Silled

ME 274 Lecture 19

1 2B2

Particle Kinetics – Work/energy Part 1

2 2B1

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2/27/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 18 (4.E and 4.F) due today!!**
2. **Exam 1 Grades were released…**
    - ***Exam 1 stats:*** Mean = 44.1/60 (73.5%); Median = 45.5/60 (75.8%)
        - **On Course website**
3. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
    - Feel free to come to OHs if you need help with HW or discussing exam problems.

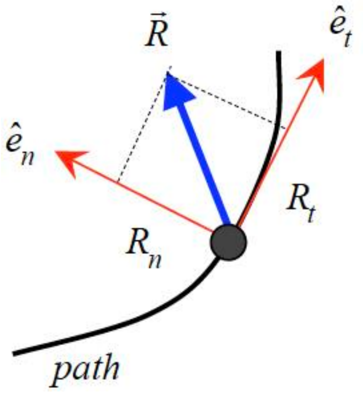
Particle Kinetics: Work-Energy Equation

- **Motivation:** Need to simplify complex force-and-acceleration problems.
    - By focusing on energy transfer, we can skip the need to calculate intermediate kinematic variables

- In the last 3 lectures, we saw that resolution of forces and acceleration into path description provides us with the following equations for a particle:

$$R_t = \sum F_t = m\frac{dv}{dt}$$

$$R_n = \sum F_n = m\frac{v^2}{\rho}$$



- Only tangential components are in charge for $\boxed{\text{speed of the particle}}$
- The normal components are responsible for $\boxed{\text{changing the direction of motion}}$

- In this lecture we will develop the **work-energy equation** for analyzing the *change in speed of a particle as a results of forces on the particle* (or systems of particles).

$$\boxed{T_1 + V_1 + U_{1\to 2}^{(nc)} = T_2 + V_2}$$

'Travail mécanique' ≈ French
Work Mechanical

Voltage
Potential

- T = Kinetic Energy.
- V = Potential Energy.
- U = Total Work done on the system

[pg. 209 content]

Where do we get the work-energy equation from?

1. Let ***R*** represent the resultant force acting on a particle as it moves along a path. $\sum \vec{R} = m\vec{a}$
2. We are interested in understanding change in speed of a particle so we only need to deal with the tangential component of the resultant force's equation (Newton's Law).
3. We project Newton onto tangential unit vector (dot product) giving:

$$\vec{R}\cdot\hat{e}_t = m\vec{a}\cdot\hat{e}_t$$

$$(R_t\hat{e}_t + R_n\hat{e}_n)\cdot\hat{e}_t = m\left(\frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n\right)\cdot\hat{e}_t$$

$$R_t\hat{e}_t\cdot\hat{e}_t + R_n\hat{e}_n\cdot\hat{e}_t = m\frac{dv}{dt}\hat{e}_t\cdot\hat{e}_t + m\frac{v^2}{\rho}\hat{e}_n\cdot\hat{e}_t$$

$$R_t(1) + R_n(0) = m\frac{dv}{dt}(1) + \frac{v^2}{\rho}(0)$$

$$R_t = m\frac{dv}{dt}$$

4. We can then **use chain rule**, with s representing the distance traveled, to produce: $R_t = mv\frac{dv}{ds}$
5. Separation of variables and integration then gives us:

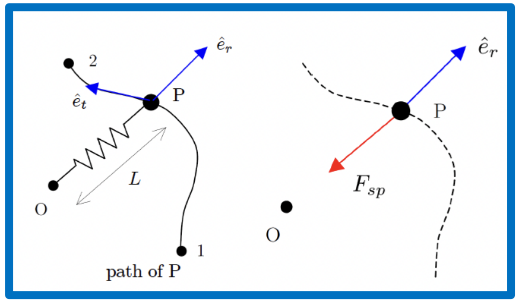
$$\int_1^2 R_t\,ds = m\int_1^2 v\,dv = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2 \rightarrow \boxed{U_{1\to 2} = T_2 - T_1}$$

