Not Filled

ME 274 Lecture 13

Moving Reference Kinematics: 3D Part 1

Eugenio “Henny” Frias-Miranda

2/11/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

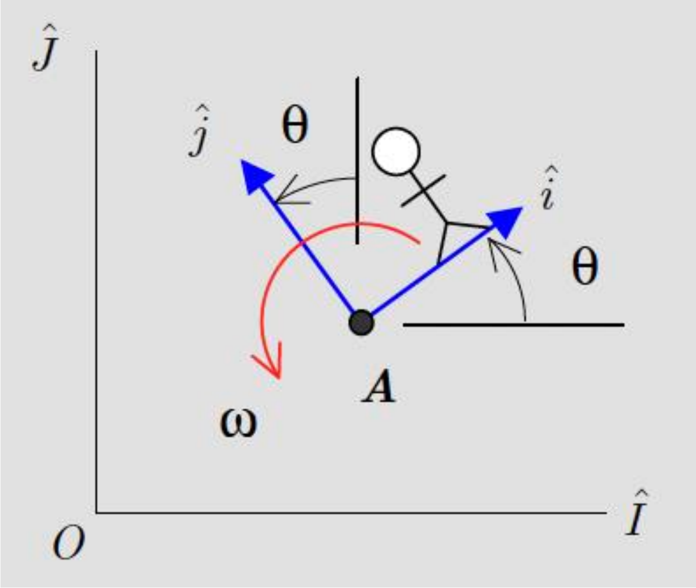
1. **Exam 1 is tomorrow!**
    - **Details on course website (https://www.purdue.edu/freeform/me274/exams-spring-2026/)**
    - **Recording of review session on course website**
2. No lecture on Friday
    - HW due on Friday

3 Advice on Exam 1

Last Class, Moving Reference Frame Eqns

$$\boxed{\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}}$$

$$\boxed{\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right]}$$



- $\vec{v}_A$ and $\vec{v}_B$ are the velocities seen by a **fixed observer** [XYZ]
- $\vec{a}_A$ and $\vec{a}_B$ are the accelerations seen by **fixed observer** [XYZ]
- $\vec{\omega}$ is angular velocity of the **moving observer** [xyz]
- $\vec{\alpha}$ is angular acceleration of the **moving observer** [xyz]

How does observer move?

- $\left(\vec{v}_{B/A}\right)_{rel}$ is the "velocity of point B as seen by the **moving observer** at A"
- $\left(\vec{a}_{B/A}\right)_{rel}$ is the "acceleration of point B as seen by the **moving observer** at A"
- $2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel}$ is known as the **"Coriolis" component** of acceleration.
  - Arises when observer has a non-zero angular velocity

What does the observer see?

[pg. 147 content]

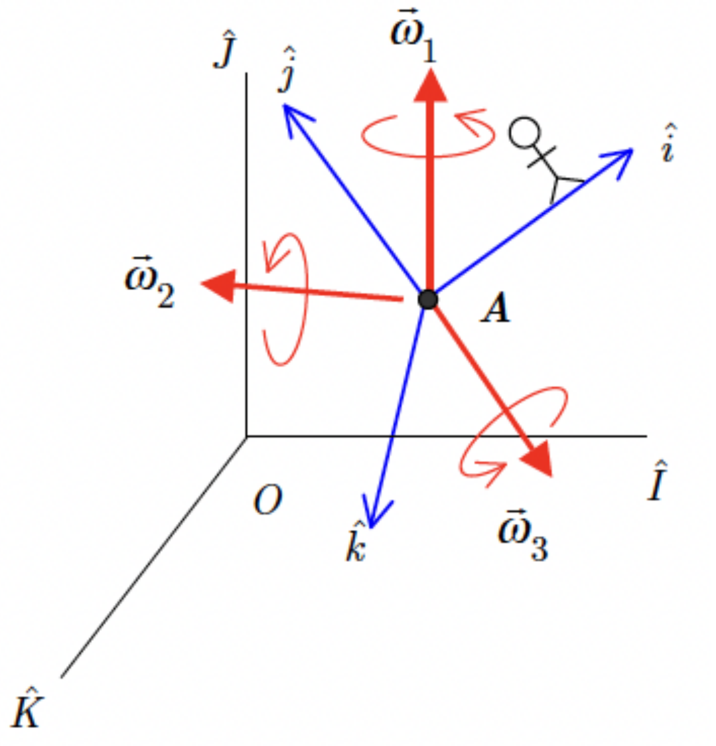
3D Rotating Reference Frames

- For a 3D Rotating Reference Frame system/problem, we will use the same Moving Reference Frame equations as before, except we now have a ‘k’ term.
  - For derivation, look at pg. 158-159.

- in 3D motion will usually be made up of several components (omega’s). With each component being about a different axis, as shown in the figure.

- For it is important to note distinction between a and a axis.
  - Why is this important?
    1. Derivative of velocity -> acceleration.
    2. When we derive the angular acceleration, we have to take derivative of i, j, and k unit vectors (product rule).
    3. For fixed axes, the derivative of I, J, and K will be equal to zero.

- Recalling derivation in pg. 144. **For a rotating axis we see that:**
  - **Straightforward to remember:**



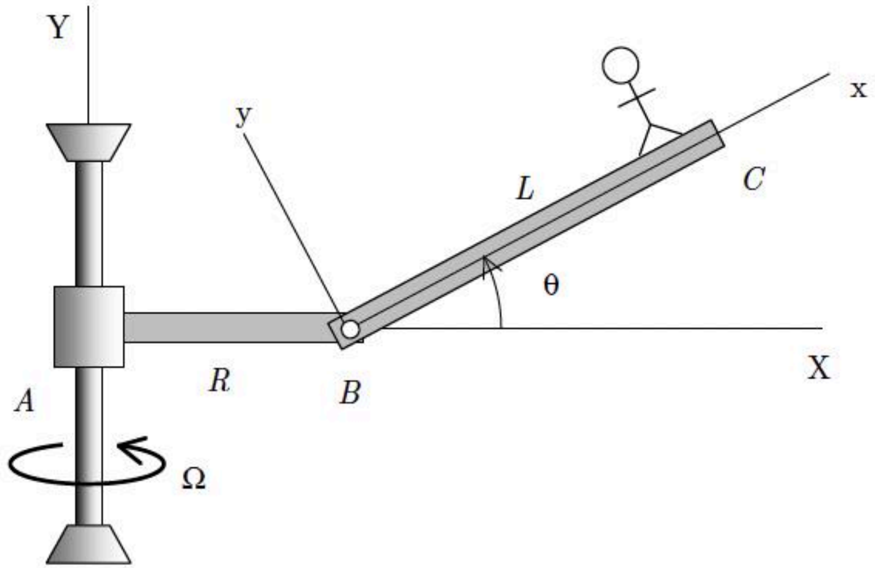
Example 3.B.1

Given: Bar BC is pinned at end B to bar AB, which in turn rotates about a fixed vertical axis at a constant rate of $\Omega = 5$ rad/s. The angle θ is increasing at a constant rate of $\dot{\theta} = 4$ rad/s. The observer and the xyz axes are attached to arm BC, and the XYZ axes are fixed.

Find: Determine:

(a) The angular velocity of the observer when $\theta = 90°$; and

(b) The angular acceleration of the observer when $\theta = 90°$.

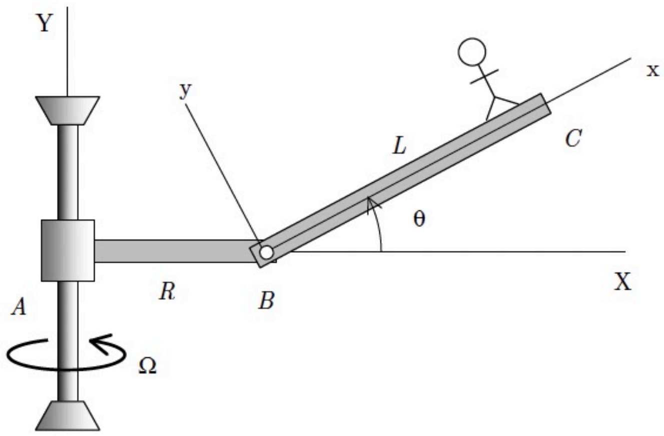


[pg. 163]

Example 3.B.1

Given: Bar BC is pinned at end B to bar AB, which in turn rotates about a fixed vertical axis at a constant rate of $\Omega = 5$ rad/s. The angle θ is increasing at a constant rate of $\dot{\theta} = 4$ rad/s. The observer and the xyz axes are attached to arm BC, and the XYZ axes are fixed.

Find: Determine:
(a) The angular velocity of the observer when $\theta = 90°$; and
(b) The angular acceleration of the observer when $\theta = 90°$.



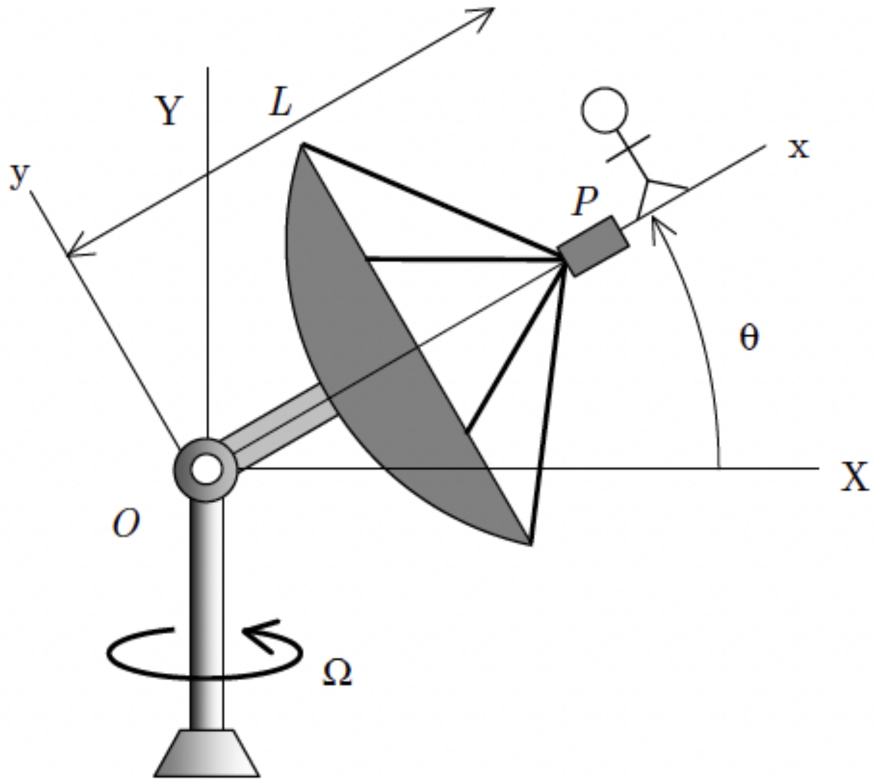
Example 3.B.2

Given: The radar antenna is rotating about a fixed vertical axis at a constant rate of $\Omega = 0.2$ rad/s. The angle θ is increasing at a constant rate of $\dot{\theta} = 0.5$ rad/s. The observer and the xyz axes are attached to the antenna dish, with the XYZ axes being fixed.

Find: Determine:

(a) The angular velocity of the observer when $\theta = 36.87°$; and

(b) The angular acceleration of the observer when $\theta = 36.87°$.



[pg. 164]

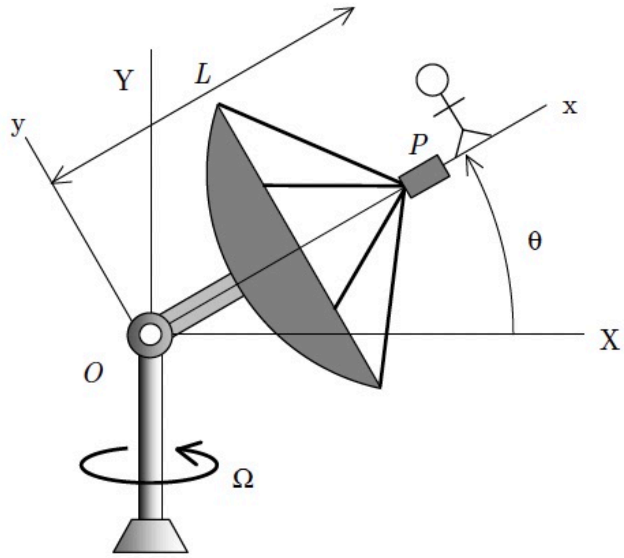
Example 3.B.2

Given: The radar antenna is rotating about a fixed vertical axis at a constant rate of $\Omega = 0.2$ rad/s. The angle θ is increasing at a constant rate of $\dot{\theta} = 0.5$ rad/s. The observer and the xyz axes are attached to the antenna dish, with the XYZ axes being fixed.

Find: Determine:

(a) The angular velocity of the observer when $\theta = 36.87°$; and

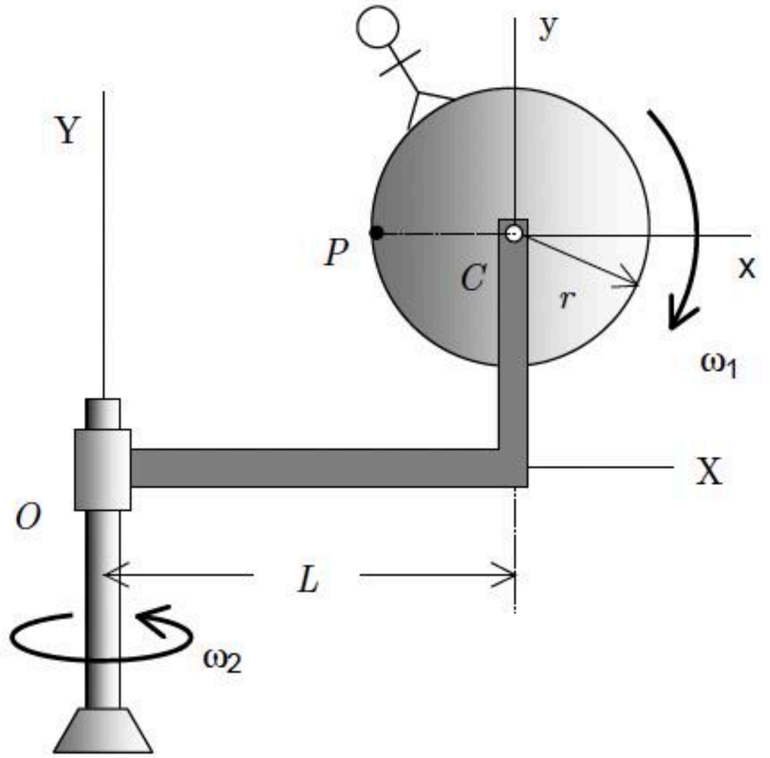
(b) The angular acceleration of the observer when $\theta = 36.87°$.



Example 3.B.3

Given: A disk rotates with a constant rate of $\omega_1 = 20$ rad/s with respect to the arm OC as the arm OC rotates about a fixed vertical axis with a constant rate of $\omega_2 = 5$ rad/s. The observer and the xyz axes are attached to the disk, while the XYZ axes are fixed. At this instant, the XYZ and xyz axes are aligned.

Find: Determine:
(a) The angular velocity of the observer at the instant shown; and
(b) The angular acceleration of the observer at the instant shown.



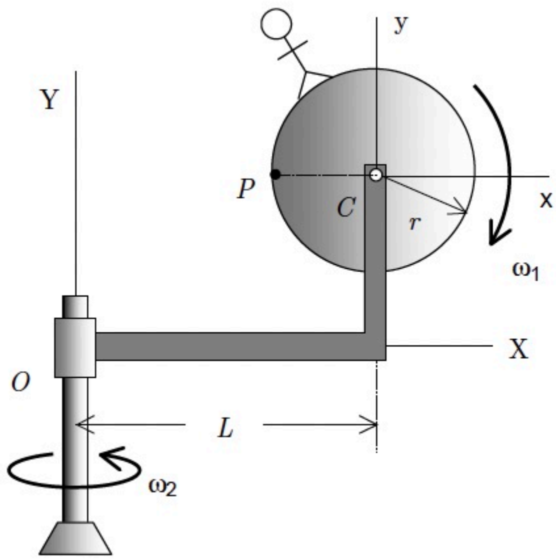
[pg. 165]

Example 3.B.3

Given: A disk rotates with a constant rate of $\omega_1 = 20$ rad/s with respect to the arm OC as the arm OC rotates about a fixed vertical axis with a constant rate of $\omega_2 = 5$ rad/s. The observer and the xyz axes are attached to the disk, while the XYZ axes are fixed. At this instant, the XYZ and xyz axes are aligned.

Find: Determine:

(a) The angular velocity of the observer at the instant shown; and
(b) The angular acceleration of the observer at the instant shown.

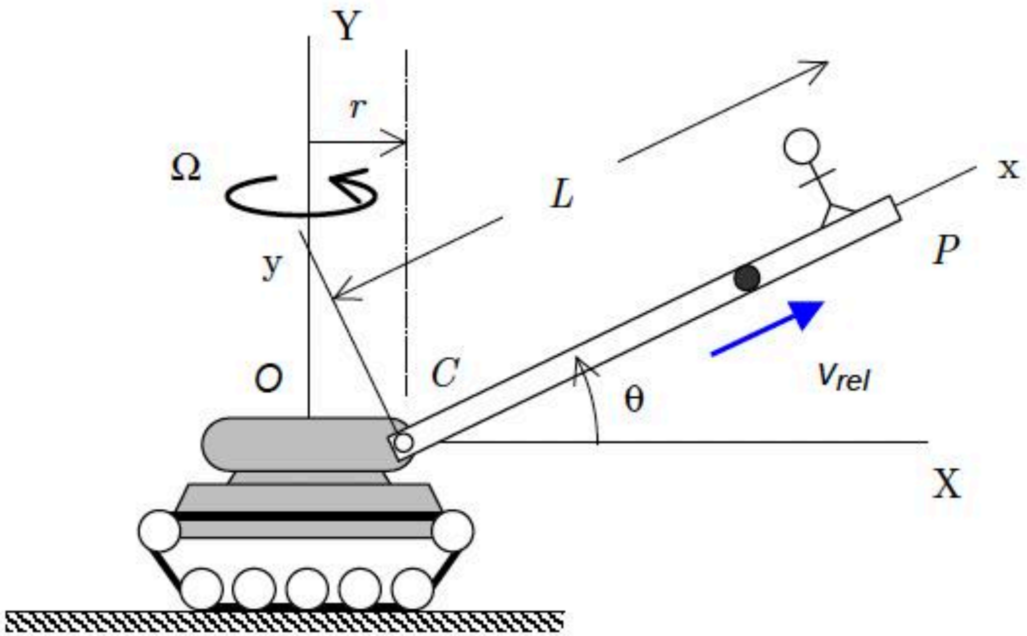


Example 3.B.4

Given: The turret on a tank is rotating about a fixed vertical axis at a constant rate of $\Omega = 0.4$ rad/s. The barrel is being raised at a constant rate of $\dot{\theta} = 0.4$ rad/s . A cannon shell is fired with a constant muzzle speed of $v_{rel} = 200$ ft/s relative to the barrel. The observer and the xyz axes are attached to the barrel, while the XYZ axes are fixed. Here, $r = 3$ ft and $L = 15$ ft.

Find:

(a) The angular velocity of the barrel at the instant shown;
(b) The angular acceleration of the barrel at the instant shown; and
(c) The acceleration of the shell as it leaves the barrel at P.



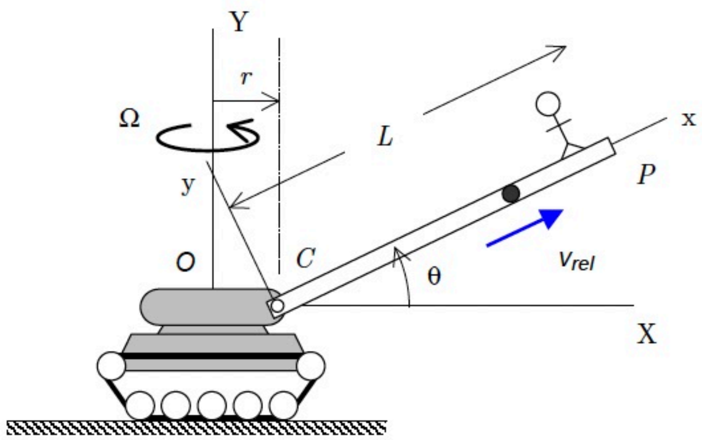
[pg. 166]

Example 3.B.4

Given: The turret on a tank is rotating about a fixed vertical axis at a constant rate of $\Omega = 0.4$ rad/s. The barrel is being raised at a constant rate of $\dot{\theta} = 0.4$ rad/s . A cannon shell is fired with a constant muzzle speed of $v_{rel} = 200$ ft/s relative to the barrel. The observer and the xyz axes are attached to the barrel, while the XYZ axes are fixed. Here, $r = 3$ ft and $L = 15$ ft.

Find:

(a) The angular velocity of the barrel at the instant shown;
(b) The angular acceleration of the barrel at the instant shown; and
(c) The acceleration of the shell as it leaves the barrel at P.



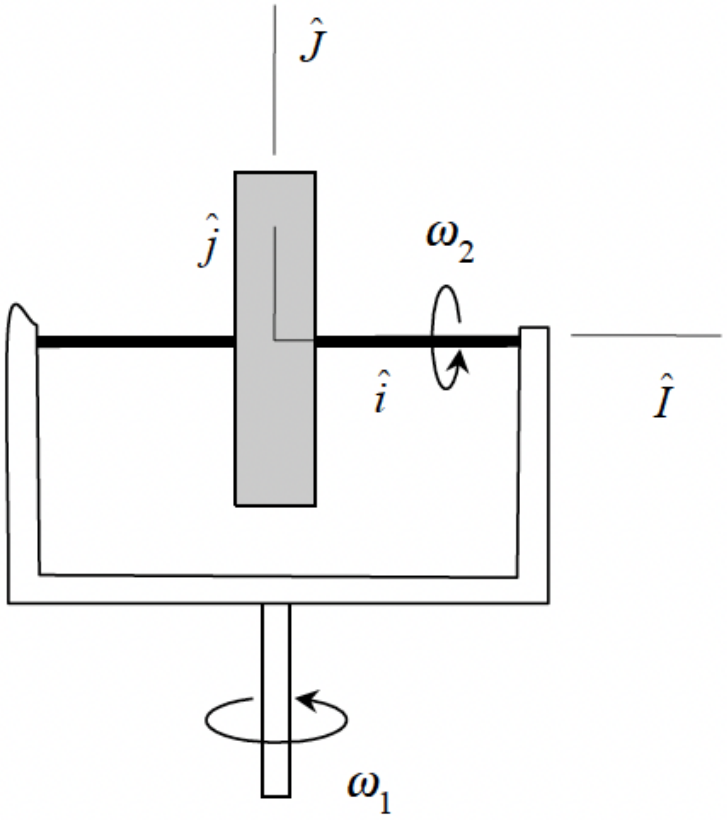
Example 3.B.5

Given: The disk of a gyroscope rotates about its own axis at a constant rate of $\omega_2 = 600$ rev/min. The gimbal support is rotating at a constant rate of $\omega_1 = 10$ rad/s about a fixed vertical axis. The observer and the xyz axes are attached to the disk. The XYZ axes are fixed in space.

Find: Determine:

(a) The angular velocity of the observer at the instant shown; and

(b) The angular acceleration of the observer at the instant shown.

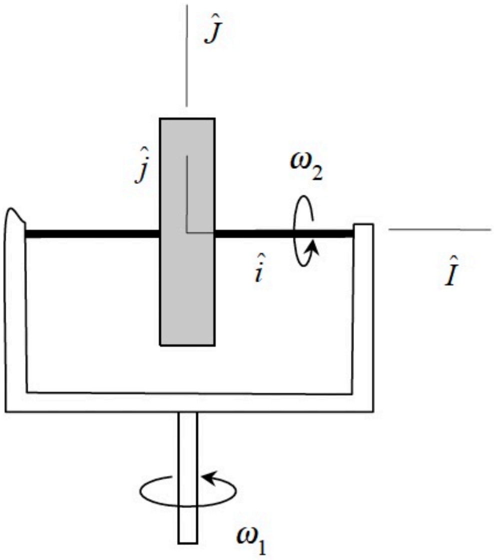


[pg. 167]

Example 3.B.5

Given: The disk of a gyroscope rotates about its own axis at a constant rate of $\omega_2 = 600$ rev/min. The gimbal support is rotating at a constant rate of $\omega_1 = 10$ rad/s about a fixed vertical axis. The observer and the xyz axes are attached to the disk. The XYZ axes are fixed in space.

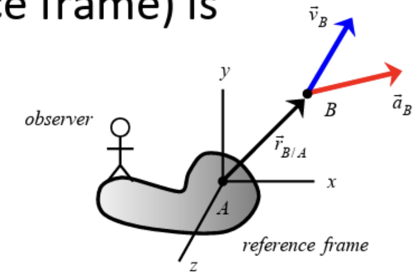
Find: Determine:
(a) The angular velocity of the observer at the instant shown; and
(b) The angular acceleration of the observer at the instant shown.



Summary: 3D Moving Reference Frame Kinematics 1

[pg. 145]

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point B.



$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha}\times\vec{r}_{B/A} + 2\vec{\omega}\times\left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{B/A}\right)$$

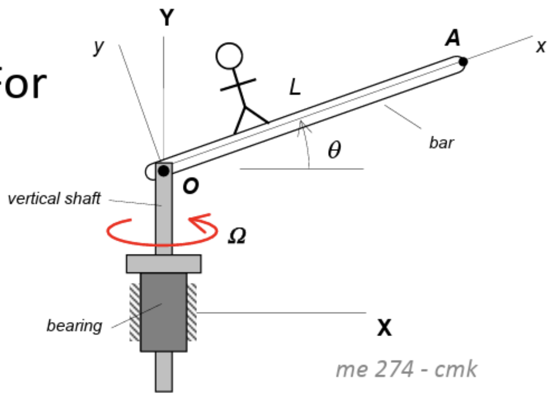
where:

- $\vec{\omega}$ and $\vec{\alpha}$ are the angular velocity/acceleration *of the observer* (*no exceptions*).
- $\left(\vec{v}_{B/A}\right)_{rel}$ and $\left(\vec{a}_{B/A}\right)_{rel}$ are the velocity/acceleration of B *as seen by the observer*.
- A is *ANY* point on the *same* reference frame as the observer.
- Generally, *you* are free to choose your observer.

QUESTION: How is this different from the 2D case? For observer on arm OA:

$$\vec{\omega} = \Omega\hat{J} + \dot{\theta}\hat{k}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \dot{\Omega}\hat{J} + \cancelto{0}{\Omega\dot{\hat{J}}} + \ddot{\theta}\hat{k} + \dot{\theta}\dot{\hat{k}} = \dot{\Omega}\hat{J} + \ddot{\theta}\hat{k} + \dot{\theta}\left(\vec{\omega}\times\hat{k}\right)$$



Lec 13 Short Feedback Form: