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ME 274 Lecture 12

Moving Reference Kinematics: 2D Part 2

Eugenio “Henny” Frias-Miranda

2/9/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 11 due tonight!!**
2. **Exam 1 Details on course website (https://www.purdue.edu/freeform/me274/exams-spring-2026/)**
3. **Lecture on Wednesday**
   - No HW due on Wednesday… This HW will be due on Friday.
4. **No lecture on Friday**

Goals: *Chapter 3 – Moving Reference Frame Kinematics*

1. Develop the **“moving reference frame”** velocity and acceleration kinematic equations

2. Apply these equations to the analysis of:
   a) More complicated **planar mechanisms**
   b) Problems in **3 dimensions**

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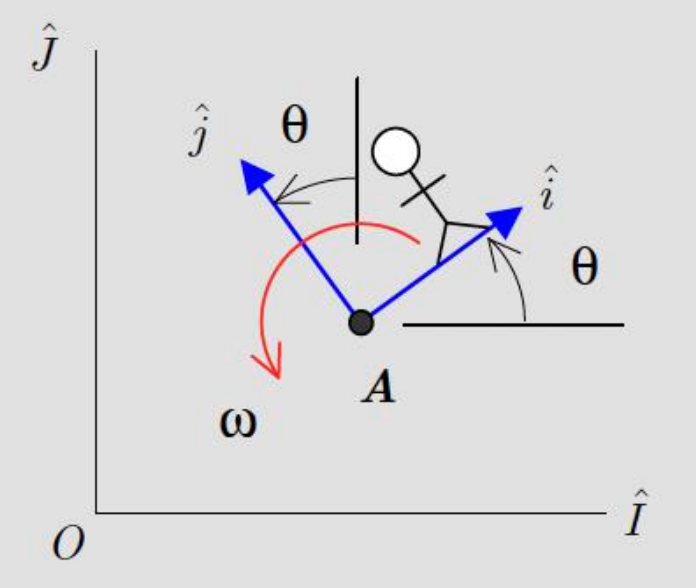
What do each of the terms mean?



$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right]$$

- $\vec{v}_A$ and $\vec{v}_B$ are the velocities seen by a **fixed observer** [XYZ]
- $\vec{a}_A$ and $\vec{a}_B$ are the accelerations seen by **fixed observer** [XYZ]
- $\vec{\omega}$ is angular velocity of the **moving observer** [xyz]
- $\vec{\alpha}$ is angular acceleration of the **moving observer** [xyz]
- $\left(\vec{v}_{B/A}\right)_{rel}$ is the “velocity of point B as seen by the **moving observer** at A”
- $\left(\vec{a}_{B/A}\right)_{rel}$ is the “acceleration of point B as seen by the **moving observer** at A”
- $2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel}$ is known as the **“Coriolis” component** of acceleration.
  - Arises when observer has a non-zero angular velocity

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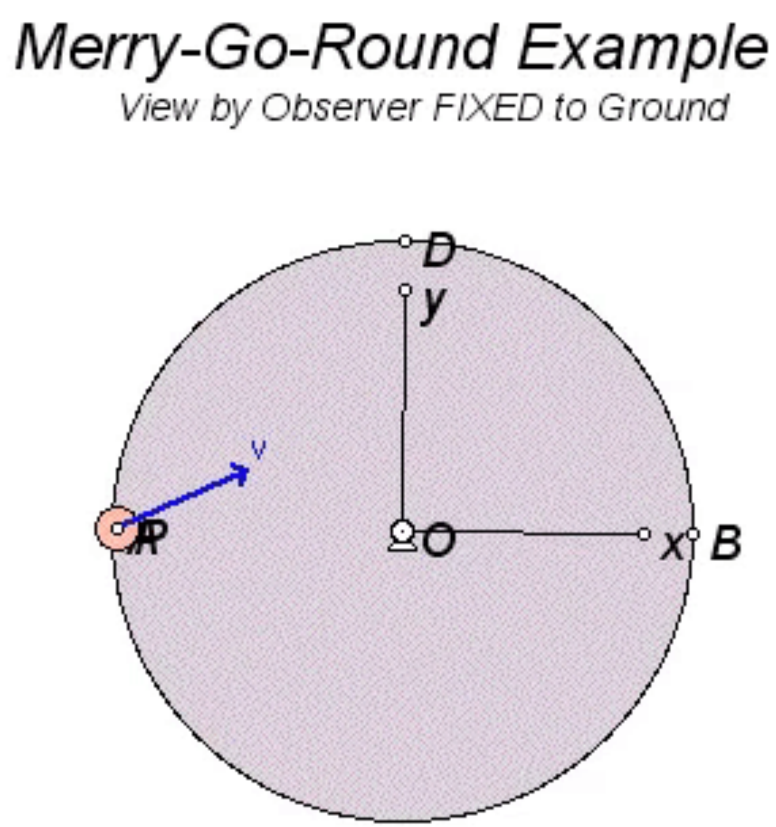
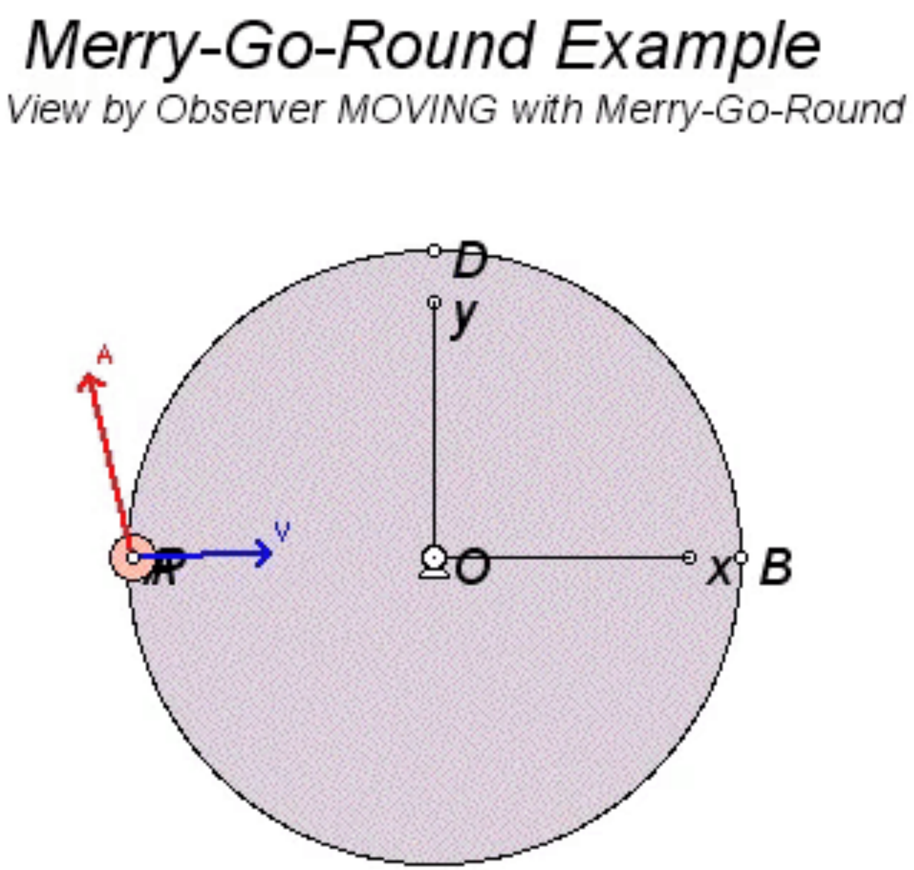
Merry-Go-Round Example: IRL Video

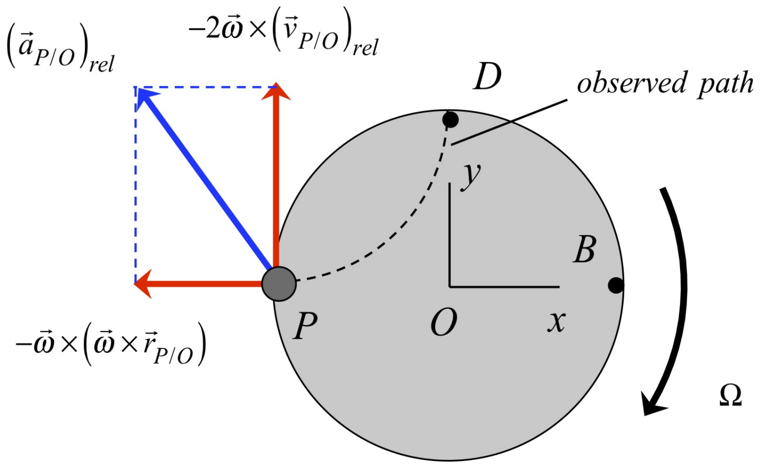
[Animation - THE MERRY-GO-ROUND AND THE CORIOLIS COMPONENT OF ACCELERATION]

