

ME 274 Lecture 14

Moving Reference Kinematics: 3D Part 2

Eugenio “Henny” Frias-Miranda

2/16/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

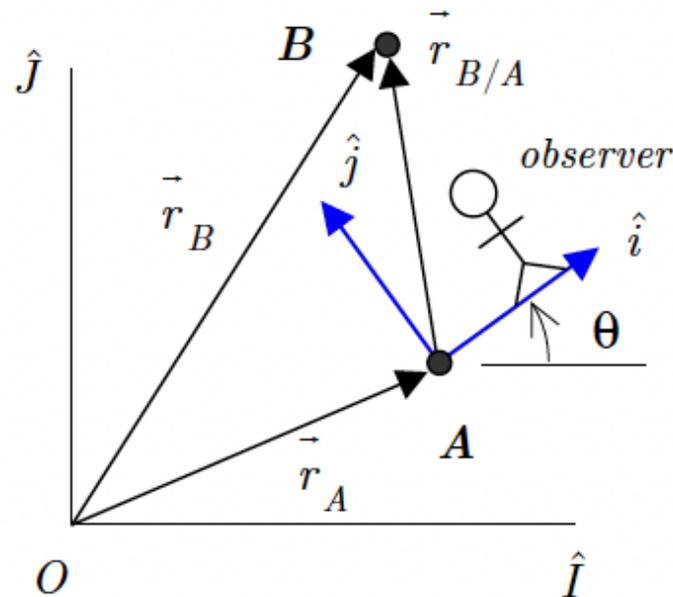
1. **HW due tonight!**
2. **Office hours are changing to ME2008B...**
 - **Second floor of renovated side of ME**

Eugenio Frias Miranda Office Hours 274

Description: Approved by Beth Hess
Confirmation status: Confirmed
Room: ME 2008 Hotel Offices - ME 2008B Private Office
Start time: 01:30:00PM - Monday 16 February 2026
Duration: 1 hours
End time: 02:30:00PM - Monday 16 February 2026
Type: Internal
Created by: ewolters
Modified by: ewolters
Last updated: 03:56:07PM - Friday 13 February 2026
Repeat type: Weekly
Repeat every: 1 week
Repeat day: Monday Wednesday Friday
Repeat end date: Friday 08 May 2026

(Reminder/refresher) Chapter 3 – Moving Reference Frame Kinematics

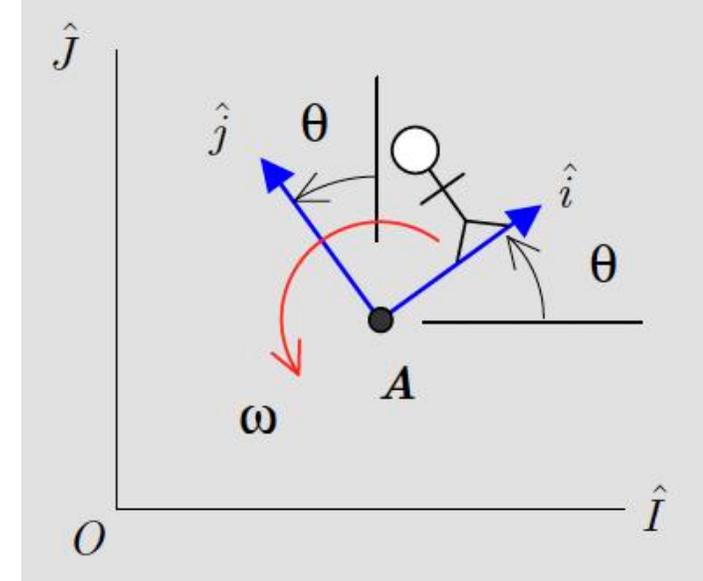
- In chapter 2, we saw that if two points A and B are on the **SAME** rigid body.
- It is often the case that we need to use the motion of two points that are **NOT** on the same rigid body.
- Many times, we can use kinematic information about the motion of B obtained from an observer on a “**moving reference frame**” [figure]



Chapter 3: Moving Reference Frame Kinematics

$$\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}]$$



- \vec{v}_A and \vec{v}_B are the velocities seen by a **fixed observer** [XYZ]
- \vec{a}_A and \vec{a}_B are the accelerations seen by **fixed observer** [XYZ]
- $\vec{\omega}$ is angular velocity of the **moving observer** [xyz]
- $\vec{\alpha}$ is angular acceleration of the **moving observer** [xyz]
- $(\vec{v}_{B/A})_{rel}$ is the “velocity of point B as seen by the **moving observer** at A”
- $(\vec{a}_{B/A})_{rel}$ is the “acceleration of point B as seen by the **moving observer** at A”
- $2\vec{\omega} \times (\vec{v}_{B/A})_{rel}$ is known as the “**Coriolis**” component of acceleration.
 - Arises when observer has a non-zero angular velocity

How does observer move?

What does the observer see?

Last Class... 3D Rotating Reference Frames

- For a 3D Rotating Reference Frame system/problem, we will use the same Moving Reference Frame equations as before, except we now have a 'k' term.
 - For derivation, look at pg. 158-159.
- **Angular Velocity** in 3D motion will usually be made up of several components (omega's). With each component being about a different axis, as shown in the figure.

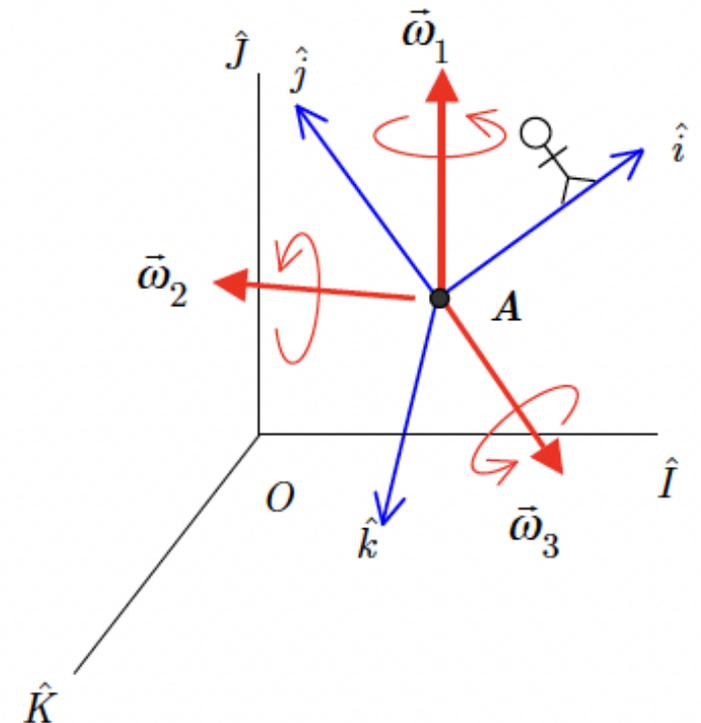
$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 + \vec{\omega}_3 + \dots$$

- For **Angular Acceleration**, it is important to note distinction between a **fixed axis** and a **rotating/moving axis**.
- Recalling derivation in pg. 144. **For a rotating axis we see that:**
 - **Straightforward to remember:**

$$\frac{d\hat{i}}{dt} = \vec{\omega} \times \hat{i}$$

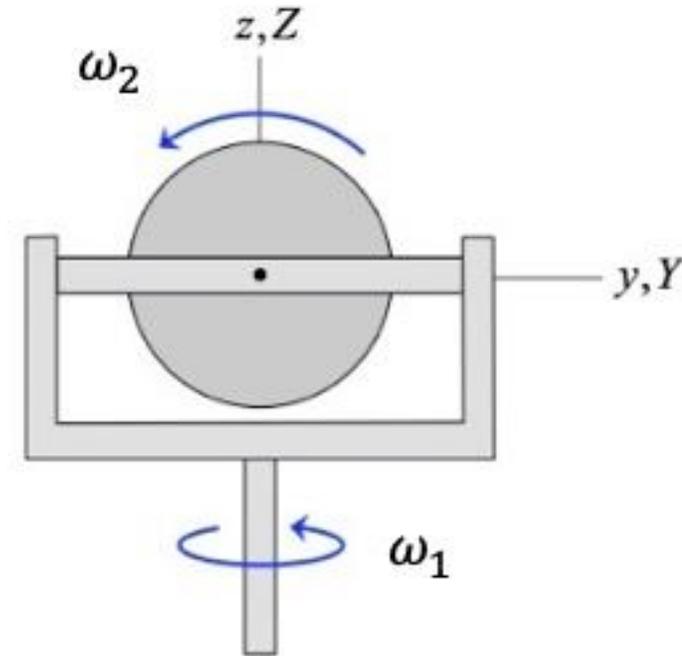
$$\frac{d\hat{j}}{dt} = \vec{\omega} \times \hat{j}$$

$$\frac{d\hat{k}}{dt} = \vec{\omega} \times \hat{k}$$



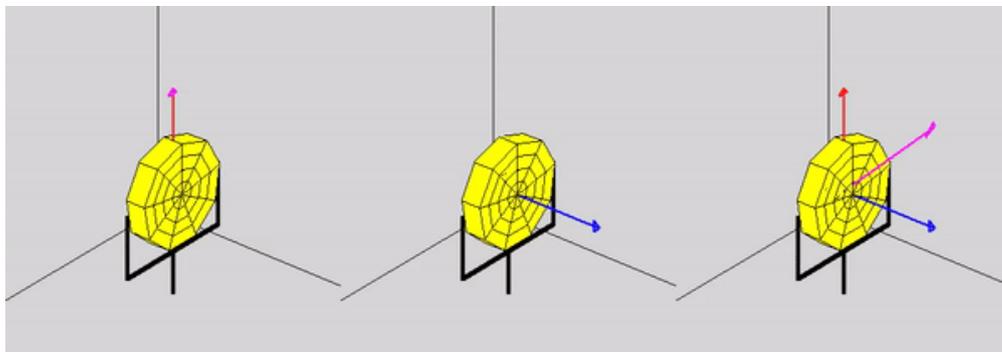
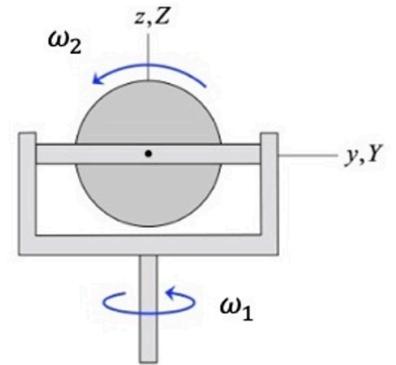
p. 161 Challenge Question and Animation:
*Does having a **Constant Angular Velocity** mean **Angular Acceleration is equal to zero...**?*

- TLDR – In 2D problems yes, but in 3D problems: **NO**
- **Why? Let's do this example problem for example:**
 - A gyro rotor has a constant rotation rate of ω_1 about a fixed vertical axis in addition to a constant rotation rate of ω_2 about the symmetry axis of the rotor. Let the xyz -axes be attached to the rotor, and the XYZ -axes be fixed to ground. **Find the angular acceleration of the rotor.**



p. 161 Challenge Question and Animation:
Does having a **Constant Angular Velocity** mean **Angular Acceleration is equal to zero...?**

- **TLDR** – In 2D problems yes, **but in 3D problems: NO**
- **Why? Let's do this example problem for example:**
 - A gyro rotor has a constant rotation rate of ω_1 about a fixed vertical axis in addition to a constant rotation rate of ω_2 about the symmetry axis of the rotor. Let the xyz -axes be attached to the rotor, and the XYZ -axes be fixed to ground. **Find the angular acceleration of the rotor.**

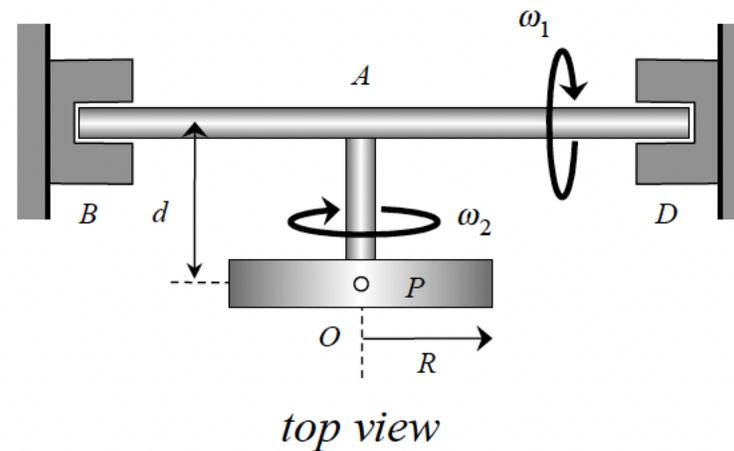
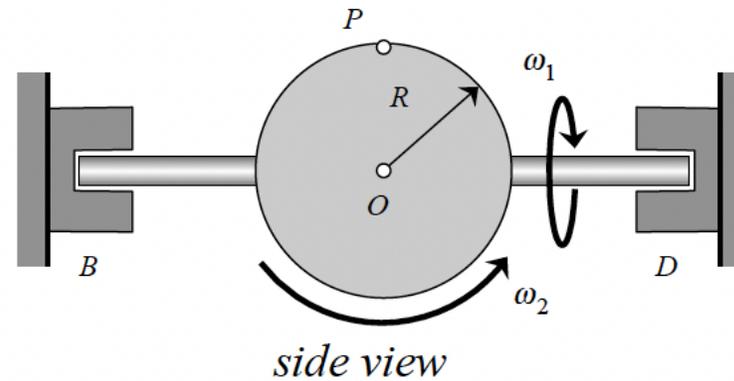


Example 3.B.6

Given: Shaft BD rotates about a fixed axis with a constant rate of ω_1 . Shaft OA is rigidly attached to shaft BD with OA being perpendicular to BD. A disk rotates about shaft OA with a constant rate of ω_2 relative to OA.

Find: The acceleration of point P on the edge of the disk for the position shown.

Use the following parameters in your analysis: $\omega_1 = 5 \text{ rad/s}$, $\omega_2 = 8 \text{ rad/s}$, $R = 6 \text{ in}$ and $d = 4 \text{ in}$.

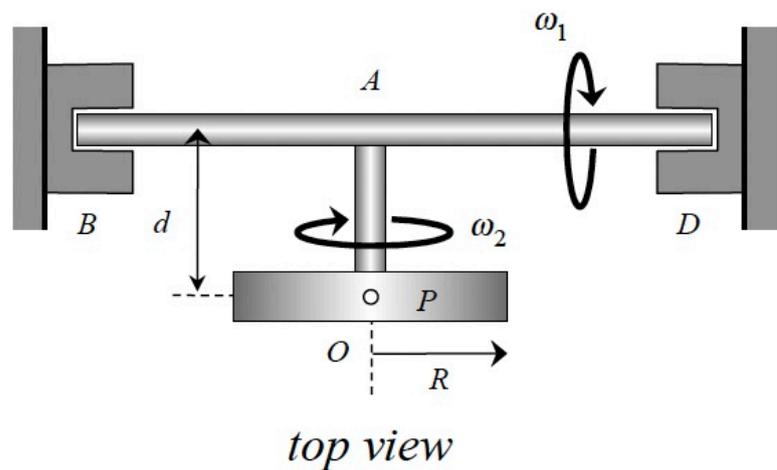
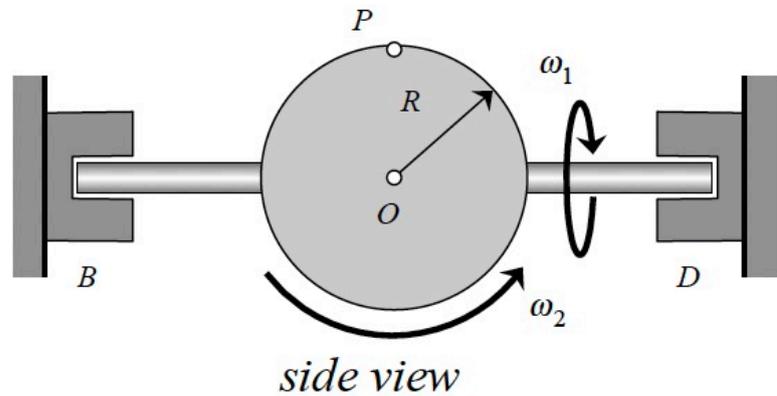


Example 3.B.6

Given: Shaft BD rotates about a fixed axis with a constant rate of ω_1 . Shaft OA is rigidly attached to shaft BD with OA being perpendicular to BD. A disk rotates about shaft OA with a constant rate of ω_2 relative to OA.

Find: The acceleration of point P on the edge of the disk for the position shown.

Use the following parameters in your analysis: $\omega_1 = 5 \text{ rad/s}$, $\omega_2 = 8 \text{ rad/s}$, $R = 6 \text{ in}$ and $d = 4 \text{ in}$.

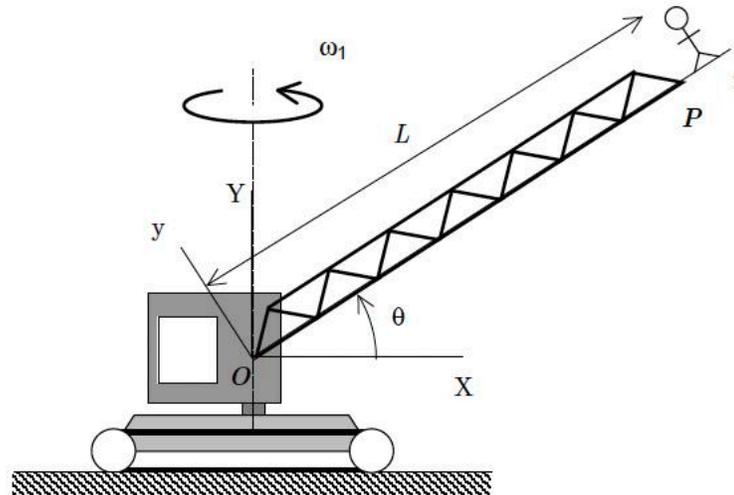


Example 3.B.7

Given: $\omega_1 = 0.30 \text{ rad/s} = \text{constant}$, $\dot{\theta} = 0.5 \text{ rad/s} = \text{constant}$ and $L = 12 \text{ m}$.

Find: When $\theta = 30^\circ$, determine:

- (a) The angular velocity of boom OP;
- (b) The angular acceleration of boom OP;
- (c) The velocity of end P of the boom; and
- (d) The acceleration of end P of the boom.

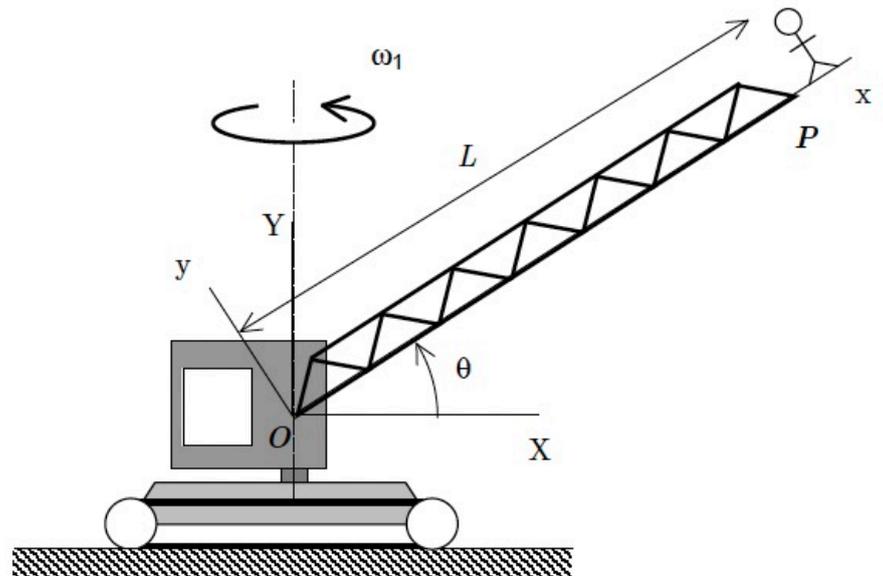


Example 3.B.7

Given: $\omega_1 = 0.30 \text{ rad/s} = \text{constant}$, $\dot{\theta} = 0.5 \text{ rad/s} = \text{constant}$ and $L = 12 \text{ m}$.

Find: When $\theta = 30^\circ$, determine:

- The angular velocity of boom OP;
- The angular acceleration of boom OP;
- The velocity of end P of the boom; and
- The acceleration of end P of the boom.

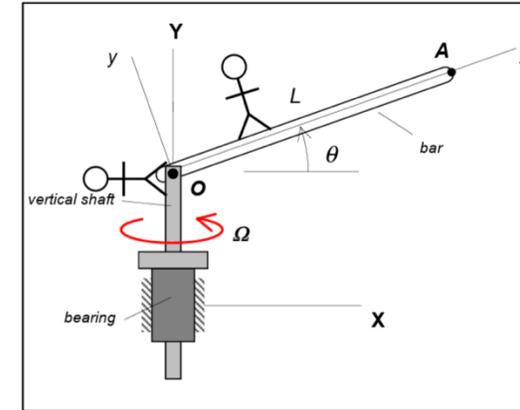


Summary: 3D Moving Reference Frame Kinematics 2

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point A.

$$\vec{v}_A = \vec{v}_O + (\vec{v}_{A/O})_{rel} + \vec{\omega} \times \vec{r}_{A/O}$$

$$\vec{a}_A = \vec{a}_O + (\vec{a}_{A/O})_{rel} + \vec{\alpha} \times \vec{r}_{A/O} + 2\vec{\omega} \times (\vec{v}_{A/O})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/O})$$



CHANGING OBSERVERS: For constant rotation rates,

Observer on vertical shaft:

$$\vec{\omega} = \Omega \hat{j}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \vec{0}$$

$$(\vec{v}_{A/O})_{rel} = L\dot{\theta} \hat{j}$$

$$(\vec{a}_{A/O})_{rel} = -L\dot{\theta}^2 \hat{i}$$

Observer on arm OA:

$$\vec{\omega} = \Omega \hat{j} + \dot{\theta} \hat{k}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \dot{\Omega} \hat{j} + \Omega \dot{\hat{j}} + \ddot{\theta} \hat{k} + \dot{\theta} \dot{\hat{k}} = \dot{\theta} (\vec{\omega} \times \hat{k})$$

$$(\vec{v}_{A/O})_{rel} = \vec{0}$$

$$(\vec{a}_{A/O})_{rel} = \vec{0}$$

These give the same result! Try it.

[pg. 158-159]

Lec 14 Short
Feedback Form:

