ME 274: Basic Mechanics II

Lecture 16: Particle Kinetics – Newton's Laws

PURDUE UNIVERSITY
School of Mechanical Engineering

In our study of kinematics, the focus has been on describing how bodies are moving

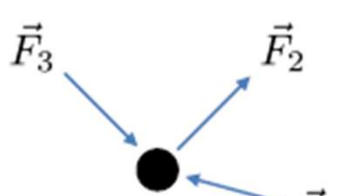
$$\textbf{Cartesian description}: \vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} \qquad \textbf{Path description}: \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\textbf{Polar description}: \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

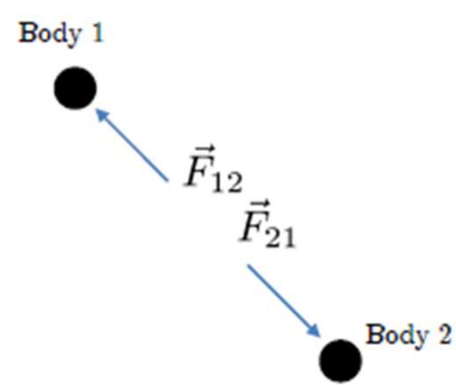
In our study of kinetics, we will now look at why bodies are moving – i.e. what external forces are acting on the system, and how they impact the motion we describe with our kinematic descriptions.

The fundamental principles that form the basis of our analysis are Newton's Laws of motion.

Newton's 2nd Law: The vector resultant of forces on a body equals the time rate of change of linear momentum.



Newton's 3rd Law: Two interacting bodies will exert equal and opposite forces on each other.



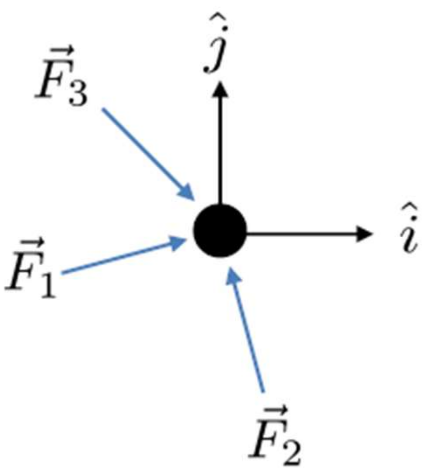
Newton's 2nd Law in 3 kinematic descriptions:

Cartesian Coordinates

$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

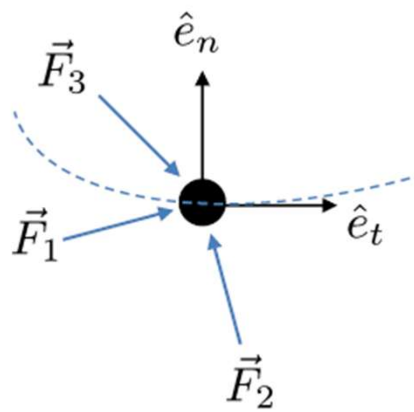


Path Coordinates

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

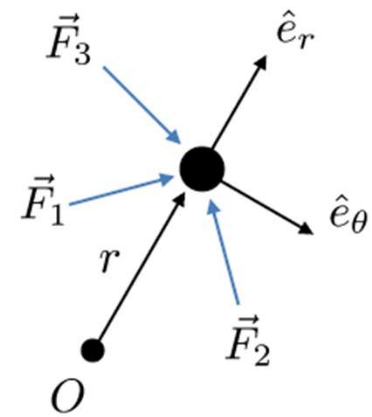


Polar Coordinates

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\,\dot{r}\dot{\theta})\hat{e}_t$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:



Kinetics: Four-Step Problem Solving Method

Suggested plan of action for solving kinetics problems:

1. ***Free body diagram(s)***. Draw the appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate system. For each FBD, draw the unit vectors corresponding to your coordinate choice.

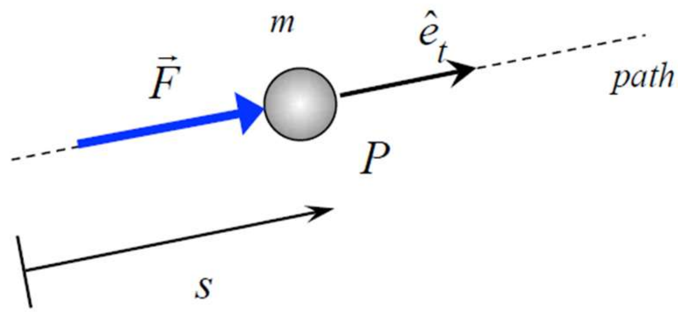
2. ***Kinetics equations***. At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.

3. ***Kinematics***. Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed for a solution of the problem.

4. ***Solve***. Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) you need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Kinetics for the rectilinear Motion of Particles – Net force dependent on time

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$



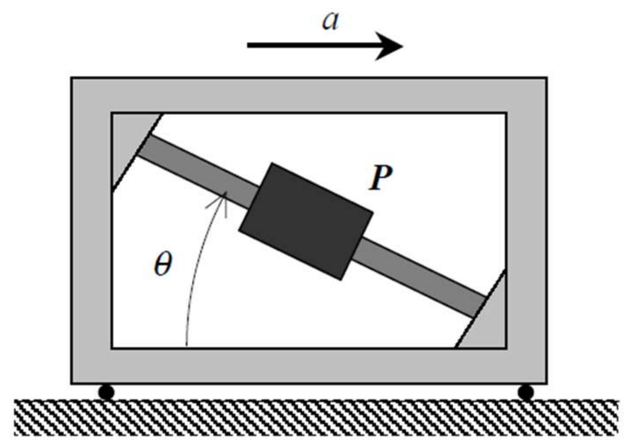
$$\sum F_t = ma_t \qquad \Rightarrow \qquad F = m\frac{dv}{dt}$$

If force is a function of time: $F = F(t)$

Example 4.A.4

Given: A collar P of mass m is free to slide along a smooth rod that is mounted at angle of $\theta = 36.87°$ in a frame. The frame is constrained to move along a horizontal surface, as shown, with a constant acceleration of a.

Find: Determine the value of a that is required for the collar to not slide along the rod as the frame accelerates to the right.



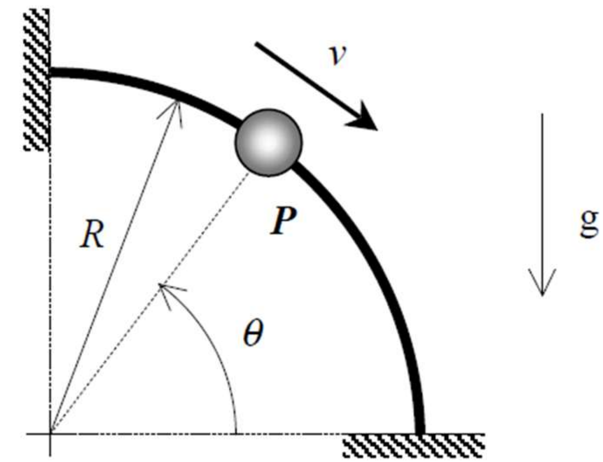
Example 4.A.7

Given: Particle P, having a weight of $W = 5$ lb, slides along a smooth, curved rod where $R = 3$ ft. At the position where $\theta = 53.13°$, the speed of P is known to be 20 ft/s.

Find: Determine:

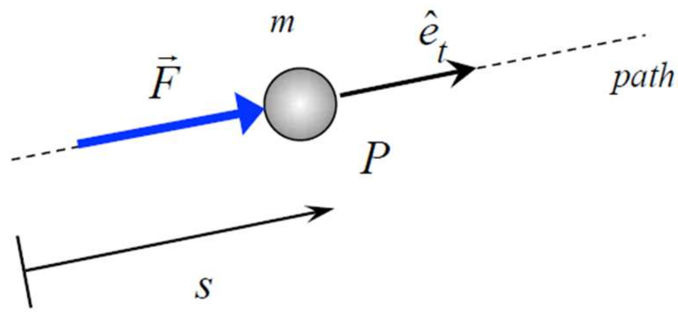
(a) The normal force acting on P by the rod at the instant shown; and

(b) The rate of change of speed of P at the instant shown.



Kinetics for the rectilinear Motion of Particles – Net force dependent on position

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$

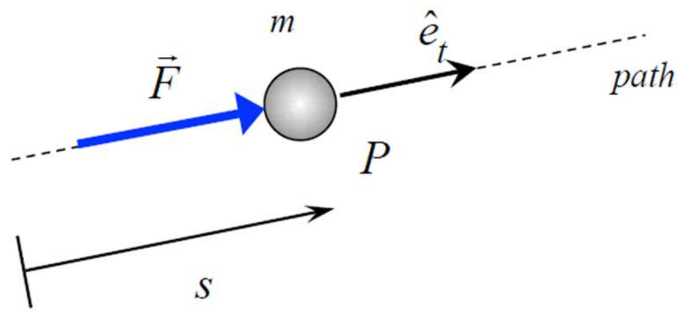


$$\sum F_t = ma_t \qquad \Rightarrow \qquad F = m\frac{dv}{dt}$$

If force is a function of position: $F = F(s)$

Kinetics for the rectilinear Motion of Particles – Net force dependent on velocity

For a particle of mass m traveling along a straight path and experiencing a force $\vec{F} = F\hat{e}_t$



$$\sum F_t = ma_t \qquad \Rightarrow \qquad F = m\frac{dv}{dt}$$

If force is a function of velocity: $F = F(v)$

Example 4.A.1

Given: As a car of mass m brakes with a constant braking force F_f, the speed of the car drops from v_1 to a speed of $v_2 = 0$ in a distance of s and in a time t.

Find: Determine:

(a) The distance s; and

(b) The time t.