Merry-Go-Round Example: Simulation





[Animation - THE MERRY-GO-ROUND AND THE CORIOLIS COMPONENT OF ACCELERATION]



$\omega = -\Omega \hat{k} = \text{constant (CW)}$

$\alpha = 0$

$$(\vec{a}_{P/O})_{rel} = -2\vec{\omega}\times(\vec{v}_{P/O})_{rel} - \vec{\omega}\times(\vec{\omega}\times\vec{r}_{P/O})$$

Find: acceleration of P as seen by observer on merry-go-round. $(a_{P/O})_{rel}$?

Solution:

① Attach observer & xyz to merry-go-round

② Solve for $(a_{P/O})_{rel}$

no net force acting on ball

$$a_P = a_O + (a_{P/O})_{rel} + \alpha \times r_{P/O} + 2\omega \times (v_{P/O})_{rel} + \omega \times (\omega \times r_{P/O})$$

($a_P = 0$, $a_O = 0$, $\alpha = 0$)

$$(a_{P/O})_{rel} = -2\omega \times (v_{P/O})_{rel} - \omega \times (\omega \times r_{P/O})$$

$$= -2(-\Omega \hat{k}) \times (v_{rel}\,\hat{\imath}) - (-\Omega \hat{k}) \times [(-\Omega \hat{k}) \times (-r\,\hat{\imath})]$$

$$= -r\Omega^2 \hat{\imath} + 2\Omega v_{rel} \hat{\jmath}$$

} observed acceleration of the ball

- originates from Coriolis component of acceleration
- say that ball tends to curve to the left of desired path

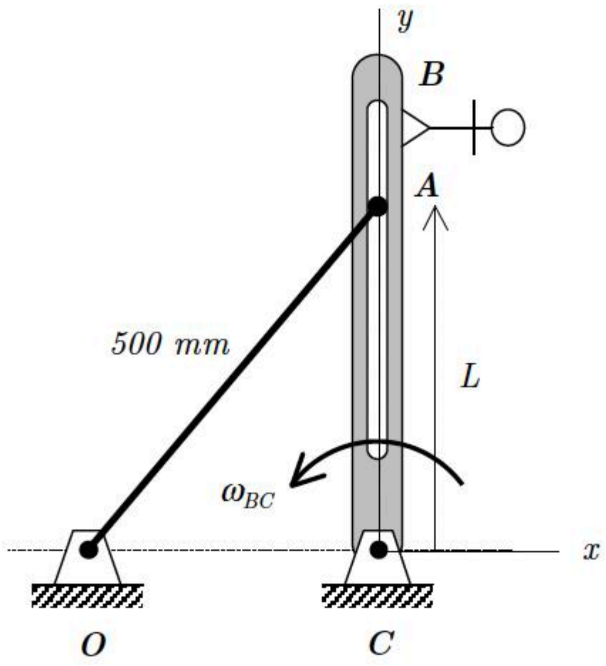
Example 3.A.5

Given: The mechanism shown below is made up of links OA and BC. Links OA and BC are pinned to ground at points O and C, respectively. Pin A of link OA is able to slide within a slot that is cut in link BC, as shown. Pins O and C are on the same horizontal line. Let the xyz axes be attached to link BC. At the instant shown:

- Link BC is vertical with $L = 400$ mm; and
- Link BC is rotating counterclockwise at a rate of $\omega = 6$ rad/s.

Find: Determine:

(a) The angular velocity of link OA at the instant shown; and
(b) The value for $\dot{L}$ at the instant shown.



