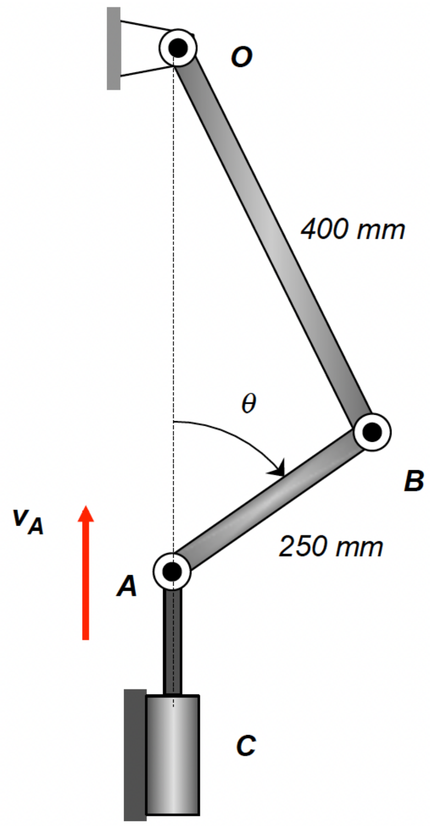


[pg. 113]

Example 2.B.5

Given: The piston rod of the hydraulic cylinder C imparts a speed to pin A of $v_A = 4$ m/s (upward).

Find: For the instant when $\theta = 45°$, using the method of instant centers, determine the angular velocity of link OB.

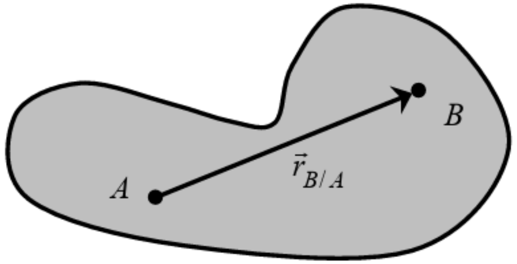


Summary: Rigid Body Kinematics 4

PROBLEM: Two points A and B on the same rigid body undergoing planar motion.

[pg. 89]

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$



Lec 9 Short Feedback Form:

SPECIAL TOPIC: "Instant center of rotation"

[pg. 104] The "center of rotation" (instant center) for a body is located at the intersection of the perpendiculars to the velocities of two points on the body.

- [pg. 106] The velocity of a point G on a link is perpendicular to the line connecting G and and the instant center C.
- [pg. 106] The speed of G is equal to the angular speed of the link times the distance from C to G.
- [pg. 104, slide 6] Where is the IC when the perpendiculars are parallel to each other?