ME 274: Basic Mechanics II

Lecture 12: Moving Reference Frame Kinematics

PURDUE UNIVERSITY | School of Mechanical Engineering

Announcements

Exam 1 info:

- Exam 1, Thurs. 2/12, 8 – 9:30, MA 175
- Students with exam accommodations: HIKS G980D, 6:30PM – reminder to email Dr. Krousgrill
- Exam review session Tuesday 2/10, 7pm over Zoom (link on course website on “Exams” page)

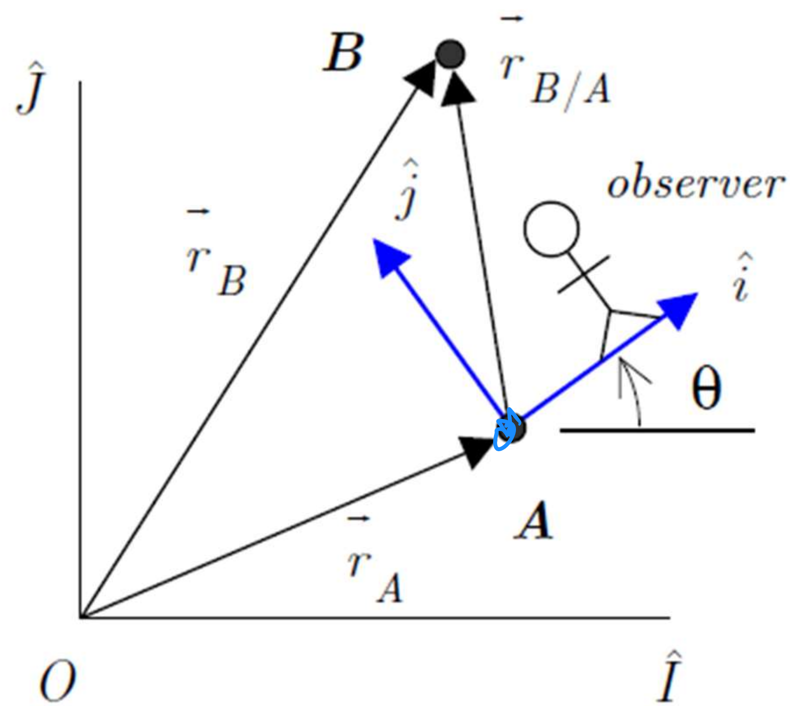
Homework and class updates:

- No class Friday, 2/13 to account for evening exam
- HW released today (2.C, 2.D) **due Friday 2/13**
- HW released on Wednesday (2.E, 2.F) **due Monday 2/16**
- **Reminder to use correct HW formatting!**
- Office hrs: T 9-10:30, W 3:30-4:20, Th 3-4:30

Moving Reference Frames – Overview

- Stationary axis ⟶ XYZ axis ⟶ point B as seen by an outside observer
- Moving reference frame ⟶ xyz axis ⟶point B as seen from an observer at point A
- The angular speed of the observer is given by the time rate of change of the angle θ between the fixed and moving reference frame ⟶$\vec{\omega} = \dot{\theta}\hat{k}$

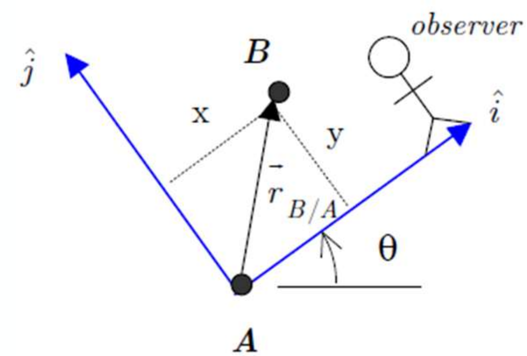
$\vec{\alpha} = \ddot{\theta}\hat{k}$



Relating velocity and acceleration of pts. B and A :

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + \boxed{2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel}} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right]$$



Where $\left(\vec{v}_{B/A}\right)_{rel}$,$\left(\vec{a}_{B/A}\right)_{rel}$ are the velocity and acceleration of point B as observed from A.

$(\vec{v}_{B/A})_{rel} = \dot{x}\hat{i} + \dot{y}\hat{j}$

Remember pt A must be on the same reference frame as the observer!

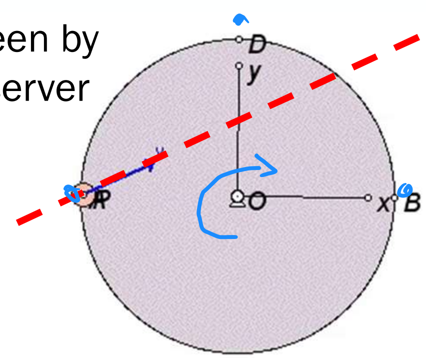
$(\vec{a}_{B/A})_{rel} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$

The Coriolis Component of Acceleration

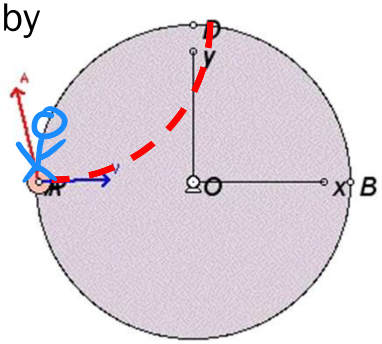
- The Coriolis component arises when a point has a nonzero velocity relative to a rotating reference frame.
- Although the point may be moving in a straight line with constant velocity in an inertial frame, the rotating observer sees the direction of that relative velocity continuously change as the reference axes rotate.

Ex 1: Merry-go-round

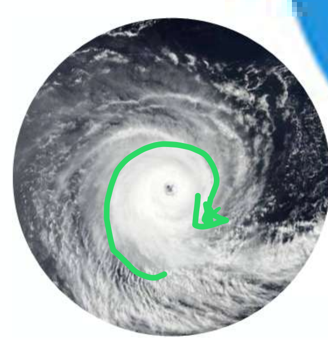
Motion of P seen by stationary observer

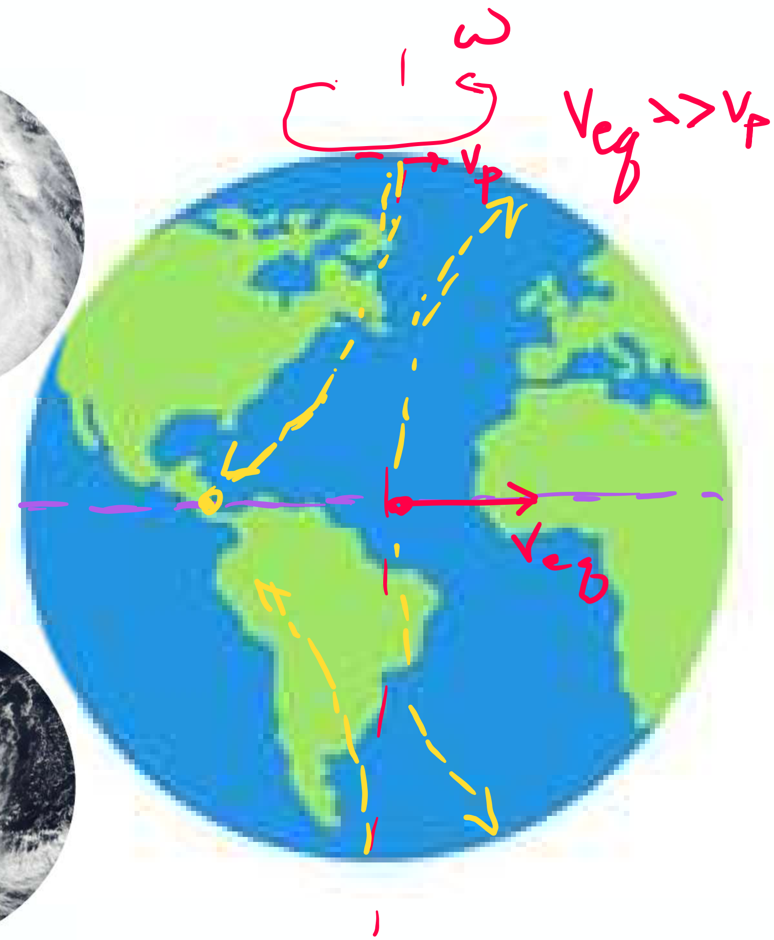


Motion of P seen by moving observer



Ex 2: Weather Systems





Question C3.3

Sprinkler arm OA is pinned to a cart at point O. The cart moves to the right with a speed of v_{cart} with $\dot{v}_{cart} = 2$ ft/s^2 $= constant$. Fluid flows through the sprinkler arm at a rate of $\dot{d}$ with $\ddot{d} = -3$ ft/s^2 $= constant$. The sprinkler arm is being raised at a constant rate of $\dot{\theta} = 4$ rad/s. An observer and xyz coordinate system are attached to the sprinkler arm, as shown in the figure below. The following equation is to be used to find the acceleration of a pellet P that flows with the fluid in the arm:

$$\vec{a}_P = \vec{a}_O + \left(\vec{a}_{P/O}\right)_{rel} + \vec{\alpha} \times \vec{r}_{P/O} + 2\vec{\omega} \times \left(\vec{v}_{P/O}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{P/O}\right]$$

Provide numerical values for the following terms when: $d = 3$ ft, $v_{cart} = 3$ ft/s, $\dot{d} = 5$ ft/s and $\theta = 90°$.

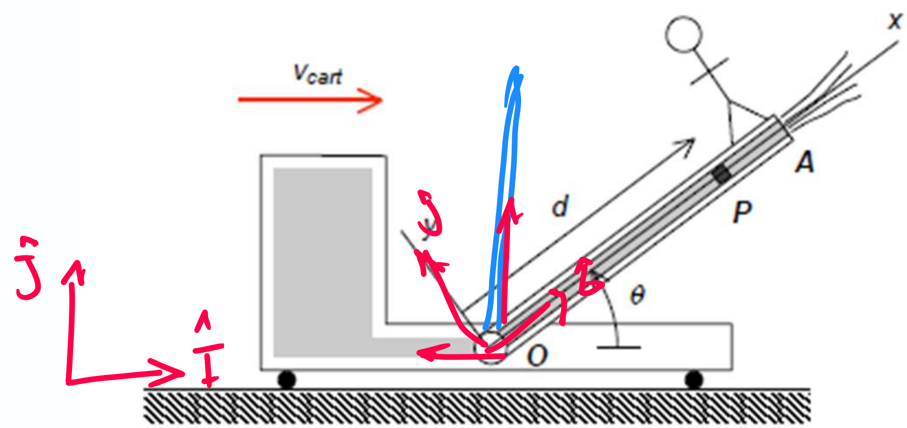
$\vec{a}_O = 2\hat{I} \rightarrow -2\hat{j}$

$\vec{\omega} = 4\hat{k}$

$\vec{\alpha} = \vec{0}$

$\left(\vec{v}_{P/O}\right)_{rel} = \dot{d}\hat{i} = 5\hat{i}$

$\left(\vec{a}_{P/O}\right)_{rel} = -3\hat{i}$

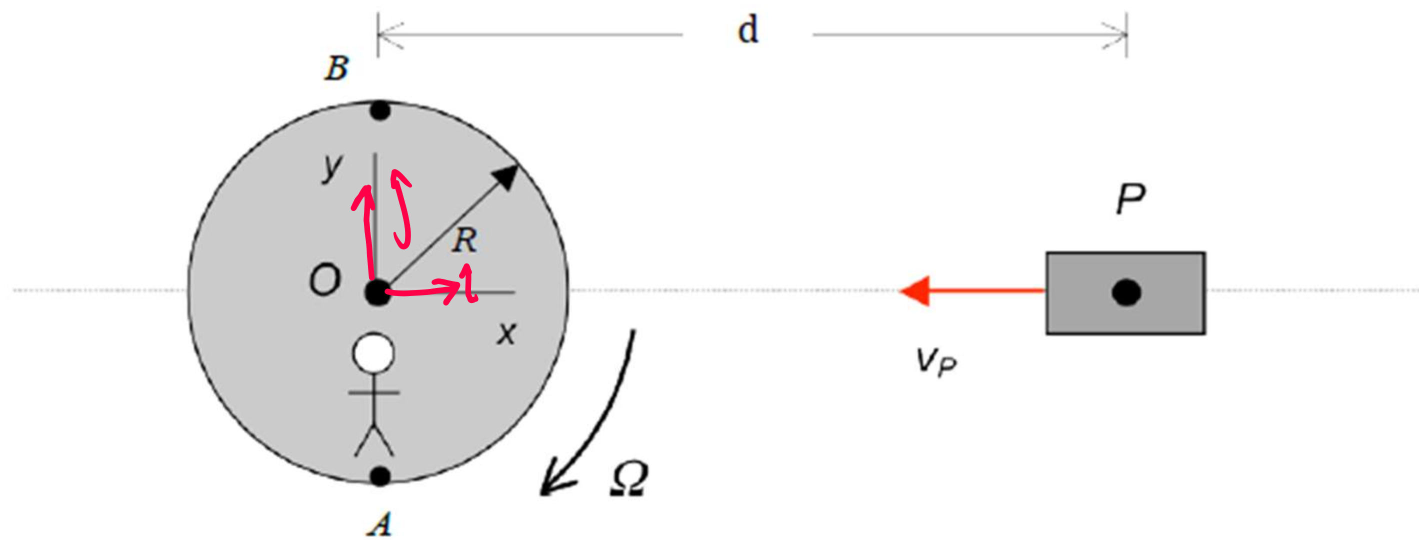


$\hat{i} = \hat{J}$ $\quad$ $\hat{j} = -\hat{I}$

Question C3.2

A disk is pinned to ground at its center O with the disk rotating clockwise at a constant rate of $\Omega = 3$ rad/s. Block P is traveling to the left along a straight path toward O with a constant speed of $v_P = 20$ ft/s. Determine the acceleration of P as seen by an observer on the disk when P is at a distance of 50 ft from O.

↳ $(a_{p/o})_{rel}$



$$\vec{a}_P = \vec{a}_O + \left(\vec{a}_{P/O}\right)_{rel} + \vec{\alpha} \times \vec{r}_{P/O} + 2\vec{\omega} \times \left(\vec{v}_{P/O}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{P/O}\right]$$

$$(\vec{a}_{p/o})_{rel} = \vec{a}_p^{\,0} - \vec{a}_o - \cancel{\vec{\alpha} \times \vec{r}_{p/o}} - 2\vec{\omega} \times (\vec{V}_{p/o})_{rel} - \omega^2 \vec{r}_{p/o}$$

$$= 0 - 0 - 0 + 2\Omega \hat{k} \times (V_{p/o})_{rel} - \Omega^2 d\, \hat{i}$$

$$\vec{V}_p = \vec{V}_a + (\vec{V}_{p/o})_{rel} + \vec{\omega} \times \vec{r}_{p/o}$$

$$\Rightarrow (\vec{V}_{p/o})_{rel} = \vec{V}_p - \cancel{\vec{V}_o}^{\,0} - \vec{\omega} \times \vec{r}_{p/o}$$

$$= -\vec{V}_p \hat{i} + \Omega \hat{k} \times (d\hat{i})$$

$$(\vec{V}_{p/o})_{rel} = -V_p \hat{i} + \Omega d \hat{j}$$

$$(a_{p/o})_{rel} = 2\Omega \hat{k} \times (-V_p \hat{i} + \Omega d \hat{j}) - \Omega^2 d \hat{i}$$

$$= -2\Omega V_p \hat{j} - 2\Omega^2 d \hat{i} - \Omega^2 d \hat{i}$$

$$= -3\Omega^2 d \hat{i} - 2\Omega V_p \hat{j}$$

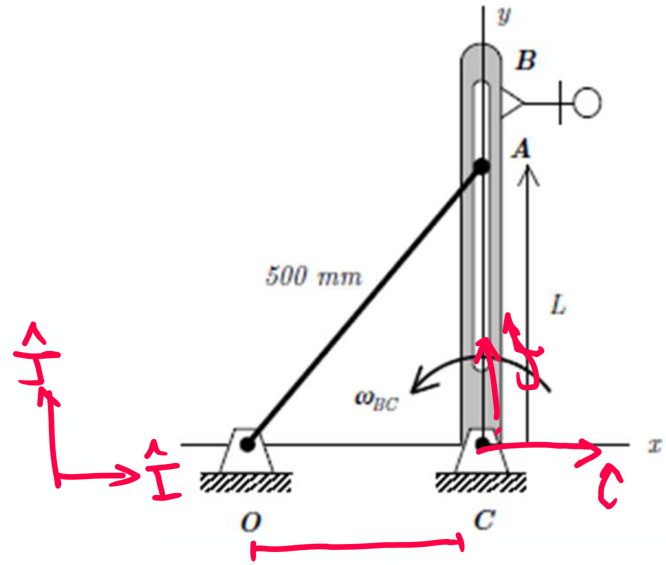
Example 3.A.5

Given: The mechanism shown below is made up of links OA and BC. Links OA and BC are pinned to ground at points O and C, respectively. Pin A of link OA is able to slide within a slot that is cut in link BC, as shown. Pins O and C are on the same horizontal line. Let the xyz axes be attached to link BC. At the instant shown:

- Link BC is vertical with $L = 400$ mm; and
- Link BC is rotating counterclockwise at a rate of $\omega = 6$ rad/s.

Find: Determine:

(a) The angular velocity of link OA at the instant shown; and
(b) The value for $\dot{L}$ at the instant shown.



$d = \sqrt{500^2 - L^2}$

1) Link OA

$$\vec{V}_A = \cancel{\vec{V}_O}^{\,0} + \vec{\omega} \times r_{A/O}$$

$$= \omega_{OA}\hat{k} \times (d\hat{I} + L\hat{J})$$

$$= \omega_{OA} d\hat{J} - \omega_{OA} L\hat{I}$$

2) A relative to C

$$\vec{V}_A = \cancel{\vec{V}_C} + (\vec{V}_{A/C})_{rel} + \vec{\omega}_{BC} \times \vec{r}_{A/C}$$

$$= \dot{L}\hat{j} + \omega_{BC}\hat{k} \times L\hat{j}$$

set =

$$= \dot{L}\hat{j} - \omega_{BC} L \hat{i}$$

$$\dot{L}\hat{J} - \omega_{BC} L \hat{I} = \omega_{OA} d \hat{j} - \omega_{OA} L \hat{i}$$

here, $\hat{J} = \hat{j}$ $\quad \hat{I} = \hat{i}$

Split into components

$\hat{i}$: $-\omega_{BC} L = -\omega_{OA} L \rightarrow \omega_{BC} = \omega_{OA}$

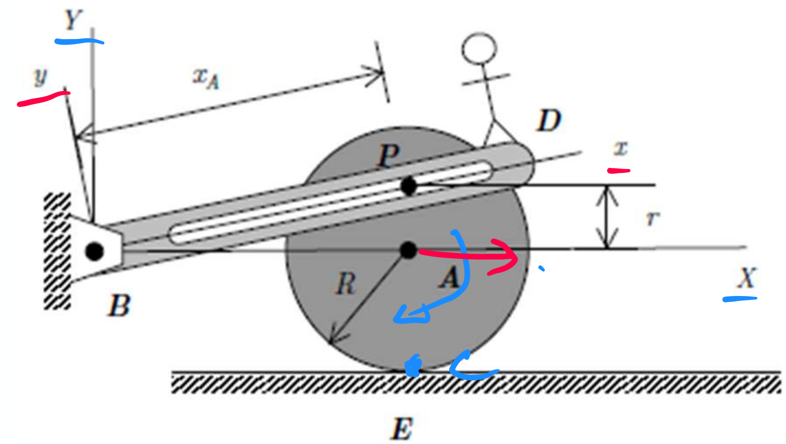
$\hat{j}$: $\dot{L} = \omega_{OA} d$

Example 3.A.7

Given: The disk rolls without slipping to the right with a constant angular speed of ω_d. At the instant shown, pin P is directly above the center A of the disk.

Find: Determine:
(a) The angular acceleration of the disk; and
(b) The acceleration of P as seen by an observer on arm BD.

Use the following parameters in your analysis: $\omega_d = 20$ rad/s (clockwise), $x_A = 0.48$ m, $r = 0.14$ m and $R = 0.2$ m.



velocity of pt P.

$$\vec{V}_P = \cancel{\vec{V}_C}^{\,0} + \vec{\omega} \times \vec{r}_{P/C}$$

$$= -\omega_d \hat{K} \times (R + r)\hat{J}$$

$$= (R + r)\omega_d \hat{I}$$

relate P to B

$$\vec{V}_P = \cancel{\vec{V}_B}^{\,0} + (V_{P/B})_{rel} + \vec{\omega}_{BD} \times r_{P/B}$$

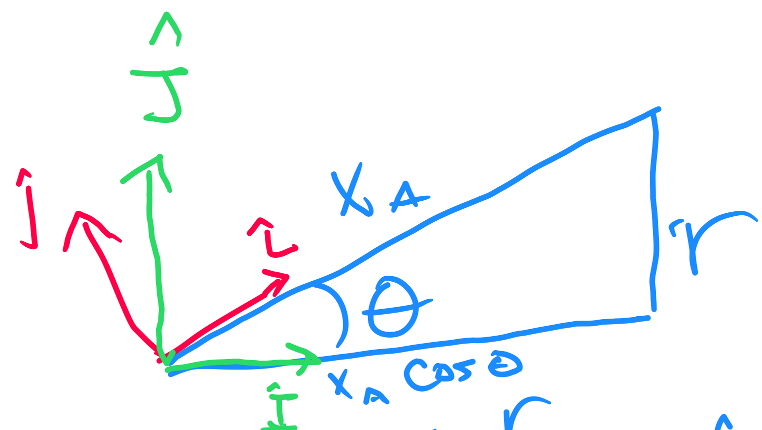
$$= \dot{x}_A \hat{i} + \omega_{BD} \hat{k} \times x_A \hat{i}$$

$$= \dot{x}_A \hat{i} + \omega_{BD} x_A \hat{j}$$

Set eqns =

$$(R+r)\omega_d \hat{I} = \dot{x}_A \hat{i} + \omega_{BD} x_A \hat{j} \quad *$$

write $\hat{i}, \hat{j}$ in terms of $\hat{I}$ + $\hat{J}$



$$\hat{i} = \cos\theta \hat{I} + \sin\theta \hat{J}$$

$$\hat{j} = -\sin\theta \hat{I} + \cos\theta \hat{J}$$

$\sin\theta = \dfrac{r}{x_A}$ → find θ

Sub in to * + split up components

$$\hat{I}: \quad (R+r)\omega_d = \dot{x}\cos\theta - \omega_{BD} x_A \sin\theta$$

$$\hat{J}: \quad 0 = \dot{x}\sin\theta + \omega_{BD} x_A \cos\theta$$

Solve for $\dot{x}$ + ω_{BD}

acceleration of pt P

$$\vec{a}_P = \cancel{\vec{a}_A} + \cancel{\vec{\alpha}_A} \times \vec{r}_{P/A} - \omega_A^2 \vec{r}_{P/A}$$

$$= -\omega_D^2 r \hat{J}$$

$\vec{\alpha}_D = \vec{0}$

$\vec{V}_A$ = const

$\vec{a}_A = 0$

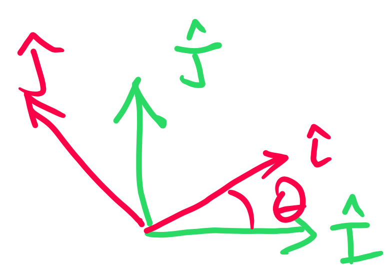
relate acceleration of P & B

$$\vec{a}_P = \vec{a}_B + (a_{P/B})_{rel} + \vec{\alpha}_{BD} \times \vec{r}_{P/B} + 2\vec{\omega} \times (v_{P/B})_{rel} - \omega^2 r_{P/B}$$

$$= 0 + \ddot{x}_A \hat{i} + \alpha_{BD} \hat{k} \times x_A \hat{i} + 2\omega_{BD} \hat{k} \times \dot{x}_A \hat{i} - \omega_{BD}^2 x_A \hat{i}$$

$$= \ddot{x}_A \hat{i} + \alpha_{BD} x_A \hat{j} + 2\omega_{BD} \dot{x} \hat{j} - \omega_{BD}^2 x_A \hat{i}$$

set $\vec{a}_P = \vec{a}_P$



$$\hat{J} = \sin\theta \hat{i} + \cos\theta \hat{j}$$

$$-\omega_D^2 r(\sin\theta \hat{i} + \cos\theta \hat{j}) = \ddot{x}_A \hat{i} + \alpha_{BD} x_A \hat{j} + 2\omega_{BD} \dot{x} \hat{j} - \omega_{BD}^2 x_A \hat{i}$$

Break into components

$\hat{i}$: $-\omega_D^2 r \sin\theta = \ddot{x}_A - \omega_{BD}^2 x_A$

$\hat{j}$: $-\omega_D^2 r \cos\theta = \alpha_{BD} x_A + 2\omega_{BD} \dot{x}$

Solve for α_{BD}, $\ddot{x}_A$