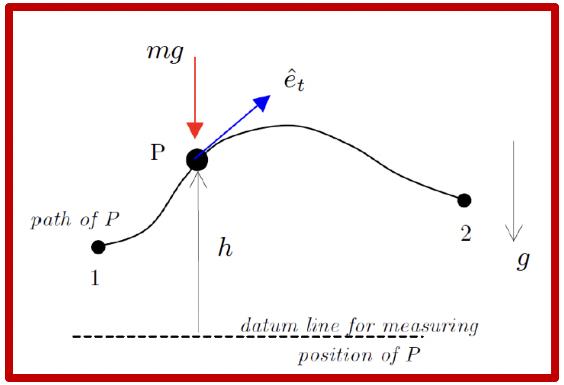
[pg. 210 content]

Conservative Forces and Potential Energy

- **Conservative Forces -** Class of forces for which *their work between positions 1 and 2* is independent of the path *over which the forces act as the particle moves from position 1 to position 2.*

- Examples of non-conservative forces:
  1. Force due to the action of a spring onto a particle
  2. Force due to the weight of a particle





- Since the work due to these conservative forces does not require us to do an integration, we can separate the total work done in a particle into **conservative and non-conservative parts:**

$$\begin{aligned} U_{1\to2} &= U^{(c)}_{1\to2} + U^{(nc)}_{1\to2} \\ &= -\left(V_2 - V_1\right) + U^{(nc)}_{1\to2} \end{aligned}$$

- We can substitute above step to the **work-energy equation from last slide** and get the one we will use extensively:

$$\boxed{T_1 + V_1 + U^{(nc)}_{1\to2} = T_2 + V_2}$$

- T = Kinetic Energy.
- V = Potential Energy.
- U = Total Work done on the system

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Discussion

- Work (U) can be computed using path and cartesian coordinates :

$$U_{1\rightarrow 2} = \int_1^2 \left(\vec{R}\cdot\hat{e}_t\right)\, ds \qquad\qquad U_{1\rightarrow 2} = \int_1^2 R_x\, dx + \int_1^2 R_y\, dy$$

- Work-energy equation is a scalar equation. Does not have vector components (ie cannot separate in x and y).
- In using the work energy equation, be careful with signs on the potential energy (V) terms:
  - ***The potential of a spring is always positive***, regardless if its stretched or compressed.
  - **The potential of the weight force** depends on whether the particle is above or below the datum line
- The Work-energy equation also applies to ***systems of particles***. If this is the case:
  - *Sum up all the kinetic/potential energy terms* to find total kinetic/potential energy
  - Calculating the work term is a little more complicated.
    - It's found by *projecting forces onto the path of the particle* on which they act *and integrating* the result over the path.
  - *Monday's Lecture (lecture 20) we will go over problems like these.*

[pg. 215, 217, 218 content]

Kinetics: Four-step problem solving method

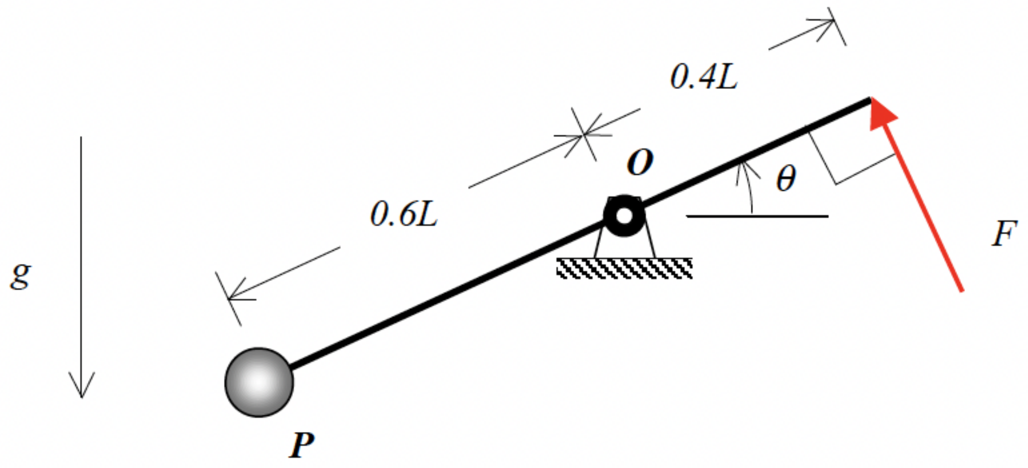
1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler (lectures 15-18)
     - Work/Energy (lectures 19-20)
     - Linear impulse/momentum
     - Angular impulse/momentum
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

[pg. 184 content]

Example 4.B.1

Given: The 6 kg particle P and the attached light rod (of length $L = 2$ m) rotate in a vertical plane about a fixed axis passing through O. The assembly is released from rest at $\theta = 0$ and moves under the action of the $F = 100$ N force (which is maintained normal to the rod).

Find: Determine the speed of the particle P when $\theta = 90°$.



[pg. 219]

Example 4.B.1 p. 219

No kinematics step on this problem

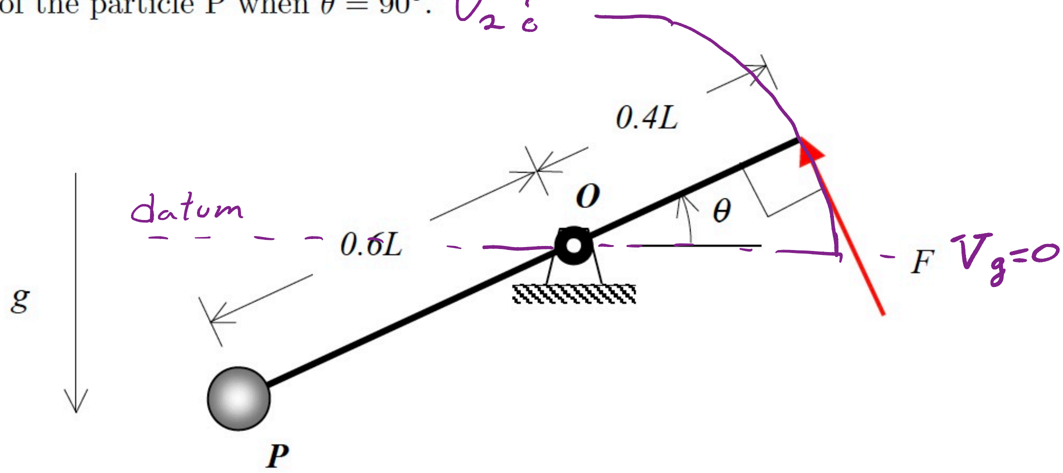
Given: The 6 kg particle P and the attached light rod (of length $L = 2$ m) rotate in a vertical plane about a fixed axis passing through O. The assembly is released from rest at $\theta = 0$ and moves under the action of the $F = 100$ N force (which is maintained normal to the rod).

Find: Determine the speed of the particle P when $\theta = 90°$. v_2?

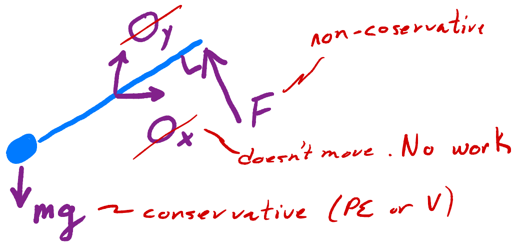
$m = 6\,kg$; $L = 2\,m$

$\theta_1 = 0°$; $\theta_2 = 90 = \frac{\pi}{2}$

$F = 100\,N$



① FBD include particle P & rigid bar



② Kinetics Work/Energy bc change in speed → change in position problem

$$T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$$

③ Define terms

$T_1 = 0$; Release from rest (RFR)

$V_1 = 0$; Mass @ datum so 0

$U_{1\to2}^{NC} = \int \vec{F}\cdot d\vec{r}$; Equivalent to applied forces magnitude (F)

$= F(0.4L)(\frac{\pi}{2})$ times distance (0.4L) and rotation of 90°.

90°

displacement of the point where F is applied

$T_2 = \frac{1}{2} m v_2^2$; Kinetic energy @ state 2

$V_2 = -mg(0.6L)$; Potential energy @ state 2

height @ 90°

④ Solve Plug-in

$\Rightarrow 0.2\pi L F = \frac{1}{2} m v_2^2 - 0.6 mgL$

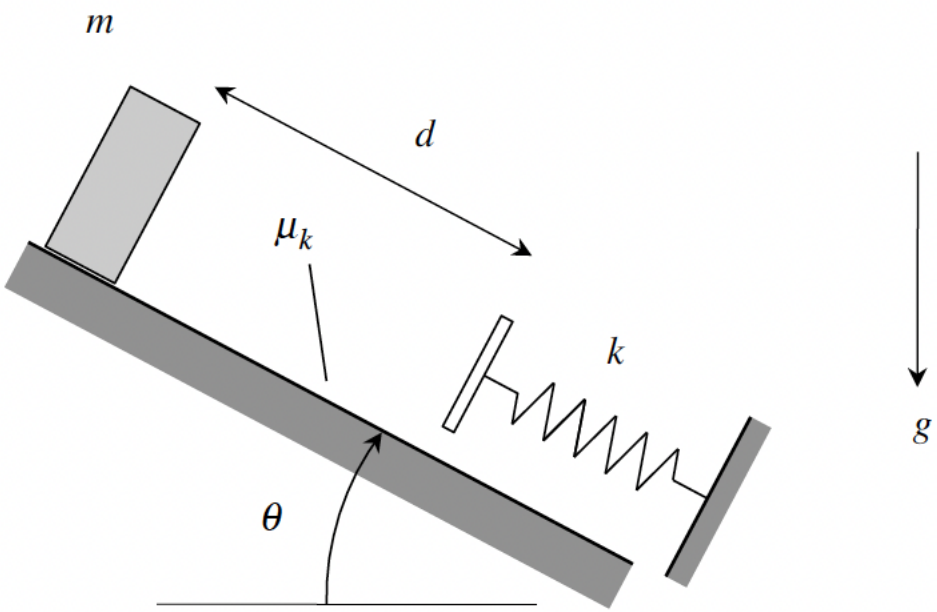
$\Rightarrow v_2$

Example 4.B.2

Given: A block of mass m is released from rest on a rough, inclined surface (with a coefficient of kinetic friction μ_k) and slides toward an uncompressed spring of stiffness k.

Find: Determine the maximum deflection of the spring.

Use the following parameters in your analysis: $\theta = 53.13°$, $d = 0.75$ m, $m = 5$ kg, $\mu_k = 0.3$ and $k = 1000$ N/m.



[pg. 220]

Example 4.B.2

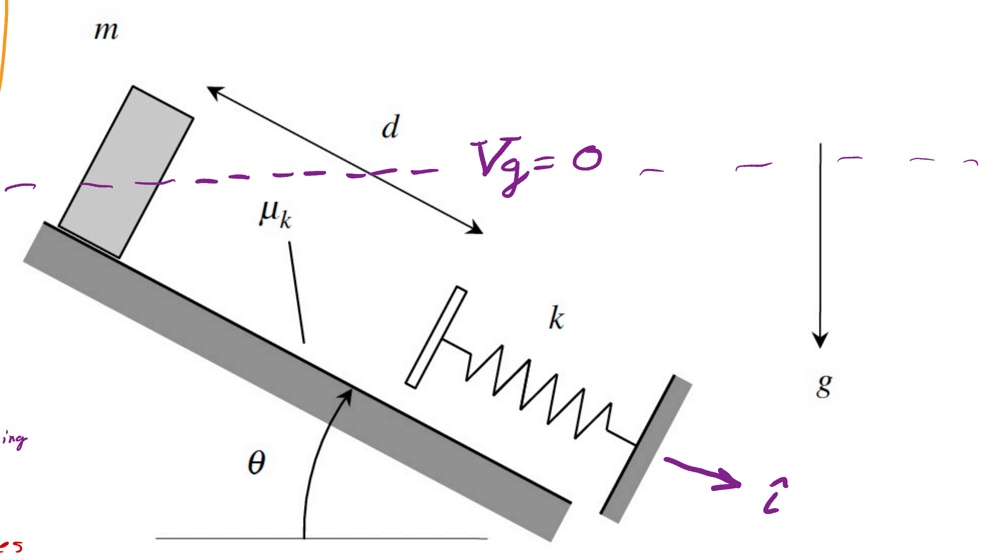
p.220

Given: A block of mass m is released from rest on a rough, inclined surface (with a coefficient of kinetic friction μ_k) and slides toward an uncompressed spring of stiffness k.

