ME 274: Basic Mechanics II

Lecture 17: Particle Kinetics – Newton's Laws

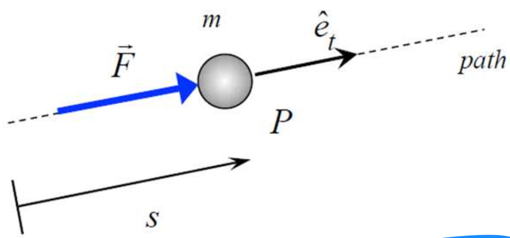
PURDUE UNIVERSITY | School of Mechanical Engineering

Kinetics: Four-Step Problem Solving Method

1) FBD
   - Draw appropriate FBD (note- you may have to draw more than 1)
   - Choose coordinate system (cartesian, path, polar)
2) Kinetics
   - Sum forces and break into components
   - Choose appropriate method to solve the problem – we will learn these in upcoming sections!
     - Newton/Euler
     - work/energy
     - linear impulse/momentum
     - angular impulse/momentum)
3) Kinematics
   - Perform a kinematic analysis of the system using techniques we have developed in the previous chapters.
   - Use your kinetics equations from step 2 to determine what information you need to solve the problem
4) Solve
   - Count the number of equations and unknowns. Do they match?
   - If not:
     - Draw more FBDs
     - Do additional kinematic analysis

Kinetics for the rectilinear Motion of Particles – Net force dependent on time

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$



$$\sum F_t = ma_t \quad \Rightarrow \quad F = m\frac{dv}{dt}$$

← already in terms of t

If force is a function of time: $F = F(t)$

$$F = m\frac{dv}{dt}$$

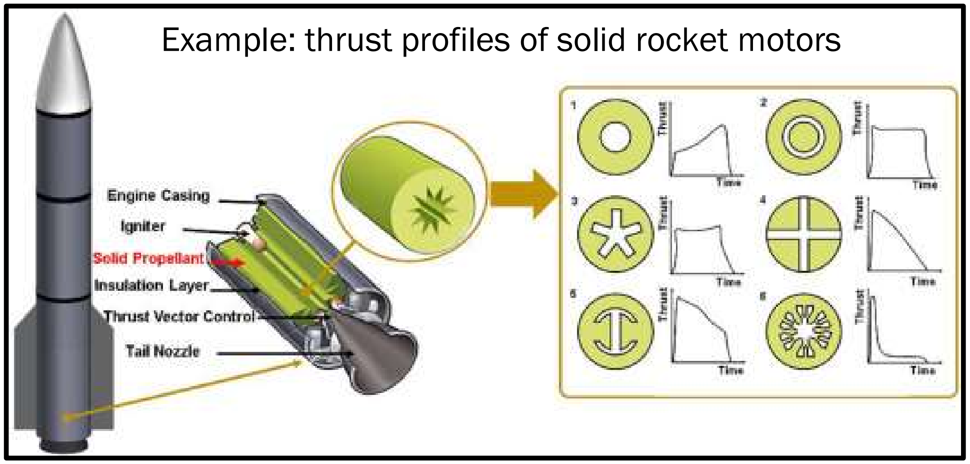
$$F(t)\,dt = m\,dv$$

$$\int_{t_1}^{t_2} F(t)\,dt = \int_{v_1}^{v_2} m\,dv$$

$$\int_{t_1}^{t_2} F(t)\,dt = m(v_2 - v_1)$$

impulse - momentum

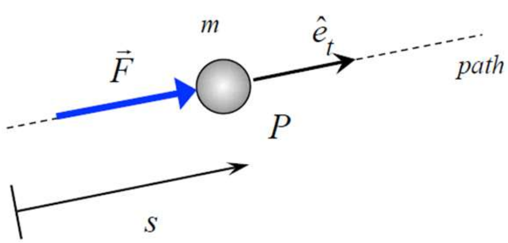
tells how speed changes with time



Example: thrust profiles of solid rocket motors

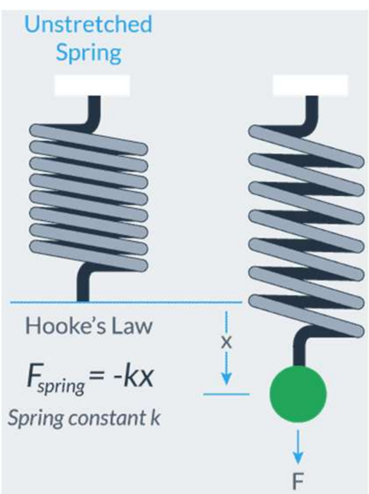
Kinetics for the rectilinear Motion of Particles – Net force dependent on **position**

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$

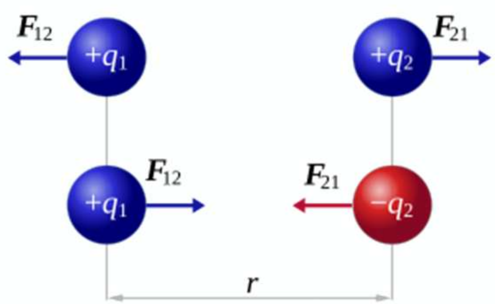


$$\sum F_t = ma_t \quad \Rightarrow \quad F = m\boxed{\frac{dv}{dt}}$$ ← need to write in terms of s

Ex: spring forces



Ex: magnetic forces



$$|F_{12}| = |F_{21}| = k_e \frac{|q_1 \times q_2|}{r^2}$$

If force is a function of position: $F = F(s)$

$F = m\frac{dv}{dt}$ → using chain rule

$= m\frac{dv}{ds}\frac{ds}{dt}$ ← velocity!

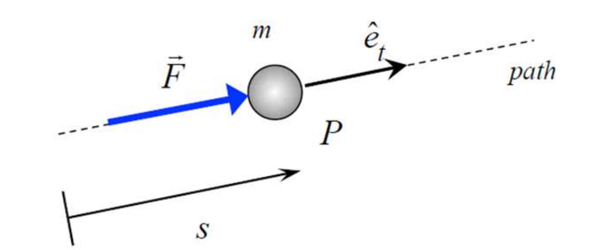
$F(s) = mv\frac{dv}{ds}$

$$\int_{s_1}^{s_2} F(s) = \int_{v_1}^{v_2} mv$$

$$\underbrace{\int_{s_1}^{s_2} F(s)}_{\text{work}} = \frac{mv^2}{2}\Big|_{v_1}^{v_2} = \underbrace{\frac{1}{2}m(v_2^2 - v_1^2)}_{\text{energy}}$$

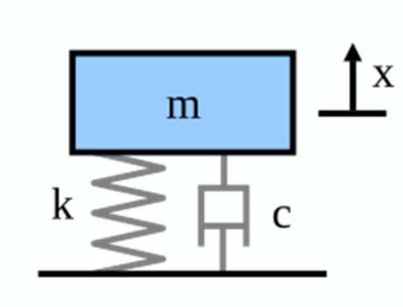
Kinetics for the rectilinear Motion of Particles – Net force dependent on **velocity**

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$



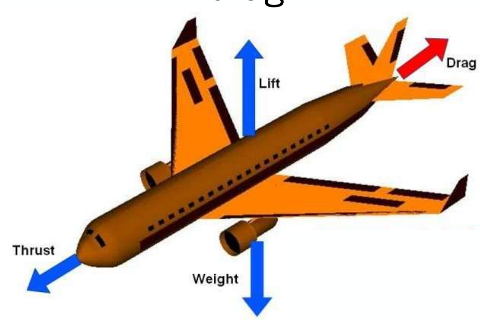
$$\sum F_t = ma_t \quad \Rightarrow \quad F = m\frac{dv}{dt}$$

Ex: dampers



$$F_{damper} = c\dot{x}$$

Ex: drag



$$F_D = \frac{1}{2}\rho v^2 C_D A$$

If force is a function of velocity: $F = F(v)$

$$F(v) = m\frac{dv}{dt}$$

$$\int_{t_1}^{t_2} dt = \int_{v_1}^{v_2} m\frac{dv}{F(v)}$$

velocity changing with time

$$(t_2 - t_1) = m\int_{v_1}^{v_2} F(v)$$

or

$$F(v) = mv\frac{dv}{ds}$$

$$\int_{s_1}^{s_2} ds = m\int_{v_1}^{v_2}\frac{v}{F(s)}dv$$

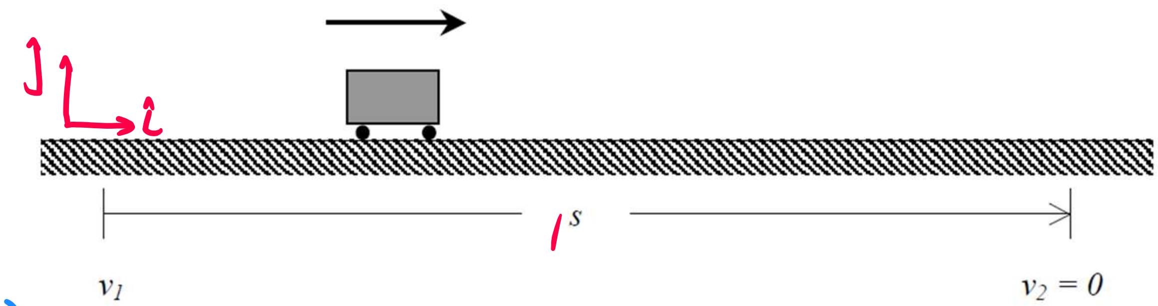
$$(s_2 - s_1) = m\int_{v_1}^{v_2}\frac{v}{F(s)}dv$$

Velocity changing with position

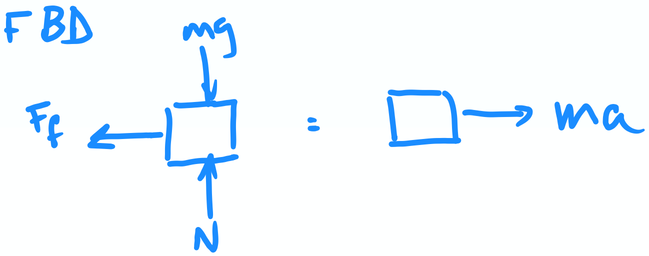
Example 4.A.1

Given: As a car of mass m brakes with a constant braking force F_f, the speed of the car drops from v_1 to a speed of $v_2 = 0$ in a distance of s and in a time t.

Find: Determine:
(a) The distance s; and
(b) The time t.



1) FBD



2) kinetics

$\Sigma F_x = -F_f = ma$

3) kinematics $\quad a = \frac{dv}{dt} = v\frac{dv}{ds}$

4) solve

b) time t to stop

$-F_f = ma = m\frac{dv}{dt}$

$\int_0^{t_2} -F_f \, dt = \int_{v_1}^{0} m \, dv$

$-F_f t \Big|_0^{t_2} = mv \Big|_{v_1}^{0}$

$-F_f t_2 = -mv_1$

$t_2 = \frac{mv_1}{F}$

a) distance s to stop

$-F_f = mv\frac{dv}{ds}$

$\int_0^{s_2} -F_f \, ds = \int_{v_1}^{0} mv \, dv$

$-F_f s \Big|_0^{s_2} = \frac{mv^2}{2}\Big|_{v_1}^{0}$

$-F_f s_2 = -\frac{mv_1^2}{2}$

$s_2 = F_f \frac{mv_1^2}{2}$

Example 4.A.2

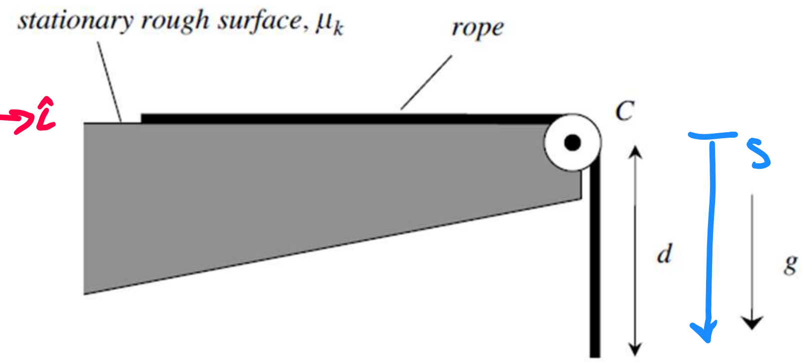
Given: A rope of length L and mass per length ρ is initially at rest on a rough, horizontal surface (with coefficient of kinetic friction μ_k) and with a portion of its length d hanging over a pulley on the right end of the resting surface. Ignore the mass of the pulley.

Find: Determine the speed of the rope when the left end of the rope reaches the pulley. Assume that the rope remains taut.

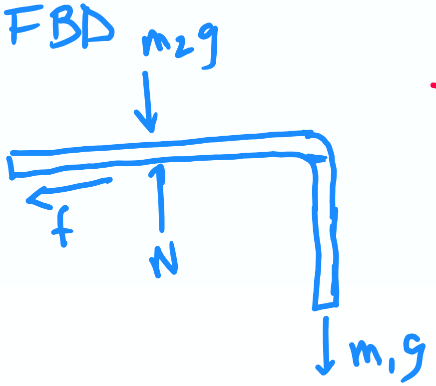
$\hookrightarrow v$ $\hookrightarrow s = L$

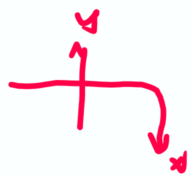
Use the following relationship in your analysis: $d = 0.2L$.

$\hat{j}$ $\hat{i}$



1) FBD





Kinematics

$m_1 = \rho s$

$m_2 = \rho(L-s)$

$m = \rho L$

$f = \mu_k N$

$\rightarrow$

$N = \rho(L-s)g$

$f = \mu_k \rho (L-s) g$

$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$

2) Kinetics

① $\sum F_x = -f + m_1 g = ma$

② $\sum F_y = N - m_2 g = 0$

Solve:

$$-\mu_k \cancel{\rho}(L-s)g + \cancel{\rho} s g = \cancel{\rho} L v \frac{dv}{ds}$$

$$g\int_{0.2L}^{L}\left[s(1+\mu_k) - \mu_k L\right]ds = L\int_{0}^{v_1} v\,dv$$

$$g\left[\frac{s^2}{2}(1+\mu_k) - \mu_k L s\right]_{0.2L}^{L} = L\frac{v^2}{2}\Big|_0^{v_1}$$

$$g\left[\frac{(L^2 - 0.04L^2)}{2}(1+\mu_k) - \mu_k(L^2 - 0.2L^2)\right] = \frac{L v_1^2}{2}$$

$$v_1 = \sqrt{2g\left[\frac{0.96L}{2}(1+\mu_k) - 0.8\mu_k L\right]}$$ simplify further