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ME 274 Lecture 11

Moving Reference Kinematics: 2D Part 1

Eugenio "Henny" Frias-Miranda

2/~~4~~ 6/26

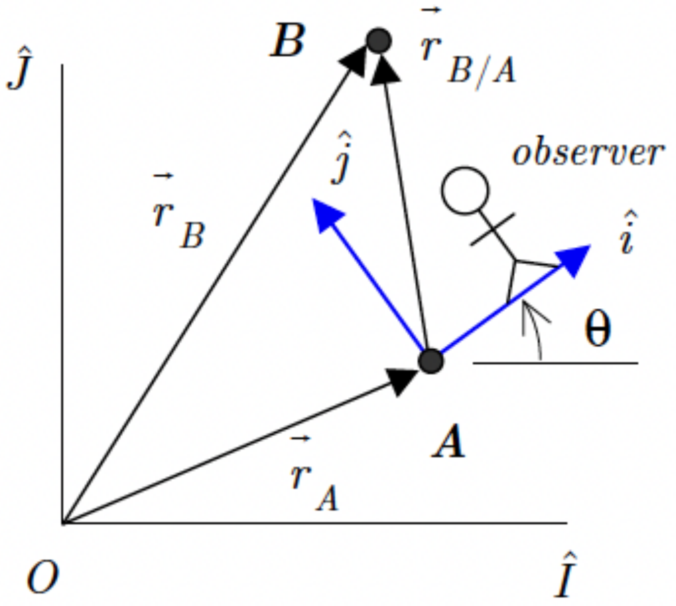
Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 10 due tonight!!**
2. Exam 1 Details on course website (https://www.purdue.edu/freeform/me274/exams-spring-2026/)

Motivation: *Chapter 3 – Moving Reference Frame Kinematics*

- In chapter 2, we saw that if two points A and B are on the rigid body.
- It is often the case that we need to use the motion of two points that are on the same rigid body.
- Many times, we can use kinematic information about the motion of B obtained from an observer on a [figure]
- A moving reference frame can be *translating* or *rotating.*
- The observer at A would describe B as:



[pg. 143 content]

Motivation (cont.): *Chapter 3 – Moving Reference Frame Kinematics*

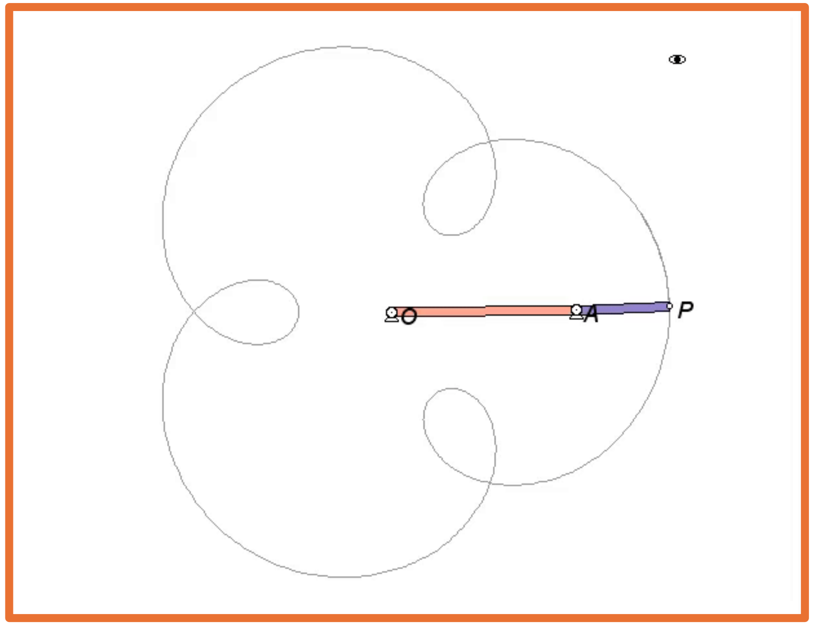


Fig 1. Observer on Fixed Ground

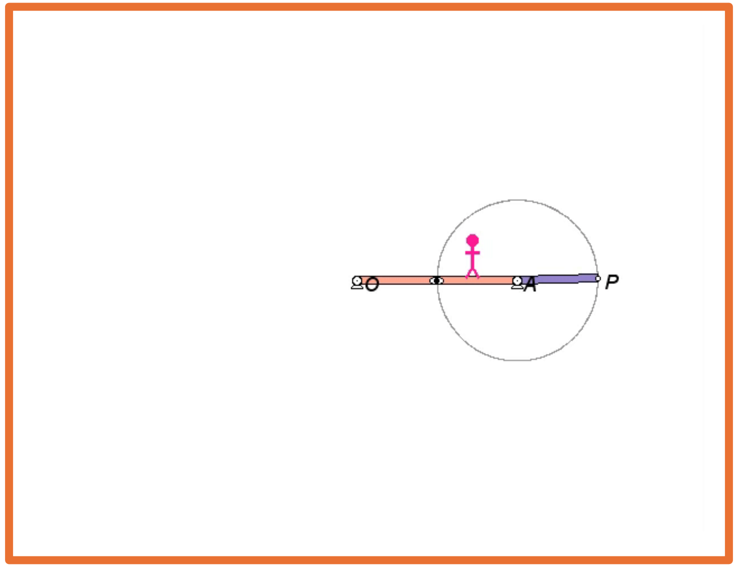


Fig 2. Observer on Link OA

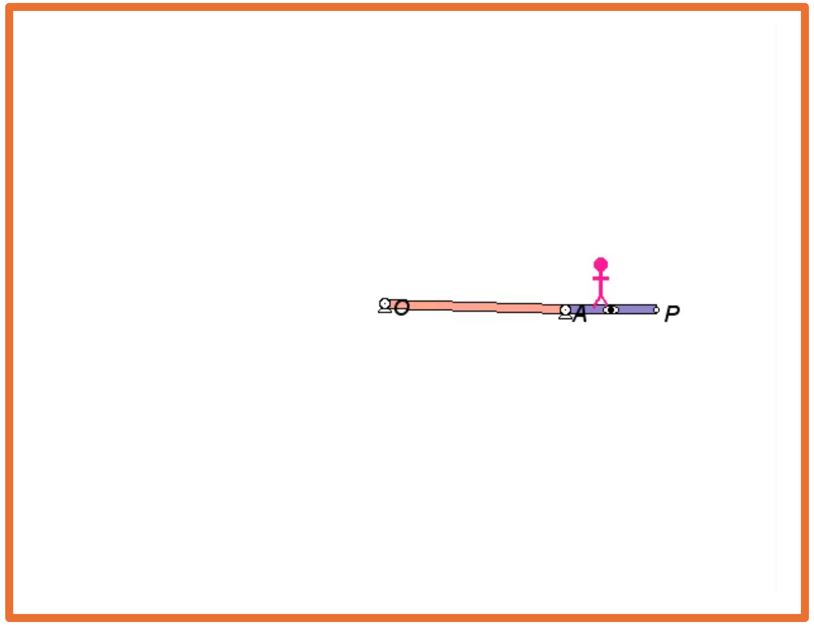


Fig 3. Observer on Link ~~OA~~ AP

- Observe *translation* and *rotation* in these videos above.
- <u>Which observer do you think would be most useful in calculating the velocity and acceleration of point P?</u>

  Answer: Link OA [Fig 2], *observer is aware of P moving and its path is observed to be in a circle. Versus a complex path in Fig 1 and not moving in Fig 3.*

[Animation: views from different reference frames]

Goals: *Chapter 3 – Moving Reference Frame Kinematics*

1. Develop the **“rotating reference frame”** velocity and acceleration kinematic equations

2. Apply these equations to the analysis of:
   a) More complicated **planar mechanisms**
   b) Problems in **3 dimensions**

[pg. 143 content]

Angular Velocity of 2D Rotating Reference Frames

- The observer is attached to the   (translating/rotating).
- We introduce a set of
- The angular velocity of the observer is therefore given by:

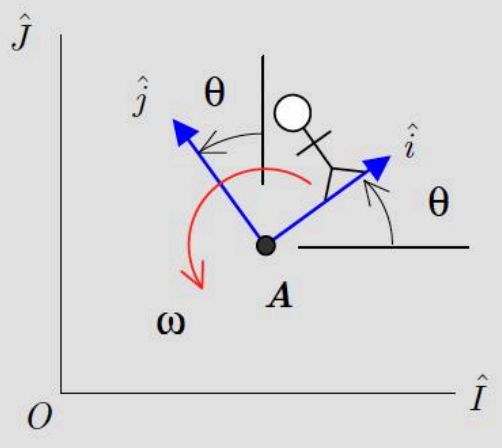
$$\vec{\omega} = \dot{\theta}\hat{k}$$

- The unit vectors i and j are constant in length, but change in direction as observer rotates, therefore:

$$\frac{d\hat{i}}{dt} = \dot{\theta}\hat{j} \quad = \dot{\theta}\left(\hat{k} \times \hat{i}\right) = \dot{\theta}\hat{k} \times \hat{i} = \vec{\omega} \times \hat{i}$$

$$\frac{d\hat{j}}{dt} = -\dot{\theta}\hat{i} \quad = -\dot{\theta}\left(-\hat{k} \times \hat{j}\right) = \dot{\theta}\hat{k} \times \hat{j} = \vec{\omega} \times \hat{j}$$

- This relationship above will be useful for deriving our moving reference frame kinematic equations (next 2 slides)….



$$\hat{i} = \cos\theta\,\hat{I} + \sin\theta\,\hat{J}$$
$$\hat{j} = -\sin\theta\,\hat{I} + \cos\theta\,\hat{J} \quad \Rightarrow$$

$$\frac{d\hat{i}}{dt} = -\dot{\theta}\sin\theta\,\hat{I} + \dot{\theta}\cos\theta\,\hat{J}$$
$$= \dot{\theta}\,\hat{j}$$
$$\frac{d\hat{j}}{dt} = -\dot{\theta}\cos\theta\,\hat{I} - \dot{\theta}\sin\theta\,\hat{J}$$
$$= -\dot{\theta}\,\hat{i}$$

[pg. 144 content]

Moving Reference Frame Kinematics – **2D Velocity Equation Derivation**

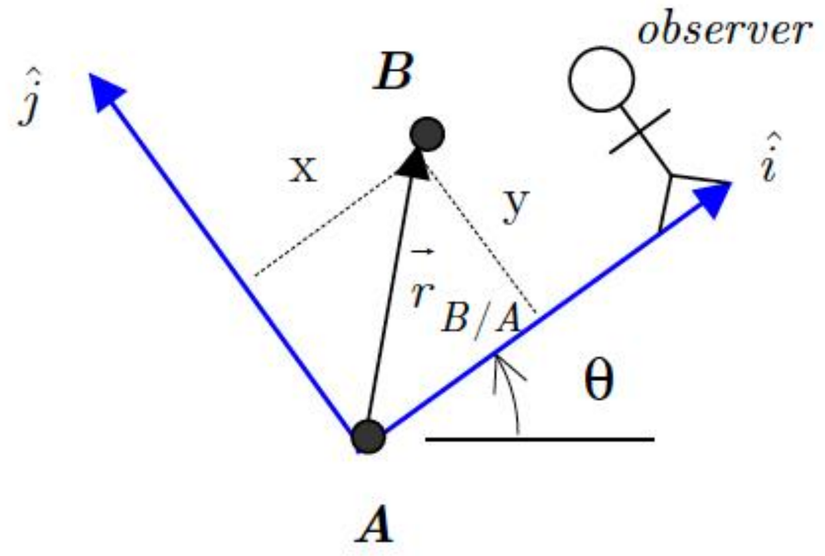
1. Position of B can be written in term of Point A: $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

2. We can write the same in terms of the observer's xy coordinates: $\vec{r}_{B/A} = x\hat{i} + y\hat{j}$

3. Differentiating the observer's position equation gives us our moving reference frame velocity equation:

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt} \qquad \Longrightarrow$$

$$\begin{aligned}
\vec{v}_B &= \vec{v}_A + \frac{d}{dt}\left(x\hat{i} + y\hat{j}\right) \\
&= \vec{v}_A + \frac{dx}{dt}\hat{i} + x\frac{d\hat{i}}{dt} + \frac{dy}{dt}\hat{j} + y\frac{d\hat{j}}{dt} \\
&= \vec{v}_A + \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + x\left(\vec{\omega} \times \hat{i}\right) + y\left(\vec{\omega} \times \hat{j}\right) \\
&= \vec{v}_A + \dot{x}\hat{i} + \dot{y}\hat{j} + \vec{\omega} \times \left(x\hat{i} + y\hat{j}\right) \\
&= \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}
\end{aligned}$$



4. Where: $\left(\vec{v}_{B/A}\right)_{rel} = \dot{x}\hat{i} + \dot{y}\hat{j} =$ "velocity of B as seen by the moving observer".

[pg. 145 content]

Moving Reference Frame Kinematics – **2D Acceleration Equation Derivation**

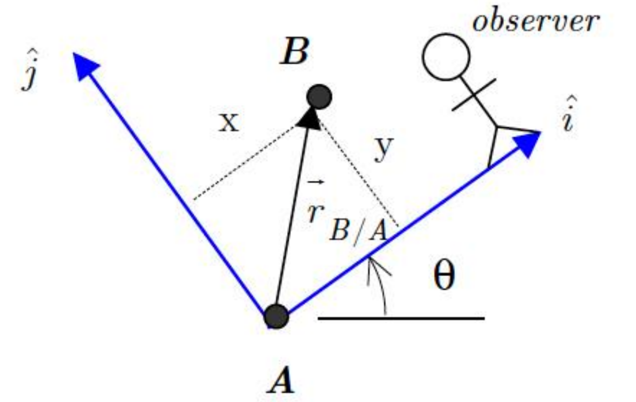
1. Velocity equation: $\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$

2. We can leverage the relationship we built from two slides ago.. $\frac{d\hat{i}}{dt} = \vec{\omega} \times \hat{i} \qquad \frac{d\hat{j}}{dt} = \vec{\omega} \times \hat{j}$

3. Differentiate velocity equation to get acceleration:

$$\frac{d\vec{v}_B}{dt} = \frac{d\vec{v}_A}{dt} + \frac{d}{dt}\left(\vec{v}_{B/A}\right)_{rel} + \frac{d}{dt}\left(\vec{\omega} \times \vec{r}_{B/A}\right) \qquad \Longrightarrow$$

$$\begin{aligned}
\vec{a}_B &= \vec{a}_A + \frac{d}{dt}\left(\vec{v}_{B/A}\right)_{rel} + \frac{d\vec{\omega}}{dt} \times \vec{r}_{B/A} + \vec{\omega} \times \frac{d\vec{r}_{B/A}}{dt} \\
&= \vec{a}_A + \frac{d}{dt}\left(\dot{x}\hat{i} + \dot{y}\hat{j}\right) + \frac{d\vec{\omega}}{dt} \times \vec{r}_{B/A} + \vec{\omega} \times \left[\left(\vec{v}_{B/A}\right)_{rel} + \omega \times \vec{r}_{B/A}\right] \\
&= \vec{a}_A + \left(\ddot{x}\hat{i} + \ddot{y}\hat{j}\right) + \left(\dot{x}\frac{d\hat{i}}{dt} + \dot{y}\frac{d\hat{j}}{dt}\right) + \frac{d\vec{\omega}}{dt} \times \vec{r}_{B/A} + \vec{\omega} \times \left[\left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}\right] \\
&= \vec{a}_A + \left(\ddot{x}\hat{i} + \ddot{y}\hat{j}\right) + \dot{x}\left(\vec{\omega} \times \hat{i}\right) + \dot{y}\left(\vec{\omega} \times \hat{j}\right) + \frac{d\vec{\omega}}{dt} \times \vec{r}_{B/A} + \vec{\omega} \times \left[\left(\vec{v}_{B/A}\right)_{rel} + \omega \times \vec{r}_{B/A}\right] \\
&= \vec{a}_A + \left(\ddot{x}\hat{i} + \ddot{y}\hat{j}\right) + \vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \frac{d\vec{\omega}}{dt} \times \vec{r}_{B/A} + \vec{\omega} \times \left[\left(\vec{v}_{B/A}\right)_{rel} + \omega \times \vec{r}_{B/A}\right] \\
&= \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right]
\end{aligned}$$

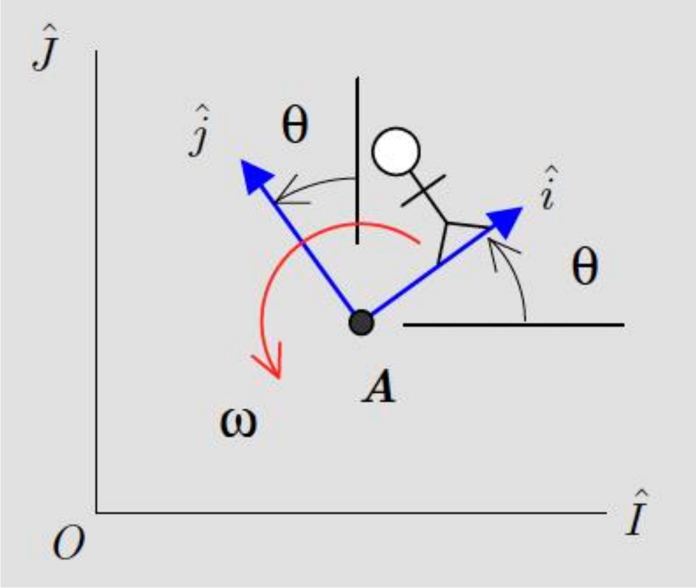


4. Where: $\left(\vec{a}_{B/A}\right) = \ddot{x}\hat{i} + \ddot{y}\hat{j} =$ "acceleration of B as seen by the moving observer". $\vec{\alpha} =$ "angular acceleration of the observer" $= \frac{d\omega}{dt}\hat{k}$.

What do each of the terms mean?

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$
$$= \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right]$$
$$= \vec{a}_A + \vec{a}_{B/A}$$



- $\vec{v}_A$ and $\vec{v}_B$ are the velocities seen by a **fixed observer** [XYZ]
- $\vec{a}_A$ and $\vec{a}_B$ are the accelerations seen by **fixed observer** [XYZ]
- $\vec{\omega}$ is angular velocity of the **moving observer** [xyz]
- $\vec{\alpha}$ is angular acceleration of the **moving observer** [xyz]
- $\left(\vec{v}_{B/A}\right)_{rel}$ is the "velocity of point B as seen by the **moving observer** at A"
- $\left(\vec{a}_{B/A}\right)_{rel}$ is the "acceleration of point B as seen by the **moving observer** at A"
- $2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel}$ is known as the **"Coriolis" component** of acceleration.
  - Arises when observer has a non-zero angular velocity

[pg. 147 content]

Brief - how to do a ***Moving Reference Frame Kinematic Equations*** Problem

1. Choose your moving reference frame (observer).

2. Draw your choice of xyz axes for the moving reference frame. Draw your choice of stationary XYZ axes.

3. Determine the angular velocity, omega, of the moving reference frame

4. Imagine yourself as the observer on the moving reference frame.
   How do I see point B move if I am that observer?
   - Based on this answer write down $\left(\vec{v}_{B/A}\right)_{rel}$ and $\left(\vec{a}_{B/A}\right)_{rel}$

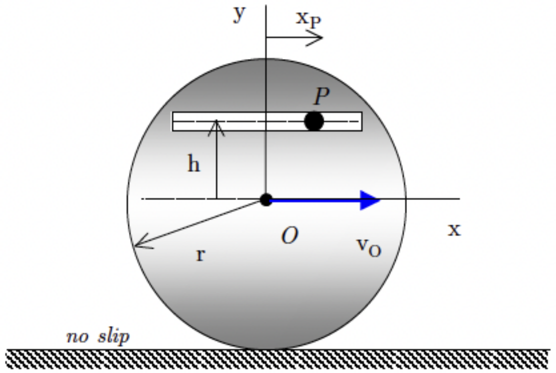
[pg. 148-149 content]

Example 3.A.1

Given: The disk shown below rolls without slipping on a horizontal surface. At the instant shown, the center O is moving to the right with a speed of $v_0 = 5$ m/s with this speed decreasing at a rate of 2 m/s^2. Also for this instant, the particle P is at a position of $x_p = 0.2$ m with $\dot{x}_p = 2$ m/s $=$ *constant*, where x_p is measured relative to the xyz coordinate system that is attached to the disk.

Find: Determine:
(a) The velocity of particle P; and
(b) The acceleration of particle P.

Use the following parameters in your analysis: $h = 0.2$ m and $r = 0.6$ m.



[pg. 150]

Example 3.A.1

Given: The disk shown below rolls without slipping on a horizontal surface. At the instant shown, the center O is moving to the right with a speed of $v_O = 5$ m/s with this speed decreasing at a rate of 2 m/s^2. Also for this instant, the particle P is at a position of $x_p = 0.2$ m with $\dot{x}_p = 2$ m/s $=$ *constant*, where x_p is measured relative to the xyz coordinate system that is attached to the disk.

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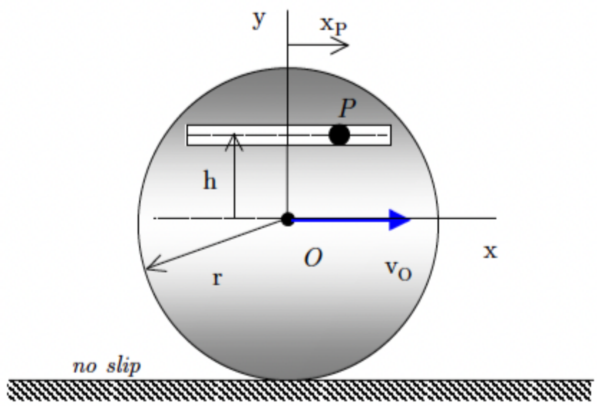


Example 3.A.2

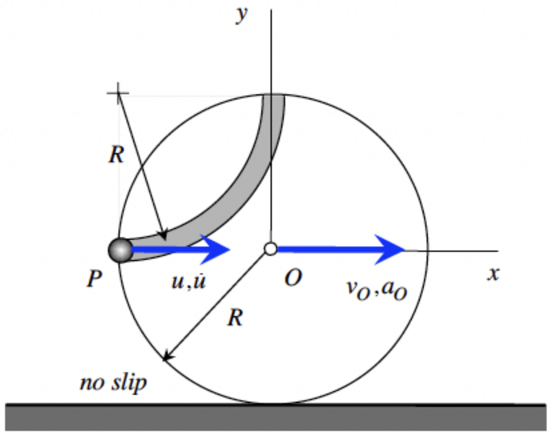
Given: The disk of radius R rolls without slipping on a horizontal surface with the speed and acceleration of the disk center O given by v_O and a_O, respectively. A particle P slides within a semi-circular slot cut in the disk with P moving with a speed of u relative to the disk and a rate of change of speed of $\dot{u}$ relative to the disk. The radius of the slot is R.

Find: Determine:

(a) The velocity of P when P is on the perimeter of the disk and is immediate to the left of O; and

(b) The acceleration of P at the same position.

Use the following parameters in your analysis: $v_O = 3$ m/s, $a_O = 5$ m/s^2, $u = 2$ m/s $\dot{u} = 7$ m/s^2, and $R = 0.75$ m.



[pg. 151]

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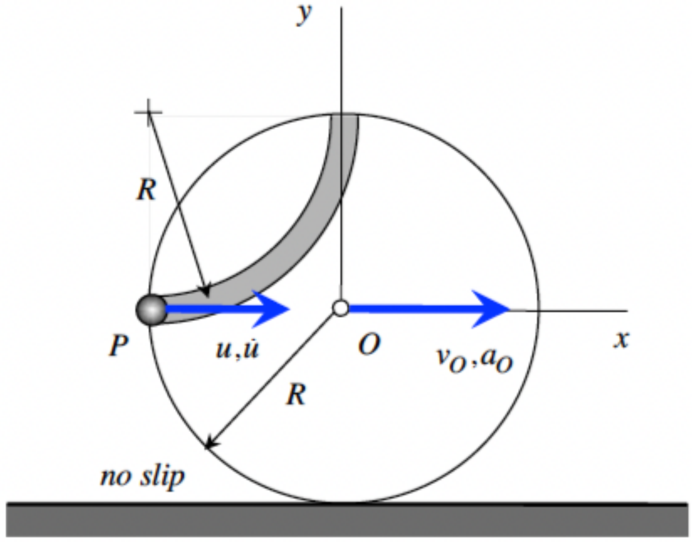
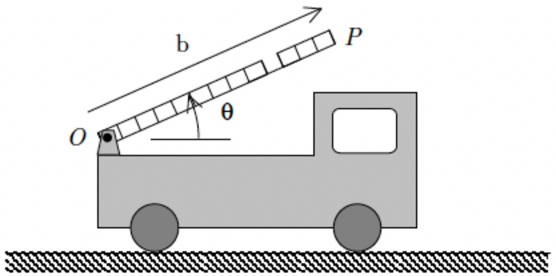


Example 3.A.3

Given: The fire truck moves forward at a constant speed of 50 ft/s. The ladder is being raised at a constant rate of $\dot{\theta} = 0.3$ rad/s. In addition, the ladder is being extended at a constant rate of $\dot{b}$ = 2 ft/s.

Find: The acceleration of end P of the ladder when $b = 25$ ft and $\theta = 30°$.



[pg. 152]

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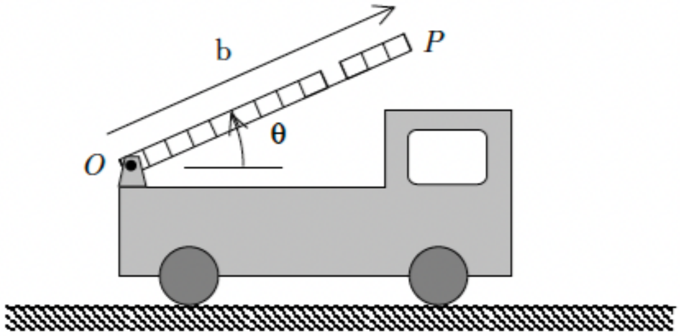
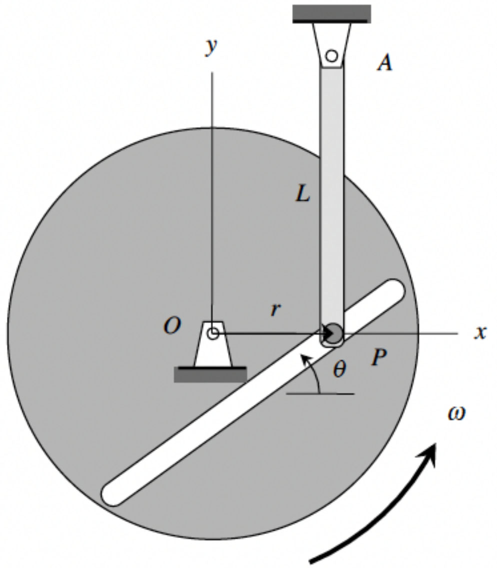


Example 3.A.4

Given: The disk shown below is rotating counterclockwise at a constant rate of ω. Link AP is vertical. Pin P slides within a straight slot cut into the disk. Let the xyz axes be attached to the disk. The slot is oriented at an angle of θ as measured from the x-axis. At the instant shown, P is on the x-axis, and the x-axis is horizontal.

Find: Determine:
(a) The angular velocity of link AP at this instant; and
(b) The angular acceleration of link AP at this instant.

Use the following parameters in your analysis: $\omega = 8$ rad/s, $r = 0.2$ m, $L = 0.3$ m and $\theta = 36.87°$.



[pg. 153]

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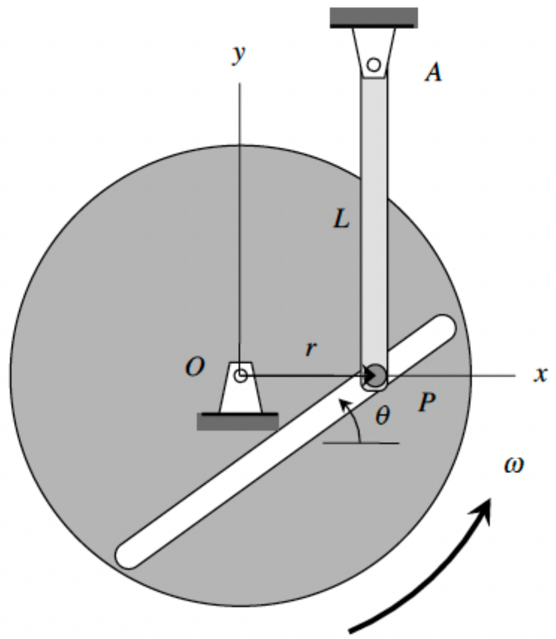
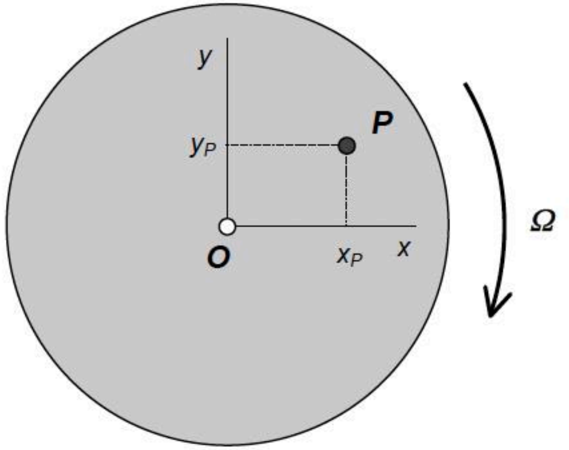


Question C3.1

Particle P moves on the top horizontal surface of a disk that is rotating in a clockwise sense about a vertical axis with a rate of $\Omega = 5$ rad/s. The position of P is described in terms of a set of Cartesian components x_P and y_P measured relative to the disk. When $(x_P, y_P) = (4, 3)$ ft, the velocity of P as seen by a stationary observer is: $\vec{v}_P = (10\hat{i} - 20\hat{j})$ ft/s. Describe the velocity of P as seen by an observer on the disk.



TOP VIEW

[pg. 176]

Question C3.1

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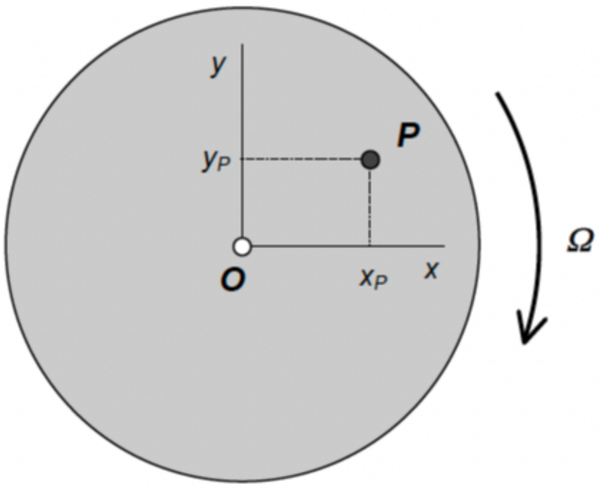
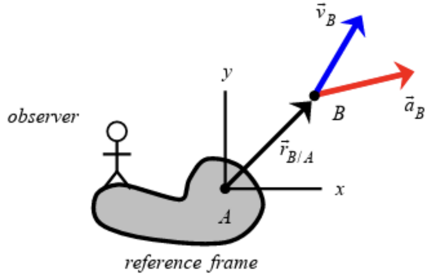


TOP VIEW

Summary: 2D Moving Reference Frame Kinematics 1

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point B.



Lec 11 Short Feedback Form:

[pg. 145]

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

[pg. 146]

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

where:

- $\vec{\omega}$ is the angular velocity *of the observer* (*no exceptions*).
- $\vec{\alpha}$ is the angular velocity *of the observer* (*no exceptions*).

[pg. 147]

- $\left(\vec{v}_{B/A}\right)_{rel}$ is the velocity of B *as seen by the observer* (*no exceptions*).
- $\left(\vec{a}_{B/A}\right)_{rel}$ is the acceleration of B *as seen by the observer* (*no exceptions*).

[pg. 148]

- A is *ANY* point on the same reference frame as the observer.
- Generally, *you* are free to choose your observer.