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Example 3.A.5 p.154

Given: The mechanism shown below is made up of links OA and BC. Links OA and BC are pinned to ground at points O and C, respectively. Pin A of link OA is able to slide within a slot that is cut in link BC, as shown. Pins O and C are on the same horizontal line. Let the xyz axes be attached to link BC. At the instant shown:

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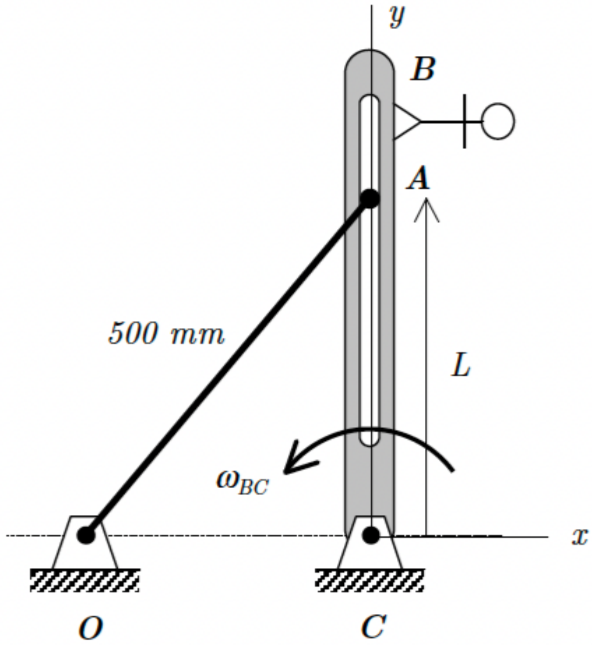
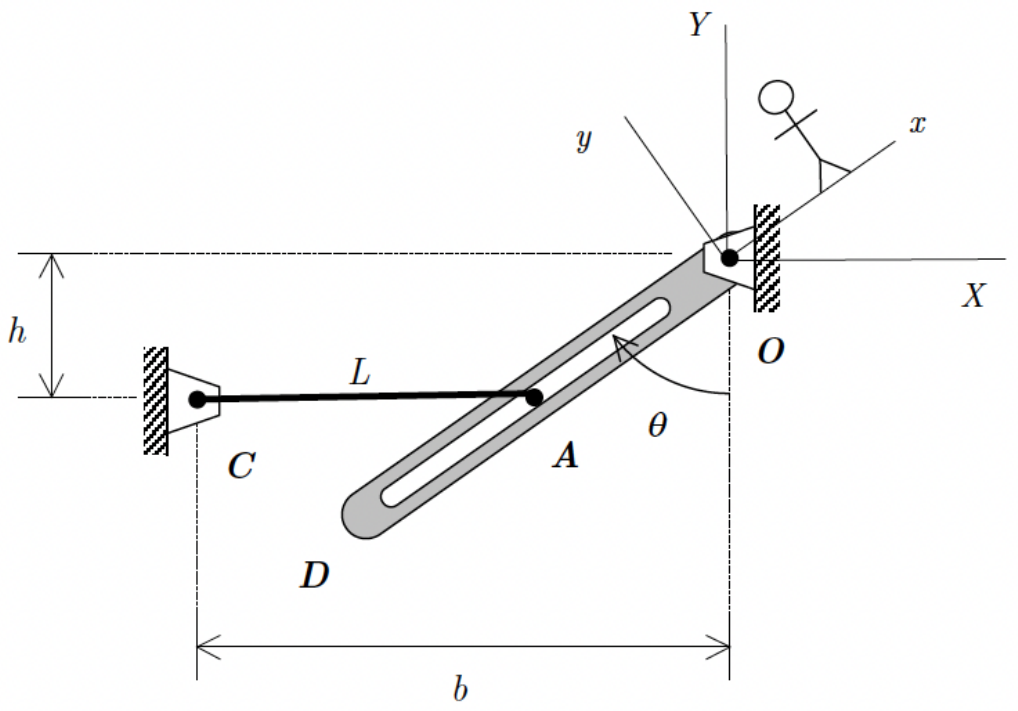


Example 3.A.6

Given: Link OD is rotating clockwise at a constant rate of $\dot{\theta} = 2$ rad/s. When $\theta = 45°$, link CA is horizontal.

Find: Determine:
(a) The velocity of A when $\theta = 45°$; and
(b) The acceleration of A at the same position

Use the following parameters in your analysis: $L = 0.225$ m, $h = 0.225$ m and $b = 0.45$ m.



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Example 3.A.6 p.155

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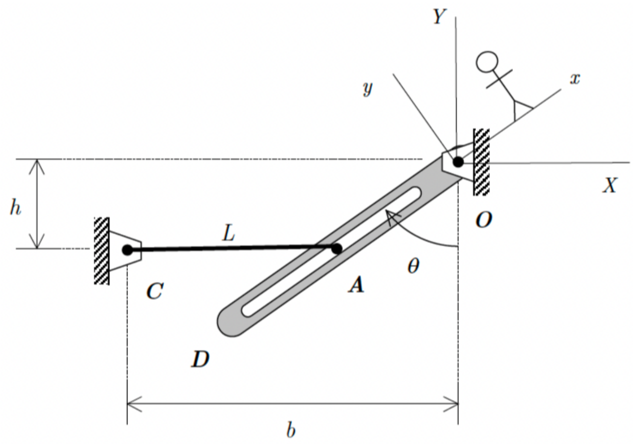


Example 3.A.7

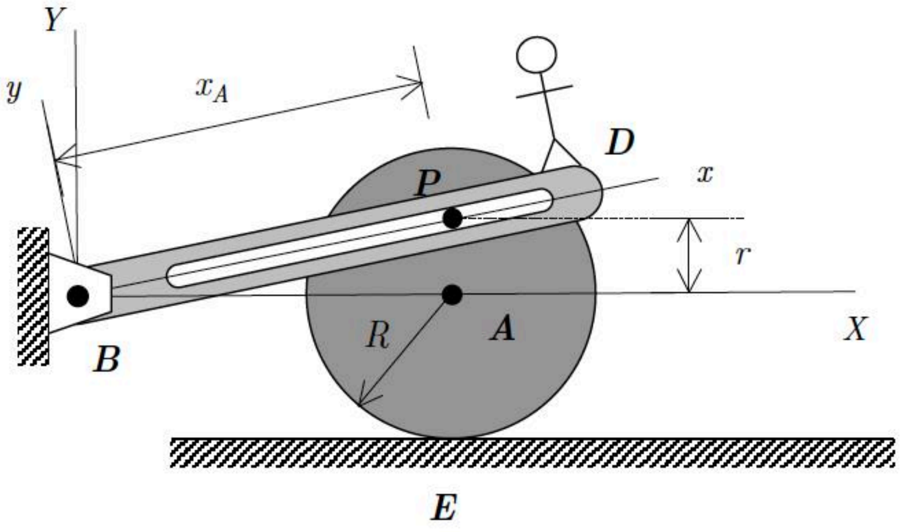
Given: The disk rolls without slipping to the right with a constant angular speed of ω_d. At the instant shown, pin P is directly above the center A of the disk.

Find: Determine:

(a) The angular acceleration of the disk; and

(b) The acceleration of P as seen by an observer on arm BD.

Use the following parameters in your analysis: $\omega_d = 20$ rad/s (clockwise), $x_A = 0.48$ m, $r = 0.14$ m and $R = 0.2$ m.



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Example 3.A.7 p.156 (one of the longer problems this semester...)

Given: The disk rolls without slipping to the right with a constant angular speed of ω_d. At the instant shown, pin P is directly above the center A of the disk.

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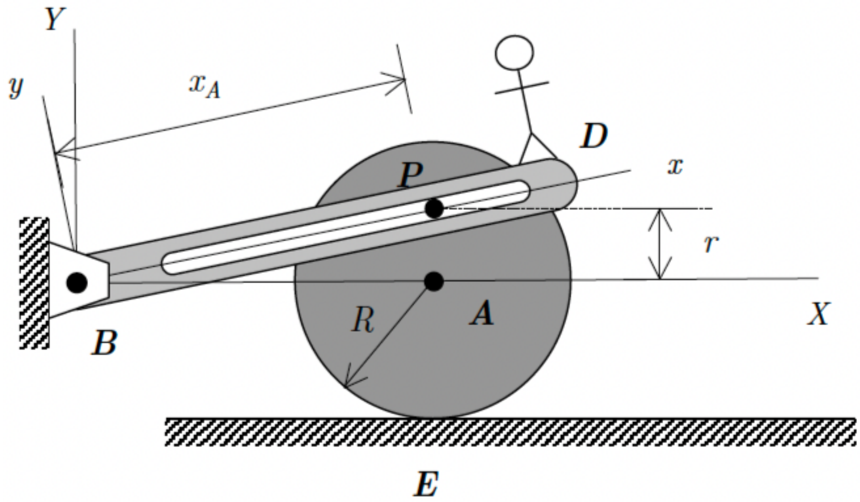
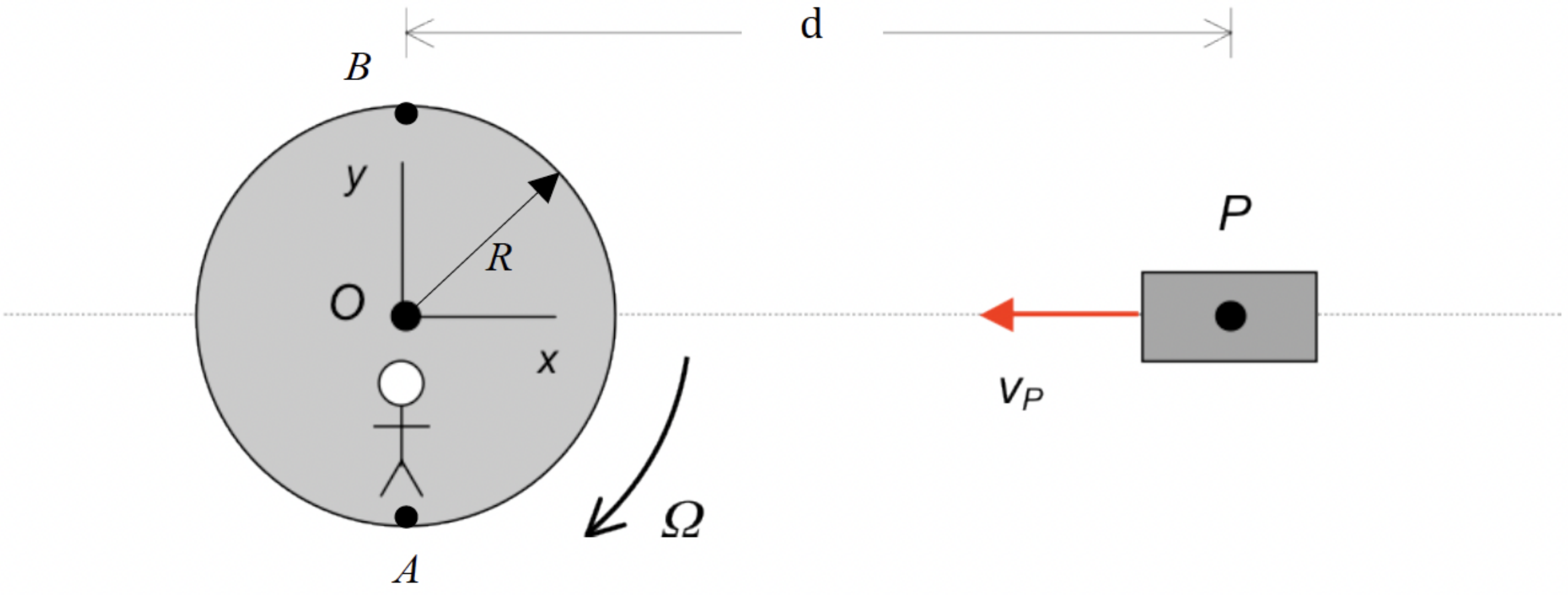


Question C3.2

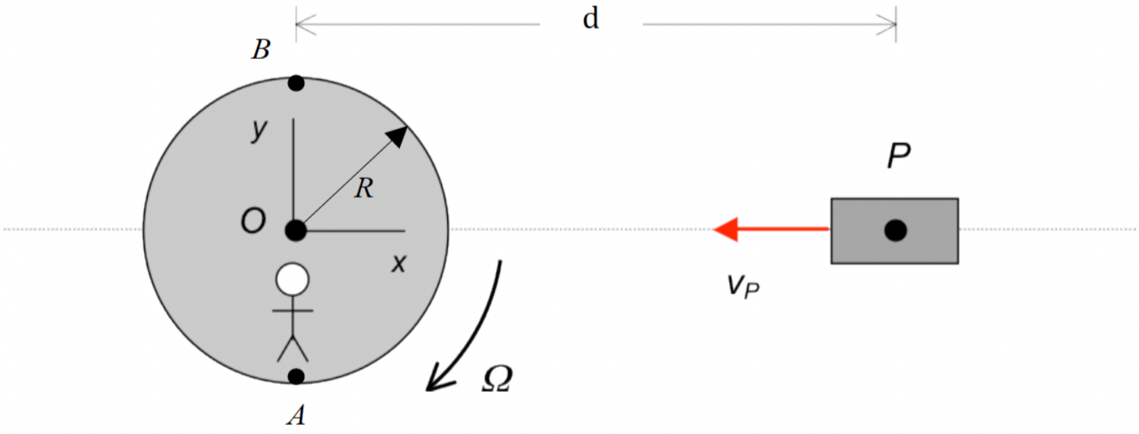
A disk is pinned to ground at its center O with the disk rotating clockwise at a constant rate of $\Omega = 3$ rad/s. Block P is traveling to the left along a straight path toward O with a constant speed of $v_P = 20$ ft/s. Determine the acceleration of P as seen by an observer on the disk when P is at a distance of 50 ft from O.



[pg. 174]

Question C3.2 p.174

A disk is pinned to ground at its center O with the disk rotating clockwise at a constant rate of $\Omega = 3$ rad/s. Block P is traveling to the left along a straight path toward O with a constant speed of $v_P = 20$ ft/s. Determine the acceleration of P as seen by an observer on the disk when P is at a distance of 50 ft from O.



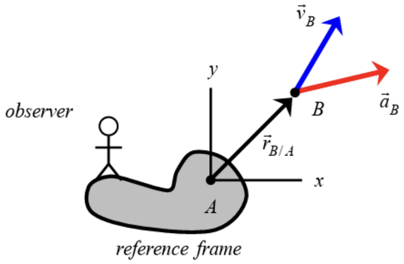
[pg. 145]

Summary: 2D Moving Reference Frame Kinematics 2

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point B.

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$



APPLICATION: Using 2D MRF equations in solving problems in the kinematics of mechanisms.

AP (rigid body):

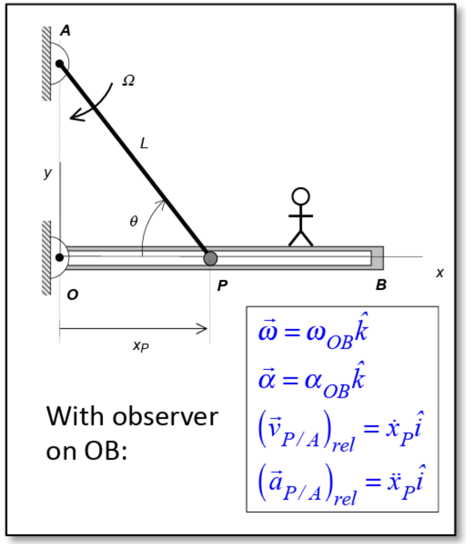
$$\vec{v}_P = \left(-\Omega\hat{k}\right) \times \vec{r}_{P/A}$$

$$\vec{a}_P = \left(-\dot{\Omega}\hat{k}\right) \times \vec{r}_{P/A} + \left(-\Omega\hat{k}\right) \times \left[\left(-\Omega\hat{k}\right) \times \vec{r}_{P/A}\right]$$

OP (<u>*not*</u> a rigid body):

$$\vec{v}_P = \dot{x}_P\hat{i} + \left(\omega_{OB}\hat{k}\right) \times \vec{r}_{P/A}$$

$$\vec{a}_P = \ddot{x}_P\hat{i} + \left(\alpha_{OB}\hat{k}\right) \times \vec{r}_{P/A} + 2\left(\omega_{OB}\hat{k}\right) \times \left(\dot{x}_P\hat{i}\right) + \left(\omega_{OB}\hat{k}\right) \times \left[\left(\omega_{OB}\hat{k}\right) \times \vec{r}_{P/A}\right]$$



Lec 12 Short Feedback Form: