

# ***ME 274: Basic Mechanics II***

Lecture 16: Particle Kinetics – Newton's Laws



School of Mechanical Engineering

In our study of kinematics, the focus has been on describing how bodies are moving

*Cartesian description* :  $\underline{\vec{a}} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$

*Path description* :  $\underline{\vec{a}} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$

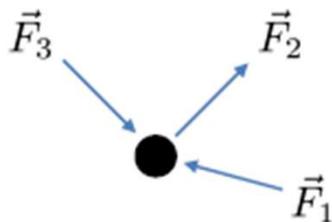
*Polar description* :  $\underline{\vec{a}} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

In our study of kinetics, we will now look at why bodies are moving – i.e. what external forces are acting on the system, and how they impact the motion we describe with our kinematic descriptions.

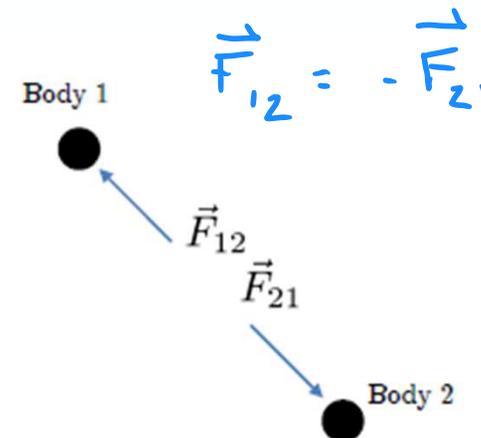
The fundamental principles that form the basis of our analysis are Newton's Laws of motion.

**Newton's 2<sup>nd</sup> Law:** The vector resultant of forces on a body equals the time rate of change of linear momentum.  $= m\vec{v}$

$$\sum \vec{F} = m\vec{a}$$



**Newton's 3rd Law:** Two interacting bodies will exert equal and opposite forces on each other.



# Newton's 2<sup>nd</sup> Law in 3 kinematic descriptions:

## Cartesian Coordinates

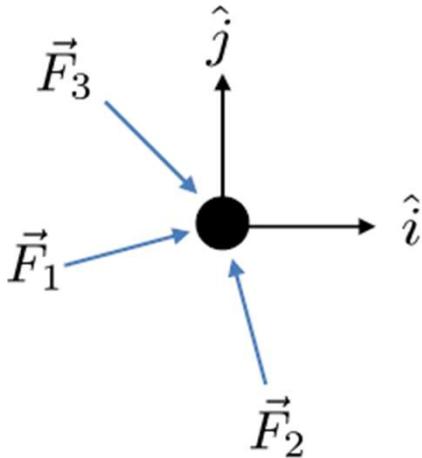
$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$\text{Sum forces: } \Sigma \vec{F} = m \vec{a}$$

Resolve into Components:

$$\Sigma F_x = m \ddot{x}$$

$$\Sigma F_y = m \ddot{y} = m \ddot{j}$$



## Path Coordinates

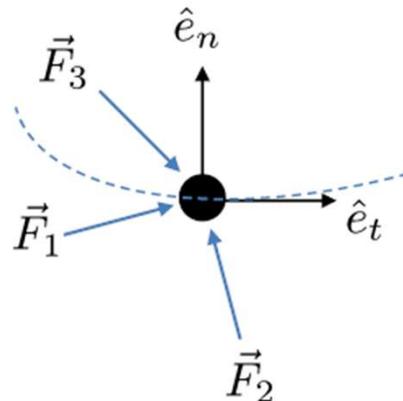
$$\rightarrow \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\text{Sum forces: } \Sigma \vec{F} = m \vec{a}$$

Resolve into Components:

$$\Sigma F_t = m a_t = m \dot{v}$$

$$\Sigma F_n = m a_n = m \frac{v^2}{\rho}$$



## Polar Coordinates

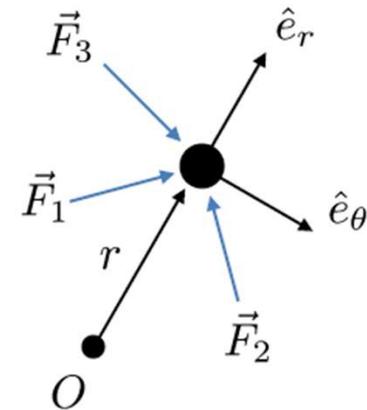
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$\text{Sum forces: } \Sigma \vec{F} = m \vec{a}$$

Resolve into Components:

$$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$



## Kinetics: Four-Step Problem Solving Method

Suggested plan of action for solving kinetics problems:

1. *Free body diagram(s)*. Draw the appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate system. For each FBD, draw the unit vectors corresponding to your coordinate choice.
2. *Kinetics equations*. At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.  $F = m\vec{a}$
3. *Kinematics*. Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed for a solution of the problem.
4. *Solve*. Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) you need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

ODE, system of eqn.

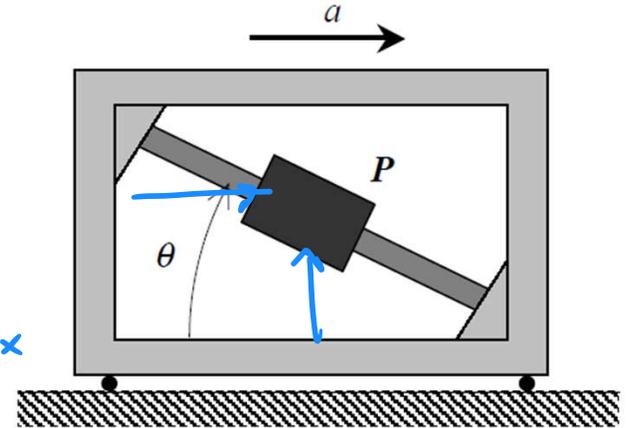
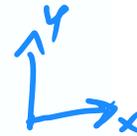
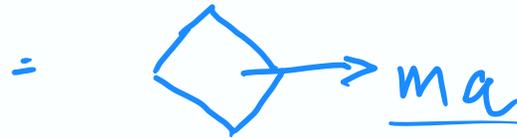
### Example 4.A.4 $x y$ - coordinates

→ no friction

**Given:** A collar P of mass  $m$  is free to slide along a smooth rod that is mounted at angle of  $\theta = 36.87^\circ$  in a frame. The frame is constrained to move along a horizontal surface, as shown, with a constant acceleration of  $a$ .

**Find:** Determine the value of  $a$  that is required for the collar to not slide along the rod as the frame accelerates to the right.

1) FBD



$$\textcircled{1} \sum F_x = ma = N \sin \theta$$

$$\textcircled{2} \sum F_y = 0 = N \cos \theta - mg \quad \rightarrow \quad N = \frac{mg}{\cos \theta}$$

$$ma = \frac{mg \sin \theta}{\cos \theta}$$

$$a = g \tan \theta$$

Example 4.A.7 path coord: nates

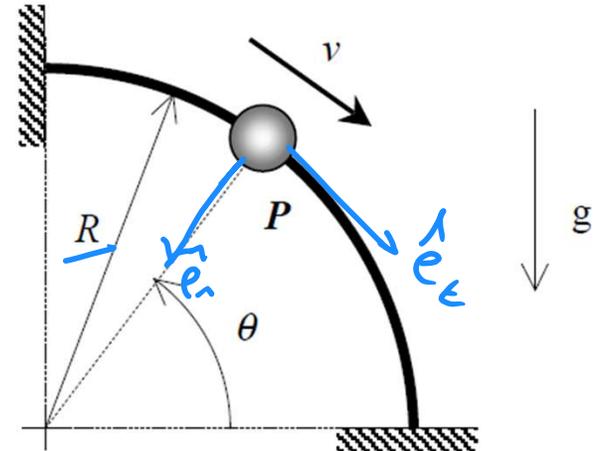
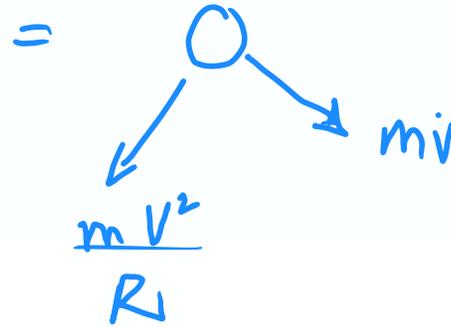
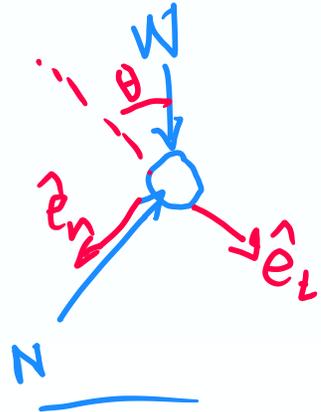
no friction

Given: Particle P, having a weight of  $W = 5$  lb, slides along a smooth, curved rod where  $R = 3$  ft. At the position where  $\theta = 53.13^\circ$ , the speed of P is known to be 20 ft/s.

Find: Determine:

- The normal force acting on P by the rod at the instant shown; and
- The rate of change of speed of P at the instant shown.

1) FBD



$$m = \frac{W}{g}$$

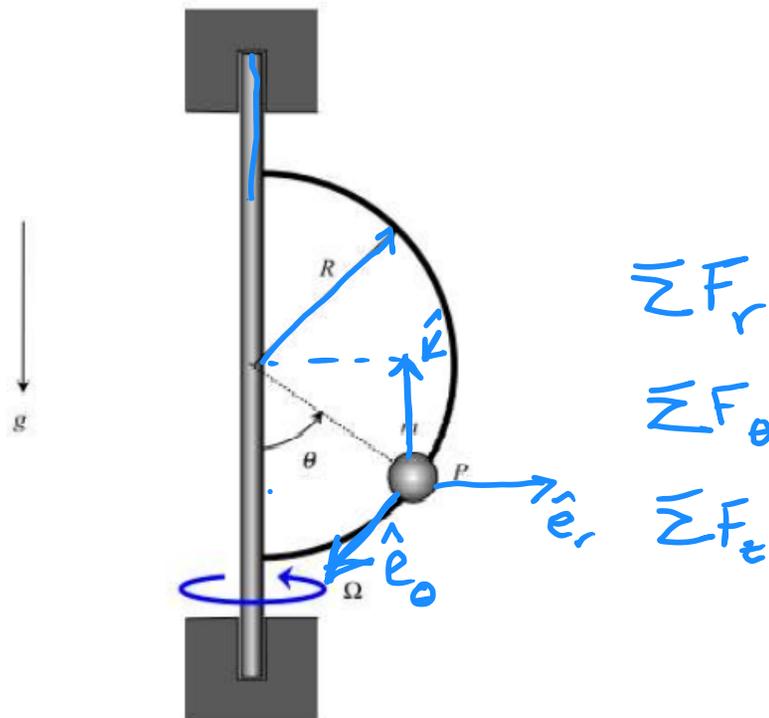
$$\textcircled{1} \sum F_t = W \cos \theta = m \dot{v} \quad \rightarrow \quad \dot{v} = \frac{W \cos \theta}{m} = \dot{v} = g \cos \theta$$

$$\textcircled{2} \sum F_n = W \sin \theta - N = \frac{m v^2}{R} \quad \vec{N} = \left( W \sin \theta - \frac{W v^2}{g R} \right) \hat{e}_n$$

**Homework 4.A.17**

**Given:** A rigid semi-circular guide of radius  $R$  is attached to a vertical shaft, with the shaft rotating with a constant rate of  $\Omega$ . Particle P (of mass  $m$ ) is able to slide on the guide.

**Find:** If the guide is smooth, determine the angle  $\theta$  at which the particle will not slide on the guide.



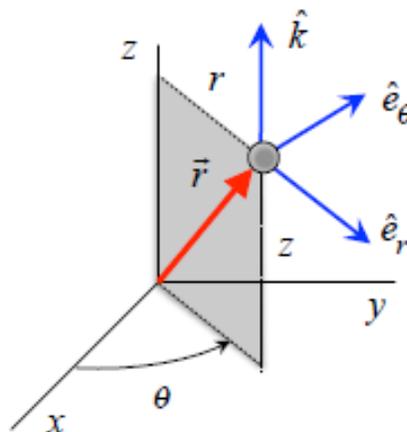
Please leave your answer in terms of, at most,  $g$ ,  $m$ ,  $R$  and  $\Omega$ .

## Cylindrical Kinematics

For the cylindrical description, the following set of unit vectors will be used:

- unit vector  $\hat{k}$  is aligned with the  $z$ -axis and is stationary
- unit vector  $\hat{e}_r$  is perpendicular to  $\hat{k}$  and is aligned with the projection of the position vector  $\vec{r}$  onto the  $xy$  plane (see the figure below)
- unit vector  $\hat{e}_\theta$  is found from  $\hat{e}_\theta = \hat{k} \times \hat{e}_r$ .

$r$  is the radial distance from the  $z$  axis to the particle.  $\theta$  is measured from the positive  $x$ -axis.



The projection of the  $\vec{r}$  vector onto the  $xy$  plane from above shows the same view as for the 2D polar description discussed earlier. From that work we saw that:

$$\frac{d\hat{e}_r}{dt} = \dot{\theta}\hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{dt} = -\dot{\theta}\hat{e}_r$$

In 3D, we write the position vector of P as:

$$\vec{r} = r\hat{e}_r + z\hat{k}$$

From this, the velocity of the particle is:

$$\vec{v} = \frac{d}{dt}(r\hat{e}_r) + \dot{z}\hat{k} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k}$$

and the acceleration is:

$$\vec{a} = \frac{d}{dt}(\dot{r}\hat{e}_r) + \frac{d}{dt}(r\dot{\theta}\hat{e}_\theta) + \ddot{z}\hat{k} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{\text{radial}} \hat{e}_r + \underbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})}_{\text{tangential}} \hat{e}_\theta + \underbrace{\ddot{z}}_{\text{axial}} \hat{k}$$

From this, we see that the velocity and acceleration equations for the cylindrical description are the same as those for the polar description with the addition of  $\dot{z}\hat{k}$  and  $\ddot{z}\hat{k}$ , respectively.