ME 274: Basic Mechanics II

Lecture 17: Particle Kinetics – Newton's Laws

PURDUE UNIVERSITY | School of Mechanical Engineering

Announcements:

- Hw 4.A, 4.B extended to Tuesday at 11:59 PM!
- Thursday office hours canceled this week – see me by appointment if needed!

Newton's 2nd Law in 3 kinematic descriptions:

Cartesian Coordinates

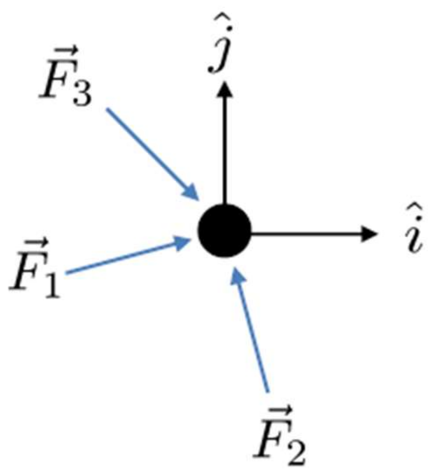
$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\sum F_x = m\ddot{x}$$

$$\sum F_y = m\ddot{y}$$



Path Coordinates

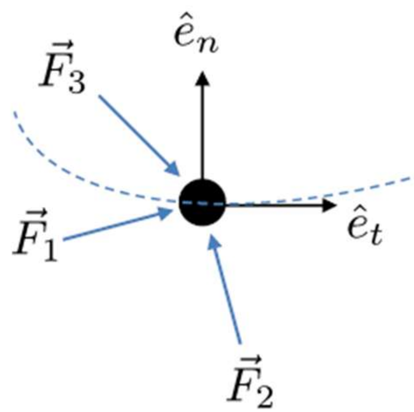
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\sum F_t = m\dot{v}$$

$$\sum F_n = m\frac{v^2}{\rho}$$



Polar Coordinates

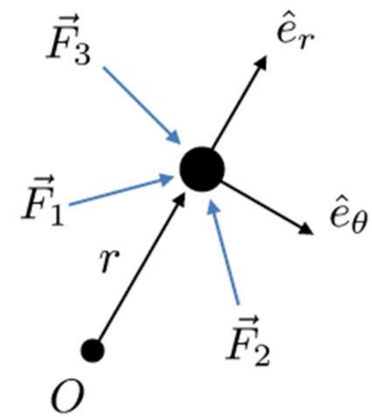
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\,\dot{r}\dot{\theta})\hat{e}_\theta$$

Sum forces: $\sum \vec{\boldsymbol{F}} = \boldsymbol{m}\,\vec{\boldsymbol{a}}$

Resolve into Components:

$$\sum F_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\sum F_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$



Kinetics: Four-Step Problem Solving Method

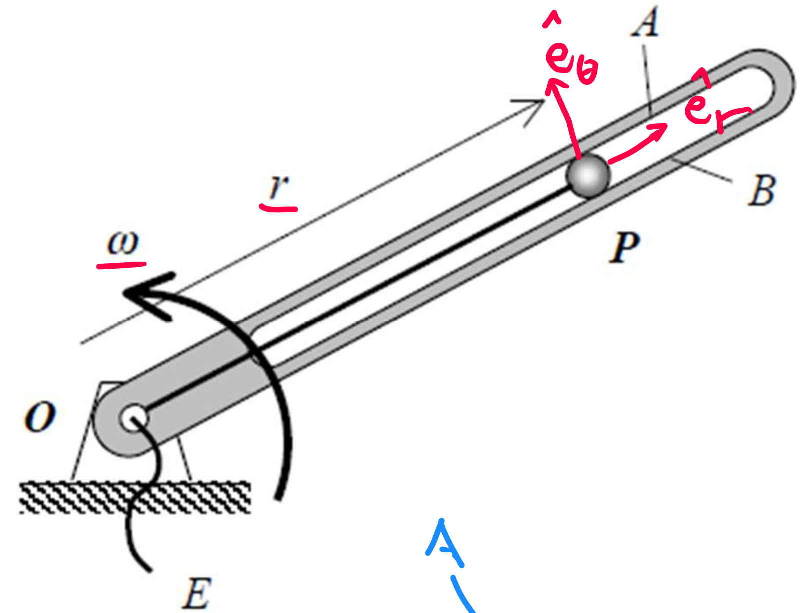
1) FBD
    - Draw appropriate FBD (note- you may have to draw more than 1)
    - Choose coordinate system (cartesian, path, polar)
2) Kinetics
    - Sum forces and break into components
    - Choose appropriate method to solve the problem – we will learn these in upcoming sections!
        - Newton/Euler
        - work/energy
        - linear impulse/momentum
        - angular impulse/momentum)
3) Kinematics
    - Perform a kinematic analysis of the system using techniques we have developed in the previous chapters.
    - Use your kinetics equations from step 2 to determine what information you need to solve the problem
4) Solve
    - Count the number of equations and unknowns. Do they match?
    - If not:
        - Draw more FBDs
        - Do additional kinematic analysis

Example 4.A.10

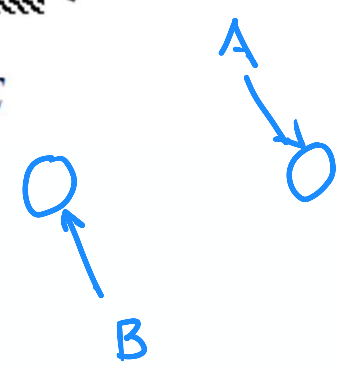
Given: Particle P (weighing W) is able to slide within a straight slot cut into an arm. The arm is rotating within a horizontal plane about end O at a constant rate of ω. The slider is being pulled toward O at a constant rate of $\dot{r}$.

Find: Determine:
- (a) The tension in the cord;
- (b) The normal contact force of the slot on P; and
- (c) Which side of the slot (A or B) is P in contact with?

Use the following parameters in your analysis:
W = 5 lb, ω = 5 rad/s, r = 0.75 ft and $\dot{r}$ = -0.5 ft/s.



1) FBD



2) Kinetics

$$\Sigma F_r = -T = m a_r$$

$$\Sigma F_\theta = N = m a_\theta$$

3) kinematics

polar form : $\vec{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r}\hat{e}_r + \underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})}_{a_\theta}\hat{e}_\theta$

① $-T = m(\ddot{r} - r\dot{\theta}^2)$

② $N = m(2\dot{r}\dot{\theta} + \underline{r\ddot{\theta}})$

4) solving → 2 eqn, 2 unknown

$r = 0.75\,ft$ $\qquad \dot{\theta} = \omega = const.$ $\qquad m = \frac{W}{g}$

$\dot{r} = \underline{-0.5 ft} = const.$ $\qquad \ddot{\theta} = 0$

$\ddot{r} = 0$

$$\boxed{\begin{aligned} T &= -\frac{W}{g}(r\omega^2) \\ N &= \frac{W}{g}(2\dot{r}\omega) \end{aligned}}$$

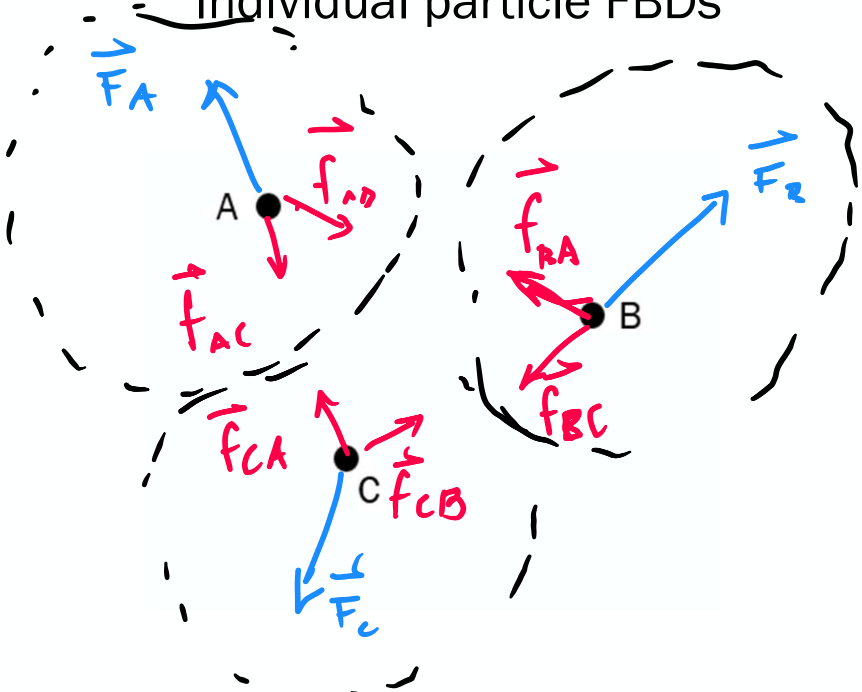
→ $N < 0$ → switch direction from FBD → side A

So far we have seen problems including a single particle where $\sum \boldsymbol{F}_{particle} = \boldsymbol{m}_{particle}\boldsymbol{a}_{particle}$

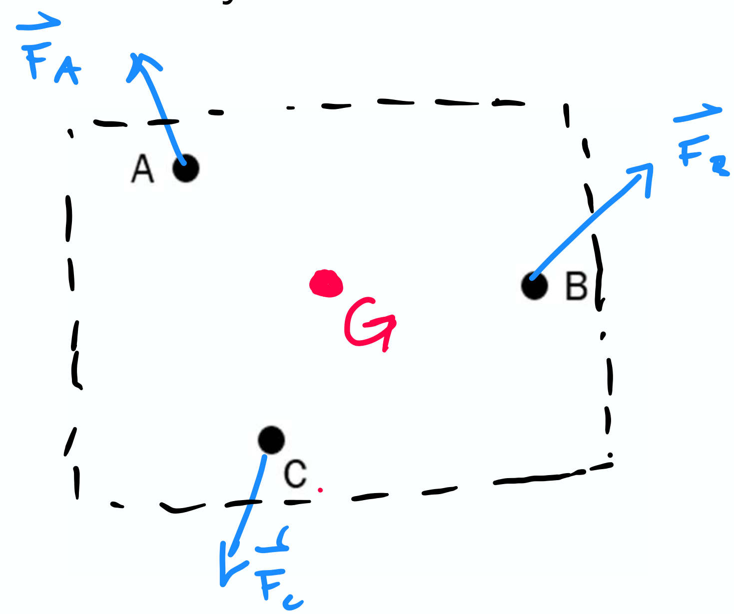
How does our kinetic analysis change when we are analyzing a system with multiple interacting bodies?

Consider these 3 interacting particles:

Individual particle FBDs



System FBDs



$$m_A \vec{a}_A = \vec{F}_A + \vec{f}_{AB} + \vec{f}_{AC}$$

$$m_B \vec{a}_B = \vec{F}_B + \vec{f}_{BA} + \vec{f}_{BC}$$

$$m_C \vec{a}_C = \vec{F}_C + \vec{f}_{CA} + \vec{f}_{CB}$$

$$\vec{m}_A \vec{a}_A + \vec{m}_B \vec{a}_B + \vec{m}_C \vec{a}_C$$

$$= \vec{F}_A + \cancel{\vec{f}_{AB}} + \cancel{\vec{f}_{AC}} + \vec{F}_B + \cancel{\vec{f}_{BA}} + \cancel{\vec{f}_{BC}}$$

$$+ \vec{F}_C + \cancel{\vec{f}_{CA}} + \cancel{\vec{f}_{CB}}$$

$$\vec{f}_{AB} = -\vec{f}_{BA} \ldots$$

$$\vec{m}_A\vec{a}_A + \vec{m}_B\vec{a}_B + \vec{m}_C\vec{a}_C = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

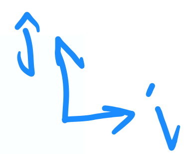
$$\sum_j m_j \vec{a}_j = \sum_j \vec{F}_j^{ext}$$

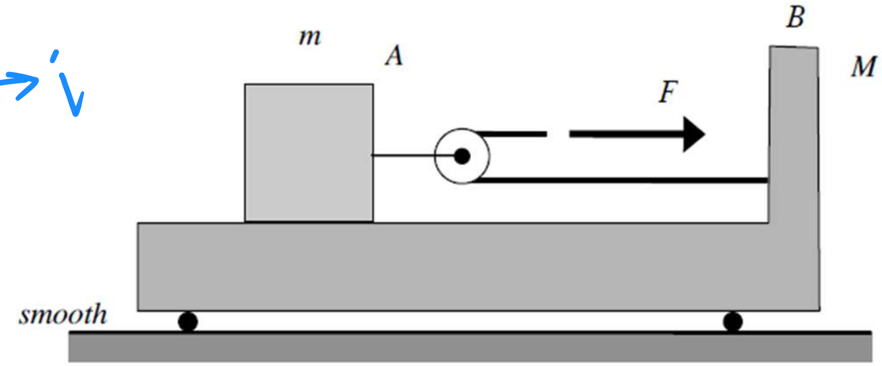
$$M\vec{a}_G = \sum F^{ext}$$

Example 4.A.6 → pulley

Given: The system of blocks A and B shown below is initially at rest when a force F is applied to the free end of the cable.

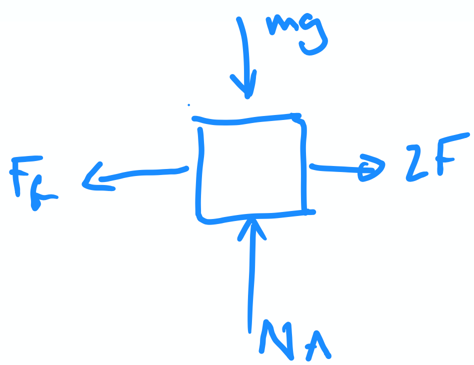
Find: Determine the accelerations of blocks A and B: (a) if the surface between A and B is smooth, and (b) if the surface between A and B is rough with a coefficient of kinetic friction being μ_k, and where it is known that block A slips on block B.



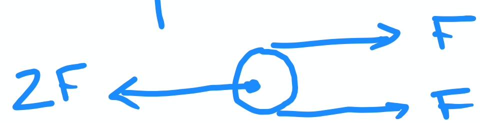


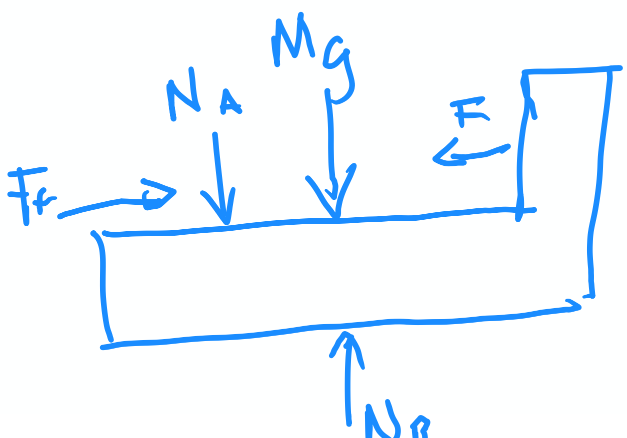
1) FBD

Block A



pulley





2) Kinetics

① $\sum F_{A,x} = 2F - F_f = m a_{Ax}$ $\qquad \sum F_{Ay} = 0 = N_A - mg$

② $\sum F_{B,x} = F_f - F = M a_{Bx}$ $\qquad \sum F_{By} = 0$

3) Kinematics $\quad a_x = \ddot{x} \quad a_y = 0$

4) solve

a) smooth $\rightarrow F_f = 0$

① $2F = m a_{Ax} \Rightarrow a_{Ax} = \frac{2F}{m}$

② $-F = M a_{Bx} \rightarrow a_{Bx} = -\frac{F}{M}$

b) rough $\rightarrow F_f = \mu_k N_A \qquad N_A = mg$

① $2F - \mu_k mg = m a_x \rightarrow a_A = \frac{2F - \mu_k mg}{m} \hat{i}$

② $\mu_k mg - F = M a_B \Rightarrow a_B = \frac{\mu_k mg - F}{M} \hat{i}$

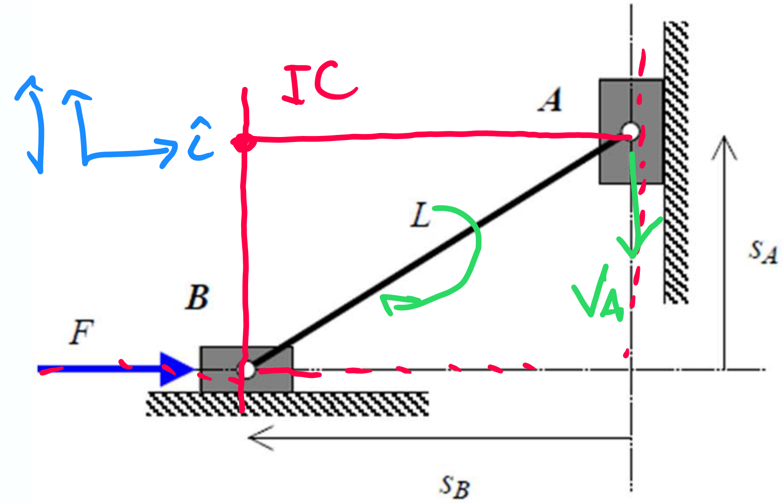
Example 4.A.8 rigid 2Force Member

Given: Blocks A and B (having masses of 10 kg and 5 kg, respectively) are constrained to move along smooth, vertical and horizontal guides, as shown in the figure. A and B are connected by a lightweight rod of length $L = 2.5$ m. A force of $F = 50$ N acts to the right on block B. At the position where $s_A = 1.5$ m, A is moving downward with a speed of 4 m/s.

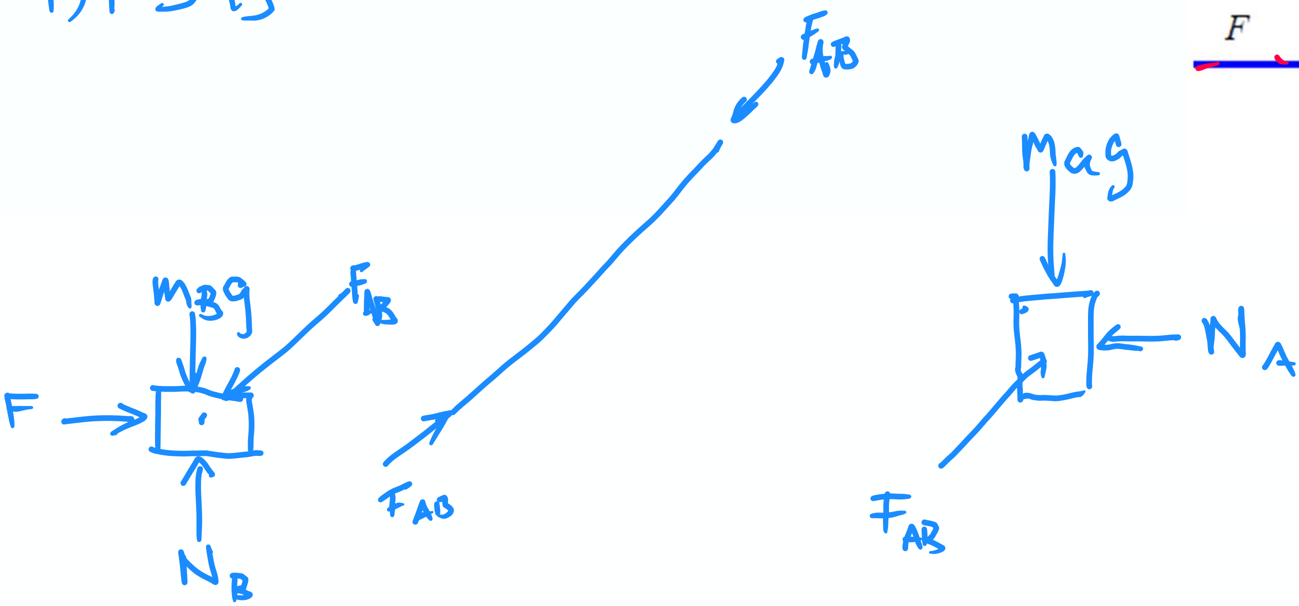
smooth = no friction

Find: Determine:

(a) The acceleration of A and B at this instant; and
(b) The force in rod AB at this instant.



1) FBD



Kinetics

Block B

$$\sum F_{x,B} = F - F_{AB}\frac{S_B}{L} = m_b a_{bx} \qquad (1)$$

$$\sum F_{y,B} = N_B - m_B g - F_{AB}\frac{S_A}{L} = \cancel{m_b a_{by}} \qquad (2)$$

Block A

$$\sum F_{x,A} = F_{AB}\frac{S_B}{L} - N_A = \cancel{m_A a_{Ax}} \qquad (3)$$

$$\sum F_{y,A} = F_{AB}\frac{S_A}{L} - m_A g = m_a a_{Ay} \qquad (4)$$

Kinematics

$a_{Ax} = 0$, $a_{By} = 0$

rigid body eqns → Link AB

relate $\underline{V_A}$, ω_{AB} using IC ↑ see above
Given

$$\omega_{AB} = \frac{V_A}{|r_{AC}|} = \frac{V_A}{S_B}$$

relate acceleration of pts A & B

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times \vec{r}_{A/B} - \omega^2 r_{AB}$$

$$a_A \hat{j} = a_B \hat{i} + \alpha_{AB} \hat{k} \times (S_B \hat{i} + S_A \hat{j}) - \left(\frac{V_A}{S_B}\right)^2 (S_B \hat{i} + S_A \hat{j})$$

$$\hat{i}: \quad 0 = a_B - S_A \alpha_{AB} - \frac{V_A^2}{S_B}$$

$$\hat{j}: \quad a_A = \alpha_{AB} S_B - \left(\frac{V_A}{S_B}\right)^2 S_A$$

Treating AB as an inextensible cable :-

$$L^2 = s_A^2 + s_B^2$$

$$\frac{d}{dt}(L^2) = 2s_A\dot{s}_A + 2s_B\dot{s}_B = 0 \rightarrow \dot{s}_B = -\frac{s_A}{s_B}\dot{s}_A$$

$$\frac{d}{dt}(s_A\dot{s}_A + s_B\dot{s}_B) = s_A\ddot{s}_A + \dot{s}_A^2 + s_B\ddot{s}_B + \dot{s}_B^2$$

$$\ddot{s}_A = -\frac{(\dot{s}_A^2 + \dot{s}_B^2 + s_B\ddot{s}_B)}{s_A} \quad 5$$

$$\ddot{s}_A = a_A, \quad \ddot{s}_B = a_B$$

5 eqn, 5 unknown

→ Solve !