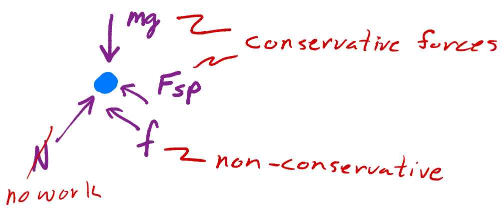
Find: Determine the maximum deflection of the spring. Δ?

Use the following parameters in your analysis: $\theta = 53.13°$, $d = 0.75$ m, $m = 5$ kg, $\mu_k = 0.3$ and $k = 1000$ N/m.

$\theta = 53.13°$; $d = 0.75\,m$
$m = 5\,Kg$; $\mu_K = 0.3$
$K = 1000\,N/m$



① FBD when in contact w/ spring



② Kinetics Work / Energy

$T_1 + V_1 + U_{1\to2}^{NC} = T_2 + V_2$

③ Define terms

$T_1 = 0$; Release from rest (RFR)

$V_1 = 0$; fcn of datum ($V_g = 0$)

$U_{1\to2}^{NC} = \int \vec{F} \cdot d\vec{r}$; friction * (displacement)

$= -f\hat{i} \cdot (d+\Delta)\hat{i}$; { d: distance traveled until you get to spring. Δ: compression of spring

$= -f(d+\Delta)$

Kinematics

$\Delta s = d + \Delta$

$T_2 = 0$; max compression speed @ this instant is 0 bc changing direction

$V_2 = -mg(d+\Delta)\sin\theta + \frac{1}{2}k\Delta^2$; Conservative forces

④ Solve

$\Rightarrow -f(d+\Delta) = -mg(d+\Delta)\sin\theta + \frac{1}{2}k\Delta^2$; Use quadratic formula $\left(\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$ to solve for 'Δ'.

⑤ To find N:

$-f = -\mu_k N$

$\hookrightarrow \Sigma F_y = -mg\cos\theta + N = 0$

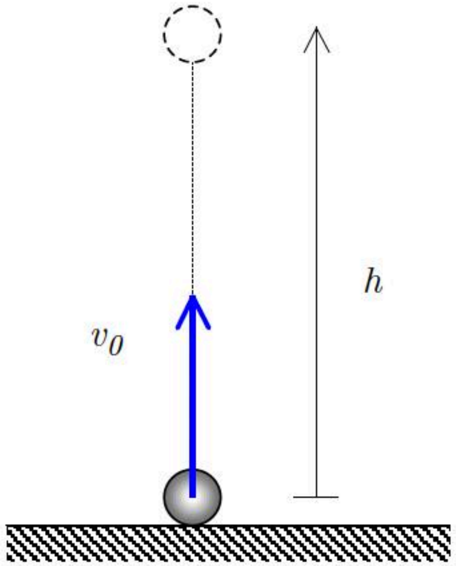
$N = mg\cos\theta$

⑥ Solve for Δ

Example 4.B.3

Given: A particle is launched vertically with a speed of v_0.

Find: Determine the maximum height h obtained by the particle. Assume that air resistance on the particle is negligible.



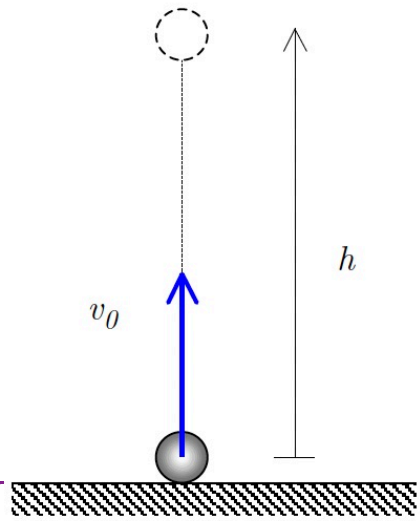
[pg. 221]

Example 4.B.3 p.221

Given: A particle is launched vertically with a speed of v_0.

