Not filled

ME 274 Lecture 19

① 2B2

Particle Kinetics – Work/energy Part 1

② 2B1

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2/27/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 18 (4.E and 4.F) due today!!**
2. **Exam 1 Grades were released…**
   - ***Exam 1 stats:*** Mean = 44.1/60 (73.5%); Median = 45.5/60 (75.8%)
      - **On Course website**
3. Office hours are changing to ME2008B…
   - Second floor of renovated side of ME.
   - Feel free to come to OHs if you need help with HW or discussing exam problems.

Particle Kinetics: Work-Energy Equation

- **Motivation:** Need to simplify complex force-and-acceleration problems.
  - By focusing on energy transfer, we can skip the need to calculate intermediate kinematic variables

- In the last 3 lectures, we saw that resolution of forces and acceleration into path description provides us with the following equations for a particle:

$$R_t = \sum F_t = m\frac{dv}{dt}$$

$$R_n = \sum F_n = m\frac{v^2}{\rho}$$
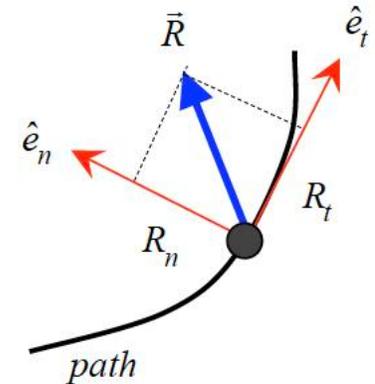
- Only tangential components are in charge for
- The normal components are responsible for

- In this lecture we will develop the **work-energy equation** for analyzing the *change in speed of a particle as a results of forces on the particle* (or systems of particles).

- T = Kinetic Energy.
- V = Potential Energy.
- U = Total Work done on the system

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Where do we get the work-energy equation from?

1. Let **R** represent the [_____] as it moves along a path.

2. We are [_____] so we only need to deal with the [_____] of the resultant force's equation (Newton's Law).

3. We project Newton onto tangential unit vector (dot product) giving:

$$\vec{R} \cdot \hat{e}_t = m\vec{a} \cdot \hat{e}_t$$

$$(R_t\hat{e}_t + R_n\hat{e}_n) \cdot \hat{e}_t = m\left(\frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n\right) \cdot \hat{e}_t$$

$$R_t\hat{e}_t \cdot \hat{e}_t + R_n\hat{e}_n \cdot \hat{e}_t = m\frac{dv}{dt}\hat{e}_t \cdot \hat{e}_t + m\frac{v^2}{\rho}\hat{e}_n \cdot \hat{e}_t$$

$$R_t(1) + R_n(0) = m\frac{dv}{dt}(1) + \frac{v^2}{\rho}(0)$$

$$R_t = m\frac{dv}{dt}$$

4. We can then **use chain rule**, with s representing the distance traveled, to produce: $R_t = mv\frac{dv}{ds}$

5. Separation of variables and integration then gives us:

$$\int_1^2 R_t\,ds = m\int_1^2 v\,dv = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2 \longrightarrow \boxed{}$$

[pg. 210 content]   4
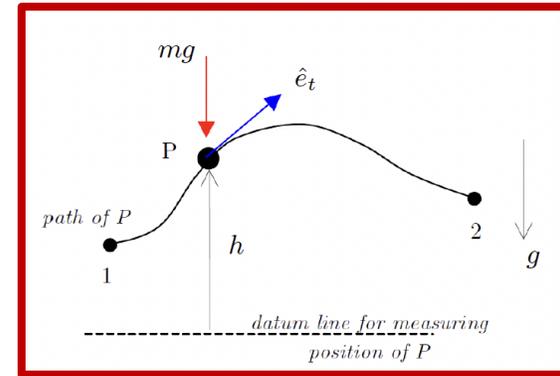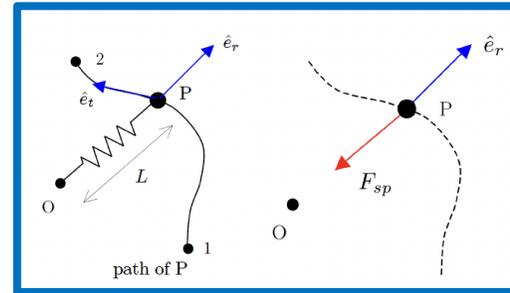
Conservative Forces and Potential Energy

- **Conservative Forces -** Class of forces for which *their work between positions 1 and 2*

  [   ] *over which the forces act as the particle moves from position 1 to position 2.*

- Examples of non-conservative forces:
  1. Force due to the action of a [   ]
  2. Force due to the [   ]

- Since the work due to these conservative forces does not require us to do an integration, we can separate the total work done in a particle into **conservative and non-conservative parts:**

$$U_{1 \to 2} = U^{(c)}_{1 \to 2} + U^{(nc)}_{1 \to 2}$$
$$= -(V_2 - V_1) + U^{(nc)}_{1 \to 2}$$

- We can substitute above step to the **work-energy equation from last slide** and get the one we will use extensively:

  - T = Kinetic Energy.
  - V = Potential Energy.
  - U = Total Work done on the system

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Discussion

- Work (U) can be computed using path and cartesian coordinates :

$$U_{1\to 2} = \int_1^2 \left(\vec{R}\cdot\hat{e}_t\right) ds$$

$$U_{1\to 2} = \int_1^2 R_x\,dx + \int_1^2 R_y\,dy$$

- Work-energy equation is a scalar equation. Does not have vector components (ie cannot separate in x and y).

- In using the work energy equation, be careful with signs on the potential energy (V) terms:
  - ***The potential of a spring is always positive***, regardless if its stretched or compressed.
  - **The potential of the weight force** depends on whether the particle is above or below the

- The Work-energy equation also applies to ***systems of particles***. If this is the case:
  - *Sum up all the kinetic/potential energy terms* to find total kinetic/potential energy
  - Calculating the work term is a little more complicated.
    - It's found by *projecting forces onto the path of the particle* on which they act *and integrating* the result over the path.
  - *Monday's Lecture (lecture 20) we will go over problems like these.*

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Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18)
     - Work/Energy (lectures 19-20)
     - Linear impulse/momentum
     - Angular impulse/momentum
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     - a) Draw more FBDs
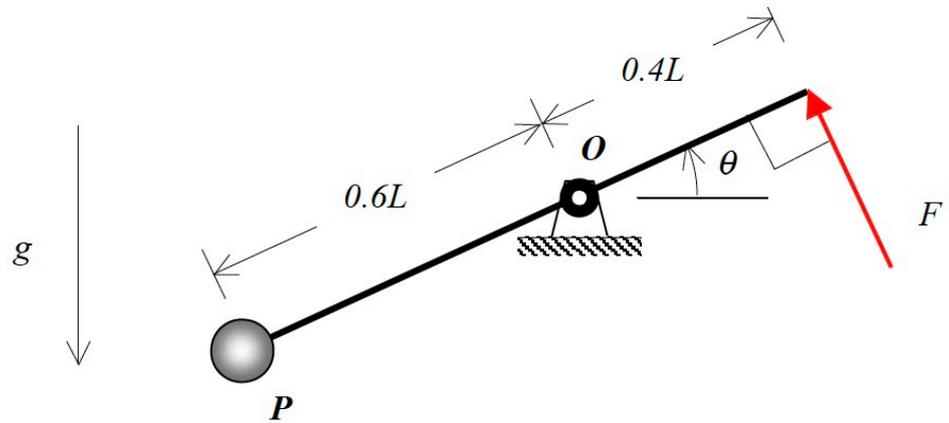   - b) You will need to do more kinematic analysis

[pg. 184 content]

Example 4.B.1

Given: The 6 kg particle P and the attached light rod (of length $L = 2$ m) rotate in a vertical plane about a fixed axis passing through O. The assembly is released from rest at $\theta = 0$ and moves under the action of the $F = 100$ N force (which is maintained normal to the rod).

**Find:** Determine the speed of the particle P when $\theta = 90°$.
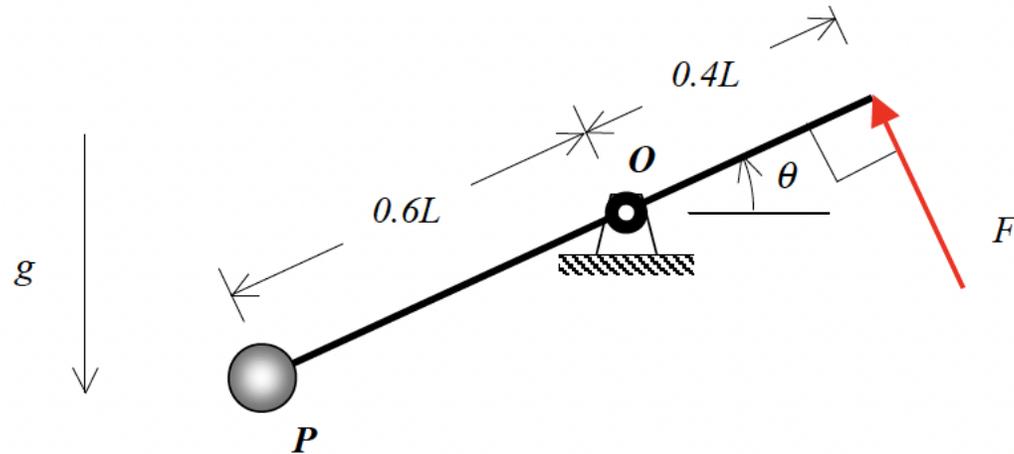
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Find: Determine the speed of the particle P when $\theta = 90°$.

Example 4.B.2

Given: A block of mass m is released from rest on a rough, inclined surface (with a coefficient of kinetic friction μ_k) and slides toward an uncompressed spring of stiffness k.

Find: Determine the maximum deflection of the spring.

Use the following parameters in your analysis: $\theta = 53.13°$, $d = 0.75$ m, $m = 5$ kg, $\mu_k = 0.3$ and $k = 1000$ N/m.
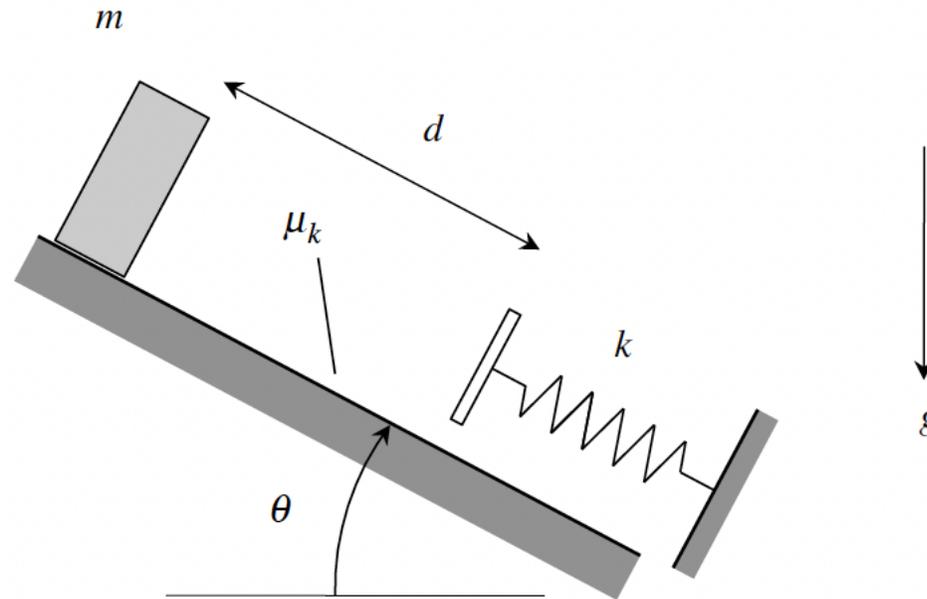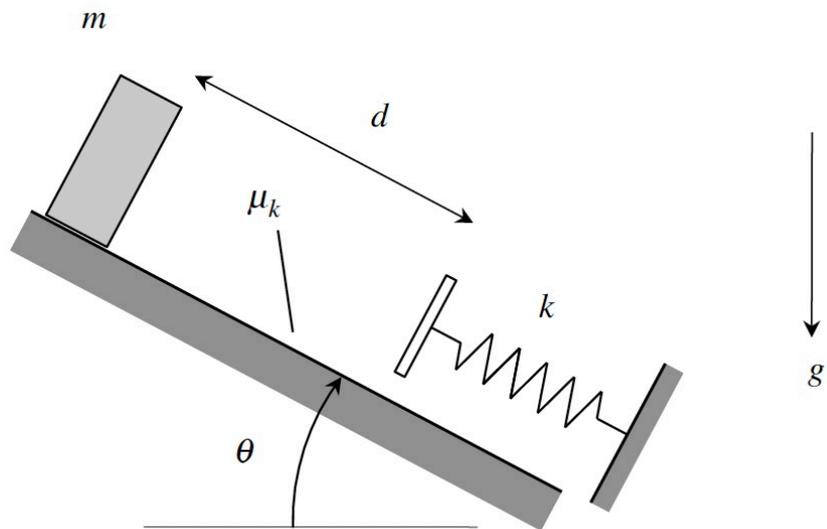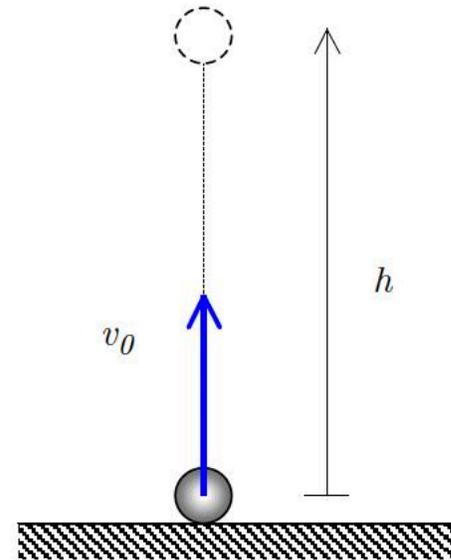
Example 4.B.2

p.220

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Example 4.B.3

Given: A particle is launched vertically with a speed of v_0.

Find: Determine the maximum height h obtained by the particle. Assume that air resistance on the particle is negligible.
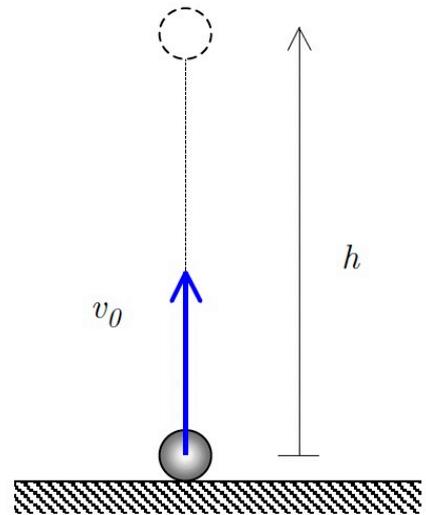
Example 4.B.3   p.221

Given: A particle is launched vertically with a speed of v_0.

Find: Determine the maximum height h obtained by the particle. Assume that air resistance on the particle is negligible.

Example 4.B.4

Given: Block C having a weight mg is traveling with a speed of v_0 when it strikes a fixed wall. A spring of stiffness k is attached to the front of the block. The spring eventually contacts the wall, resulting in a maximum spring compression of Δ_{max}.

Find: Determine the stiffness k of the spring.

Use the following parameters in your analysis: $mg = 3200$ lb, $v_0 = 10$ ft/sec and $\Delta_{max} = 6$ in.

Example 4.B.4   p.222

Given: Block C having a weight mg is traveling with a speed of v_0 when it strikes a fixed wall. A spring of stiffness k is attached to the front of the block. The spring eventually contacts the wall, resulting in a maximum spring compression of Δ_{max}.

Find: Determine the stiffness k of the spring.
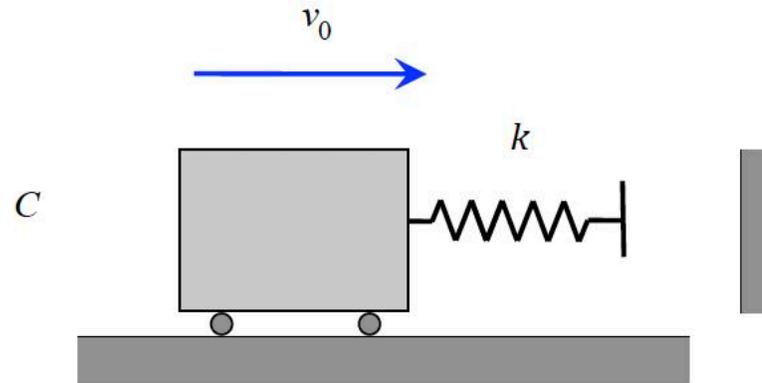
Use the following parameters in your analysis: $mg = 3200$ lb, $v_0 = 10$ ft/sec and $\Delta_{max} = 6$ in.
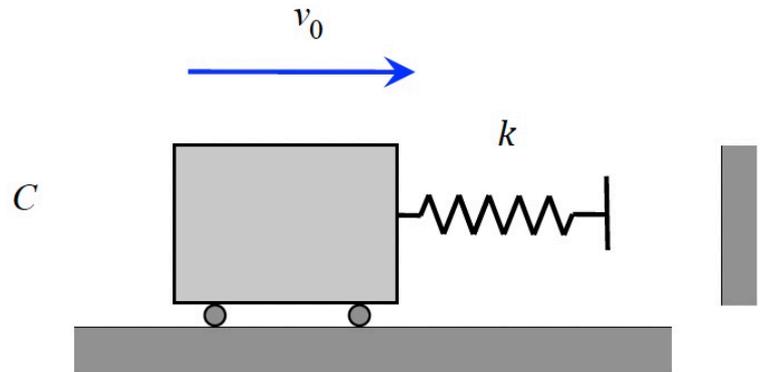
Summary: Work-Energy Equation 1

FUNDAMENTAL equation: the work-energy equation

$$T_1 + V_1 + U_{1 \to 2}^{(nc)} = T_2 + V_2$$

where:

$$T = \frac{1}{2}mv^2 \, (\geq 0) = kinetic \; energy$$

$$U_{1 \to 2}^{(nc)} = \int_1^2 \left(\sum \vec{F}\right) \bullet \hat{e}_t \; ds = work \; done \; by \; non-conservative \; forces$$

$$= \int_1^2 \left[\left(\sum \vec{F}\right)_x dx + \left(\sum \vec{F}\right)_y dy\right]$$

$$V = V_{gr} + V_{sp} = potential \; energy$$
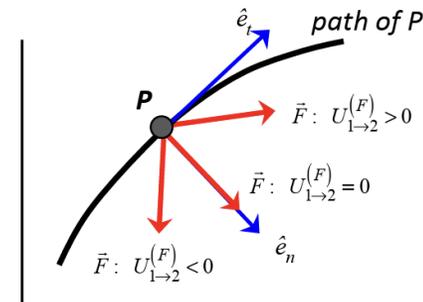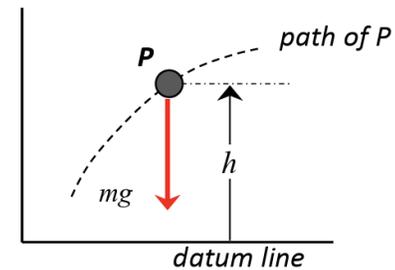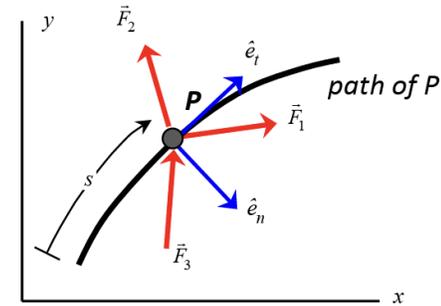
$$V_{gr} = mgh \; (sign \; depends \; on \; h)$$

$$V_{sp} = \frac{1}{2}k\Delta^2 \geq 0 \; (ALWAYS)$$

SIGN of work term: The direction of a force relative to the $\hat{e}_t$ *dictates the sign of the work, U.*

Lec 19 Short Feedback Form:

me 274 - cmk