ME 274 Lecture 15

Moving Reference Kinematics: 3D Part 3

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2/18/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 14 (3.G and 3.H) due tonight!**
2. **Office hours are changing to ME2008B…**
    - **Second floor of renovated side of ME**

Eugenio Frias Miranda Office Hours 274

Description:	Approved by Beth Hess
Confirmation status:	Confirmed
Room:	ME 2008 Hotel Offices - ME 2008B Private Office
Start time:	01:30:00PM - Monday 16 February 2026
Duration:	1 hours
End time:	02:30:00PM - Monday 16 February 2026
Type:	Internal
Created by:	ewolters
Modified by:	ewolters
Last updated:	03:56:07PM - Friday 13 February 2026
Repeat type:	Weekly
Repeat every:	1 week
Repeat day:	Monday Wednesday Friday
Repeat end date:	Friday 08 May 2026

(3) Course Evals, bonus points on hwk !

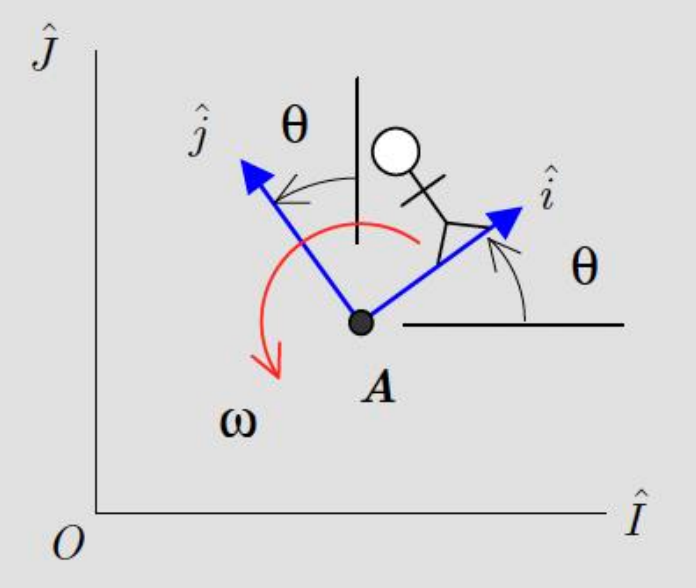
(4) Sending Email w/ above (2) & (3)

(5) Exams tell you where observer is

Chapter 3: Moving Reference Frame Kinematics

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right]$$



- $\vec{v}_A$ and $\vec{v}_B$ are the velocities seen by a **fixed observer** [XYZ]
- $\vec{a}_A$ and $\vec{a}_B$ are the accelerations seen by **fixed observer** [XYZ]
- $\vec{\omega}$ is angular velocity of the **moving observer** [xyz]
- $\vec{\alpha}$ is angular acceleration of the **moving observer** [xyz]

How does observer move?

- $\left(\vec{v}_{B/A}\right)_{rel}$ is the “velocity of point B as seen by the **moving observer** at A”
- $\left(\vec{a}_{B/A}\right)_{rel}$ is the “acceleration of point B as seen by the **moving observer** at A”
- $2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel}$ is known as the **“Coriolis” component** of acceleration.
  - Arises when observer has a non-zero angular velocity

What does the observer see?

[pg. 147 content]

3D Rotating Reference Frames

- For a 3D Rotating Reference Frame system/problem, we will use the same Moving Reference Frame equations as before, except we now have a ‘k’ term.
  - For derivation, look at pg. 158-159.

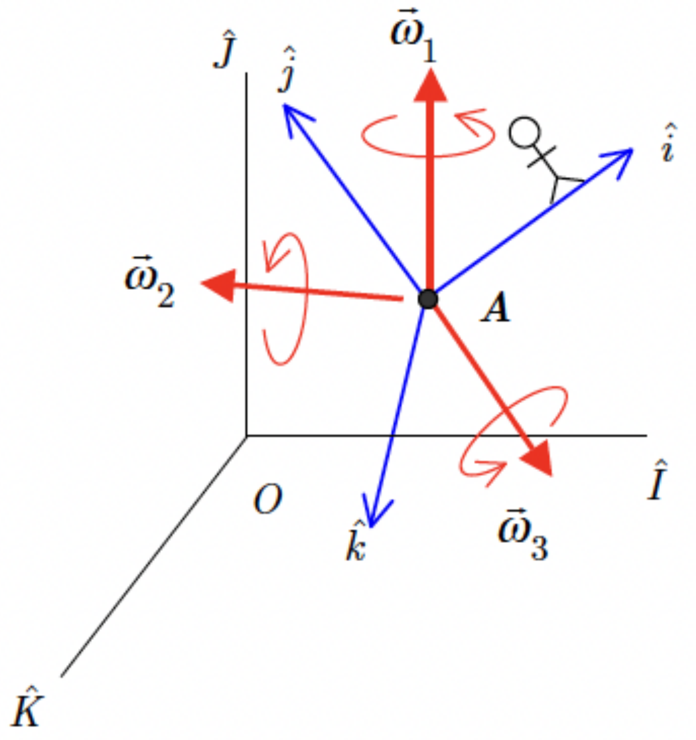
- **Angular Velocity** in 3D motion will usually be made up of several components (omega’s). With each component being about a different axis, as shown in the figure.

$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 + \vec{\omega}_3 + \ldots$$

- For **Angular Acceleration**, it is important to note distinction between a **fixed axis** and a **rotating/moving** axis.

- Recalling derivation in pg. 144. **For a rotating axis we see that:**
  - **Straightforward to remember:**

$$\frac{d\hat{i}}{dt} = \vec{\omega} \times \hat{i} \qquad \frac{d\hat{j}}{dt} = \vec{\omega} \times \hat{j} \qquad \frac{d\hat{k}}{dt} = \vec{\omega} \times \hat{k}$$

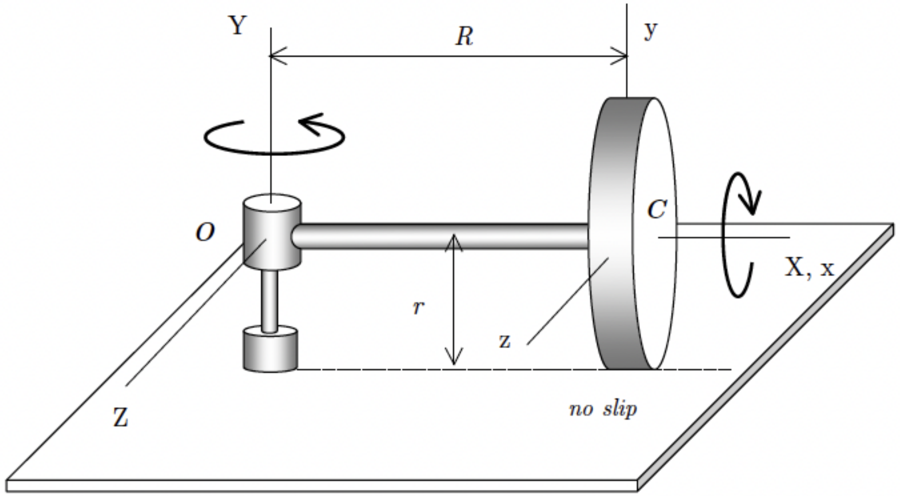


[pg. 157 content]

Example 3.B.9

Given: Arm OC rotates about the fixed Y-axis at a constant rate Ω. The disk at C, having a radius of R, is able to rotate about arm OC and rolls without slipping on a fixed horizontal surface. Let the xyz axes be attached to the disk.

Find: Determine the angular acceleration of the disk.

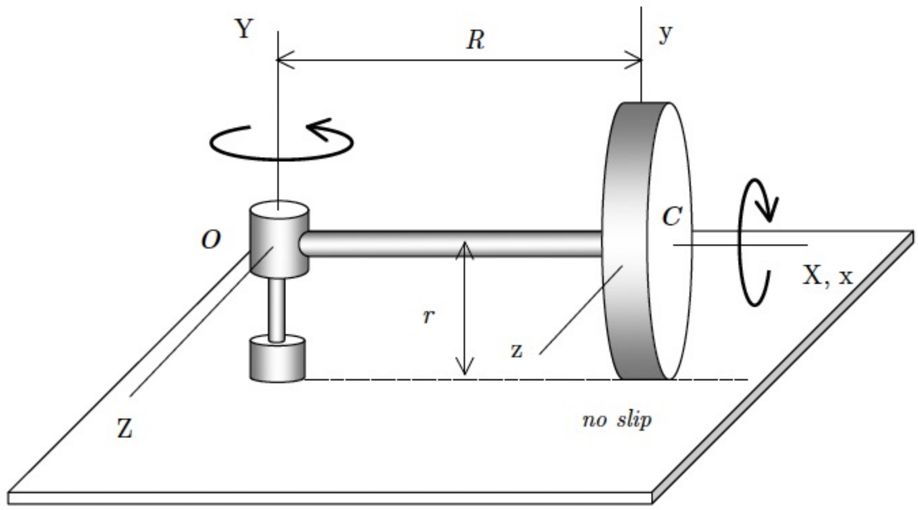


[pg. 171]

Example 3.B.9 p.171

Given: Arm OC rotates about the fixed Y-axis at a constant rate Ω. The disk at C, having a radius of R, is able to rotate about arm OC and rolls without slipping on a fixed horizontal surface. Let the xyz axes be attached to the disk.

Find: Determine the angular acceleration of the disk.



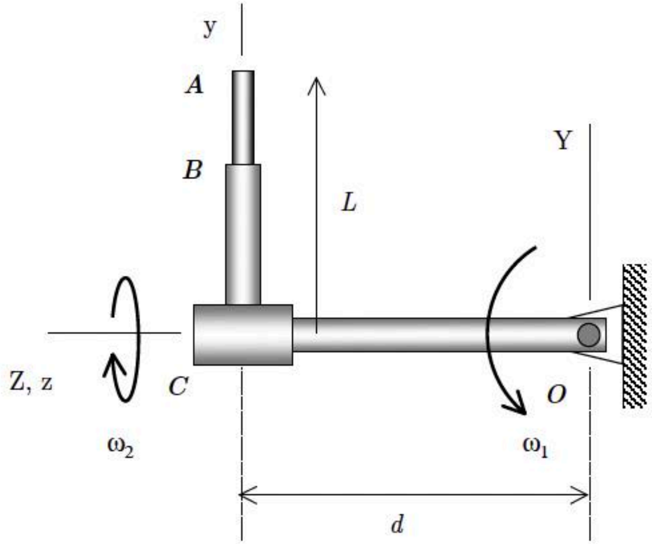
Example 3.B.10

Given: $\dot{L} = 0.06$ m/s $=$ *constant*, $\omega_1 = 1.2$ rad/s $=$ *constant* and $\omega_2 = 1.5$ rad/s $=$ *constant*. At the position shown, AC is aligned with the fixed Y-axis, $L = 0.12$ m, and $d = 0.02$ m.

Find: Determine:

(a) The velocity of end A of the telescoping rod AC; and

(b) The acceleration of the same point.

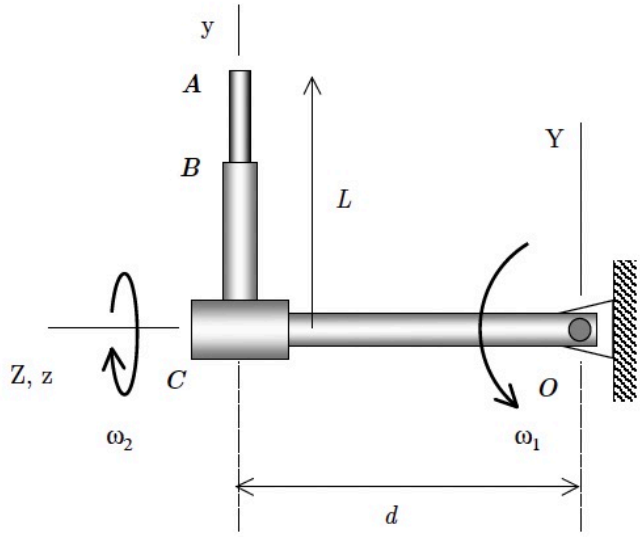


[pg. 172]

Example 3.B.10 p.172

Given: $\dot{L} = 0.06$ m/s $=$ *constant*, $\omega_1 = 1.2$ rad/s $=$ *constant* and $\omega_2 = 1.5$ rad/s $=$ *constant*. At the position shown, AC is aligned with the fixed Y-axis, $L = 0.12$ m, and $d = 0.02$ m.

Find: Determine:
(a) The velocity of end A of the telescoping rod AC; and $\vec{v}_A$?
(b) The acceleration of the same point. $\vec{a}_A$?



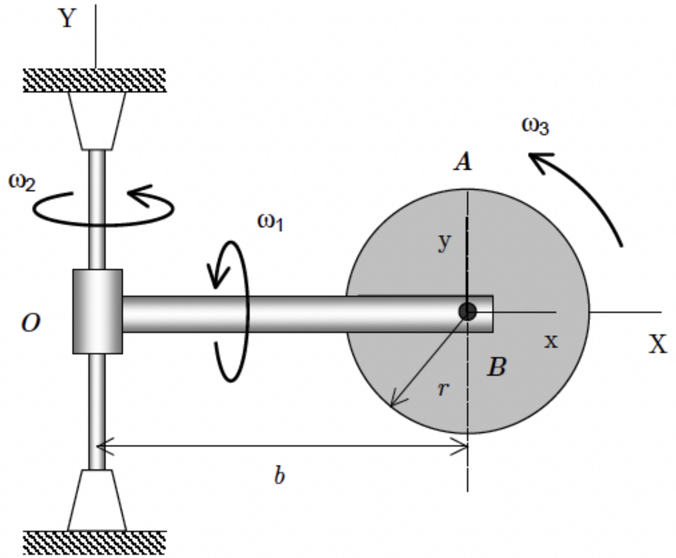
Example 3.B.11

Given: Rotation rates ω_1, ω_2 and ω_3 are all constant. The XYZ axes are fixed, and the xyz axes are attached to the disk. At the instant shown, A is directly above the center B of the disk and the xyz and XYZ axes are aligned.

Find: Determine:

(a) The velocity of point A; and

(b) The acceleration of point A.

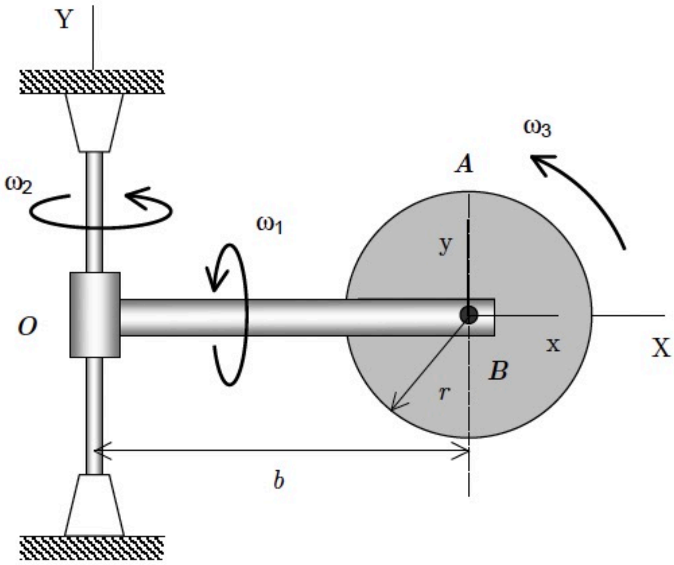


[pg. 173]

Example 3.B.11 p.173

Given: Rotation rates ω_1, ω_2 and ω_3 are all constant. The XYZ axes are fixed, and the xyz axes are attached to the disk. At the instant shown, A is directly above the center B of the disk and the xyz and XYZ axes are aligned.

Find: Determine:
(a) The velocity of point A; and $\vec{v}_A$?
(b) The acceleration of point A. $\vec{a}_A$?



$$a = \omega^2 r$$

$$a = \frac{v^2}{r}$$

Where:
"a" is acceleration
"ω" is angular velocity
"v" is scalar velocity
"r" is radius

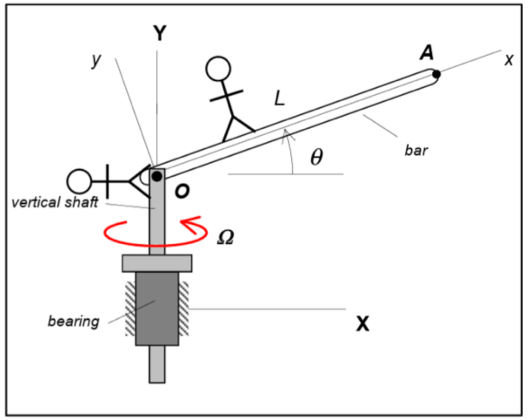
Summary: 3D Moving Reference Frame Kinematics 2

[pg. 158-159]

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point A.

$$\vec{v}_A = \vec{v}_O + \left(\vec{v}_{A/O}\right)_{rel} + \vec{\omega} \times \vec{r}_{A/O}$$

$$\vec{a}_A = \vec{a}_O + \left(\vec{a}_{A/O}\right)_{rel} + \vec{\alpha} \times \vec{r}_{A/O} + 2\vec{\omega} \times \left(\vec{v}_{A/O}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{A/O}\right)$$



Lec 15 Short Feedback Form:

CHANGING OBSERVERS: For constant rotation rates,

Observer on vertical shaft:

$$\vec{\omega} = \Omega \hat{J}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \vec{0}$$

$$\left(\vec{v}_{A/O}\right)_{rel} = L\dot{\theta}\hat{j}$$

$$\left(\vec{a}_{A/O}\right)_{rel} = -L\dot{\theta}^2\hat{i}$$

Observer on arm OA:

$$\vec{\omega} = \Omega \hat{J} + \dot{\theta}\hat{k}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \dot{\Omega}\hat{J} + \Omega\dot{\hat{J}} + \ddot{\theta}\hat{k} + \dot{\theta}\dot{\hat{k}} = \dot{\theta}\left(\vec{\omega} \times \hat{k}\right)$$

$$\left(\vec{v}_{A/O}\right)_{rel} = \vec{0}$$

$$\left(\vec{a}_{A/O}\right)_{rel} = \vec{0}$$

These give the same result! Try it.