Find: Determine the maximum height h obtained by the particle. Assume that air resistance on the particle is negligible.

h?



$V_g = 0$

① FBD

$\downarrow mg$ conservative

② Kinetics Work / Energy

$$T_1 + V_1 + U_{1\to 2}^{NC} = T_2 + V_2$$

③ Define terms

$T_1 = \frac{1}{2} m v_0^2$; Bc particle initially has speed v_0

$V_1 = 0$; Based on our datum ($V_g = 0$).

$U_{1\to 2}^{NC} = 0$; No Force here. 0.

$T_2 = 0$; zero

$V_2 = mgh$; gravitational potential only

④ Solve Plug-in

$$\Rightarrow \frac{1}{2} \cancel{m} v_0^2 = \cancel{m} g h$$

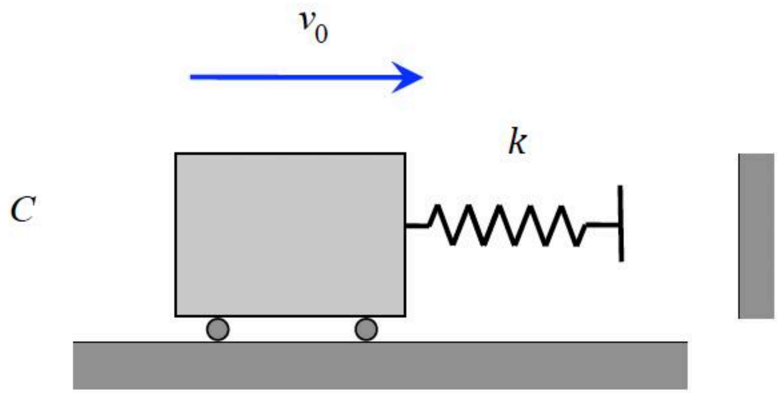
$$\Rightarrow h = \frac{v_0^2}{2g}$$

Example 4.B.4

Given: Block C having a weight mg is traveling with a speed of v_0 when it strikes a fixed wall. A spring of stiffness k is attached to the front of the block. The spring eventually contacts the wall, resulting in a maximum spring compression of Δ_{max}.

Find: Determine the stiffness k of the spring.

Use the following parameters in your analysis: $mg = 3200$ lb, $v_0 = 10$ ft/sec and $\Delta_{max} = 6$ in.



[pg. 222]

Example 4.B.4 p. 222 Classic bumper design problem in a car.

Given: Block C having a weight mg is traveling with a speed of v_0 when it strikes a fixed wall. A spring of stiffness k is attached to the front of the block. The spring eventually contacts the wall, resulting in a maximum spring compression of Δ_{max}.

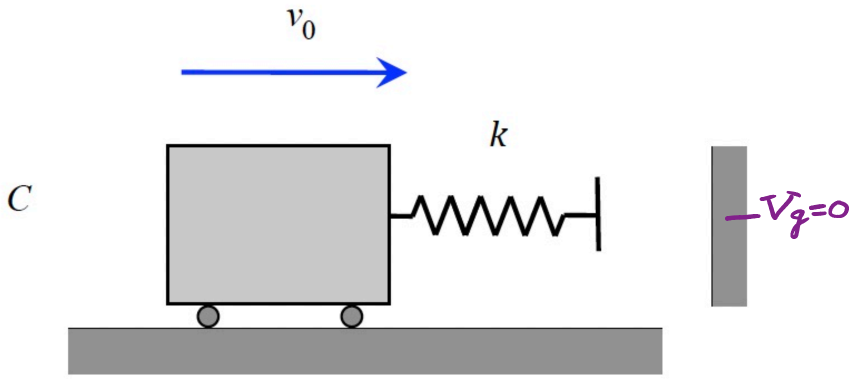
Find: Determine the stiffness k of the spring. k ?

Use the following parameters in your analysis: $mg = 3200$ lb, $v_0 = 10$ ft/sec and $\Delta_{max} = 6$ in.

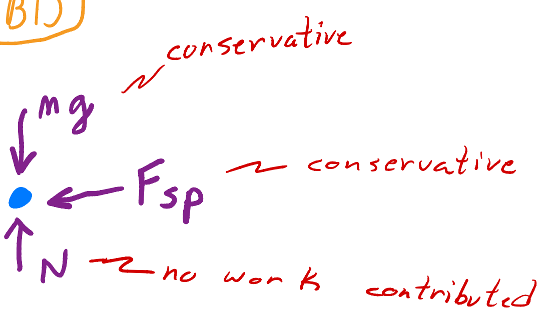
$mg = 3200$ lb

$v_0 = 10$ ft/sec

$\Delta_{max} = 6$ in



① FBD



② Kinetics Work / Energy

$$T_1 + V_1 + U_{1\to2}^{NC} = T_2 + V_2$$

③ Define terms

$T_1 = \frac{1}{2} m v_0^2$; moving @ speed v_0

$V_1 = 0$; 0 bc where i put datum.

$U_{1\to2}^{NC} = 0$; No work

$T_2 = 0$; Intantaneously change direction. So zero vel & speed

$V_2 = \frac{1}{2} k \Delta_{max}^2$; amount of deflection on spring

④ Solve Plug-in

$$\Rightarrow \cancel{\tfrac{1}{2}} m v_0^2 = \cancel{\tfrac{1}{2}} k \Delta_{max}^2$$

$$\Rightarrow k = \frac{m v_0^2}{\Delta_{max}^2}$$

Summary: Work-Energy Equation 1

FUNDAMENTAL equation: the work-energy equation

[pg. 212]

$$T_1 + V_1 + U_{1\rightarrow 2}^{(nc)} = T_2 + V_2$$

where:

$$T = \frac{1}{2}mv^2 \left(\geq 0\right) = kinetic\ energy$$

$$U_{1\rightarrow 2}^{(nc)} = \int_1^2 \left(\sum \vec{F}\right) \bullet \hat{e}_t\ ds = work\ done\ by\ non-conservative\ \ forces$$

$$= \int_1^2 \left[\left(\sum \vec{F}\right)_x dx + \left(\sum \vec{F}\right)_y dy\right]$$

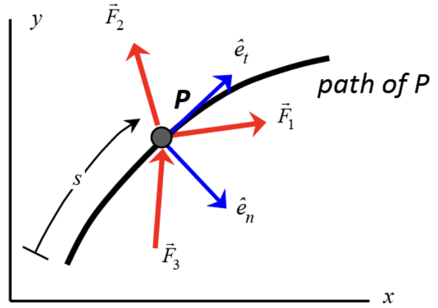
[pg. 213]

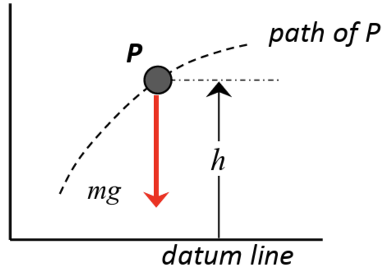
$$V = V_{gr} + V_{sp} = potential\ energy$$

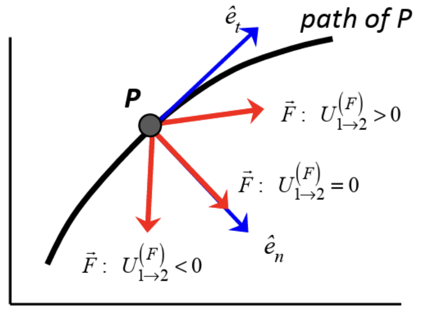
$$V_{gr} = mgh \quad \left(sign\ depends\ on\ h\right)$$

$$V_{sp} = \frac{1}{2}k\Delta^2 \geq 0 \quad \left(ALWAYS\right)$$

SIGN of work term: The direction of a force relative to the $\hat{e}_t$ *dictates the sign of the work, U.*







Lec 19 Short Feedback Form: