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ME 274 Lecture 18

Newton's Laws Part 3

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2/25/26

Housekeeping/Announcements

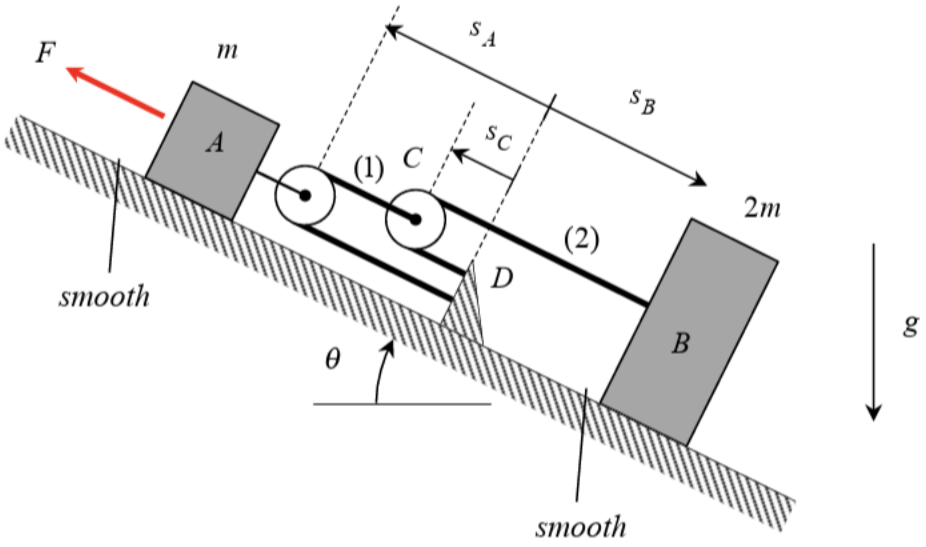
***Reminder for Henny to wear a mic during the lecture.

1. **HW 17 (4.C and 4.D) due today!!**
2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME.
    - Feel free to come to OHs if you need help with HW or discussing exam problems.

Homework H.4.C

Given: Blocks A and B (having masses of m and $2m$, respectively) are constrained to move along a smooth inclined surface. Cable (1) is connected to fixed ground at D and to the center of pulley C, as shown, with cable (1) being wrapped around a pulley connected to block A. A second cable (2) is connected between the fixed ground at D and block B. The pulleys are to be assumed to be of negligible mass, and the cables are assumed to be inextensible and not allowed to go slack. The sections of the cables not wrapped around pulleys are parallel to the incline on which blocks A and B move. A force F acts along the direction of the incline on block A.

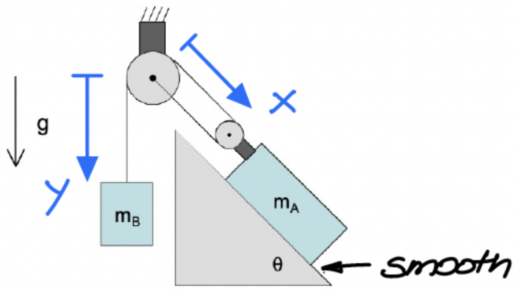
Find: For this problem, determine the accelerations of blocks A and B.



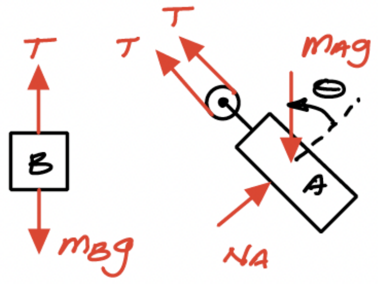
In-class Example 4.1

Given: The system shown below is released from rest in the configuration shown.

Find: Determine:
(a) The acceleration of block A;
(b) The acceleration of block B;
(c) The tension in the cable.



1. FBDs (one each for A and B)



2. Newton

(1) A: $\Sigma F_x = -2T + m_A g \sin\theta = m_A a_A$

(2) B: $\Sigma F_y = m_B g - T = m_B a_B$

Note: 2 equations/3 unknowns (a_A, a_B, T)

3. Kinematics

L = length of cable = $y + 2x + \text{const}$

(3) $\frac{d^2L}{dt^2} = 0 = \ddot{y} + 2\ddot{x} = a_B + 2a_A \Rightarrow a_B = -2a_A$

4. Solve: 3 equations/3 unknowns (T, a_A, a_B)

(1) and (2) $\Rightarrow T = \frac{1}{2} m_A (g \sin\theta - a_A) = m_B (g - a_B)$

(3) $\Rightarrow \frac{1}{2} m_A (g \sin\theta - a_A) = m_B (g + 2a_A)$

$\hookrightarrow a_A = \left[\frac{m_A \sin\theta - 2m_B}{m_A + 4m_B}\right] g \longleftarrow a_A$

$a_B = -2a_A = -2\left[\frac{m_A \sin\theta - 2m_B}{m_A + 4m_B}\right] g \longleftarrow a_B$

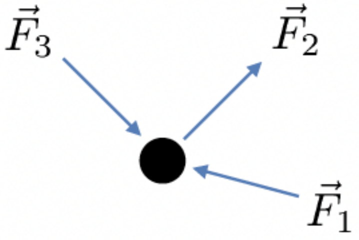
$T = m_B (g - a_B) = m_B g \left[1 + 2\frac{m_A \sin\theta - 2m_B}{m_A + m_B}\right] \longleftarrow T$

Chapter 4: Particle Kinetics

- **Kinetics is the study of the relationship between *kinematics (position/vel/accel)* and force.**
- The principles of **kinetics** derived from the following two laws of Newton:

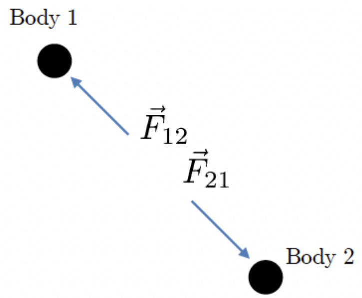
Newtons 2nd law:

$$\sum \vec{F} = m\vec{a}$$



Newtons 3rd law:

$$\vec{F}_{12} = -\vec{F}_{21}$$



- In the first part of this course, we developed sets of kinematic expressions using 3 different coordinate systems. For accelerations these were:

$$\textit{Cartesian description} : \vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$\textit{Path description} : \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\textit{Polar description} : \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

- We want to develop relationships between forces and accelerations of particles **in these three descriptions**

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Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler
     - Work/Energy
     - Linear impulse/momentum
     - Angular impulse/momentum
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

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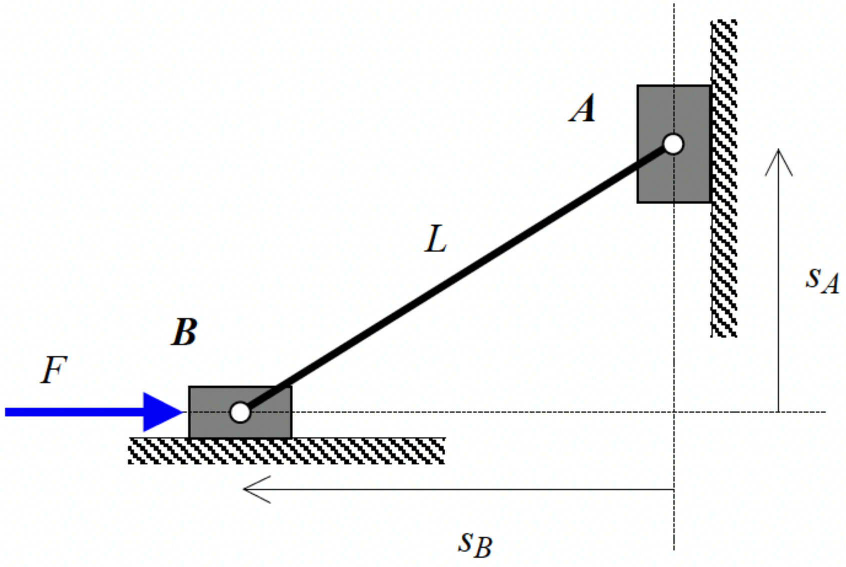
Example 4.A.8

Given: Blocks A and B (having masses of 10 kg and 5 kg, respectively) are constrained to move along smooth, vertical and horizontal guides, as shown in the figure. A and B are connected by a lightweight rod of length $L = 2.5$ m. A force of $F = 50$ N acts to the right on block B. At the position where $s_A = 1.5$ m, A is moving downward with a speed of 4 m/s.

Find: Determine:

(a) The acceleration of A and B at this instant; and

(b) The force in rod AB at this instant.



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Finish from last class

Example 4.A.8 p.201 Similar to a hw problem

Given: Blocks A and B (having masses of 10 kg and 5 kg, respectively) are constrained to move along smooth, vertical and horizontal guides, as shown in the figure. A and B are connected by a lightweight rod of length $L = 2.5$ m. A force of $F = 50$ N acts to the right on block B. At the position where $s_A = 1.5$ m, A is moving downward with a speed of 4 m/s.

Find: Determine:

(a) The acceleration of A and B at this instant; and

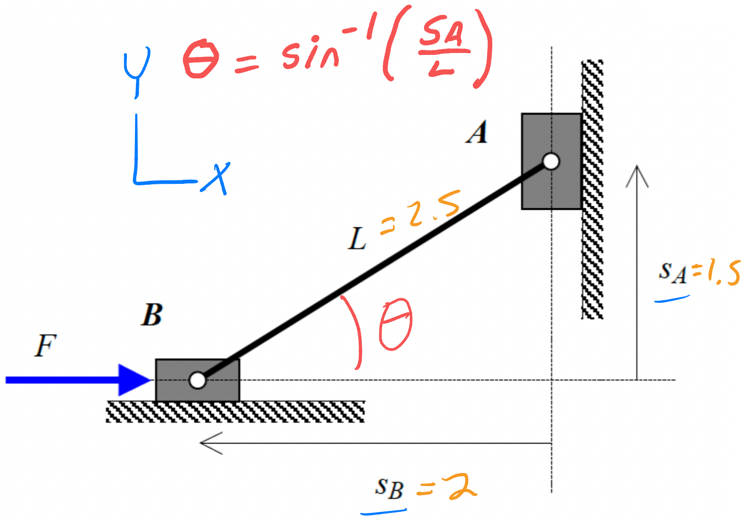
(b) The force in rod AB at this instant.

$m_A = 10\,kg$; $m_B = 5\,kg$

$L = 2.5\,m$; $F = 50\,N$;

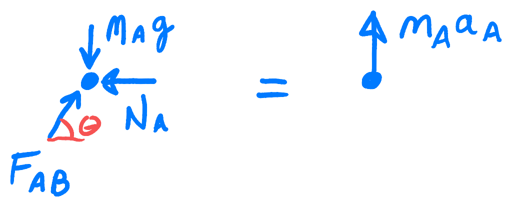
$s_A = 1.5\,m$; $v_A = -4\,m/s$

$s_B = $ trig... $= 2\,m$

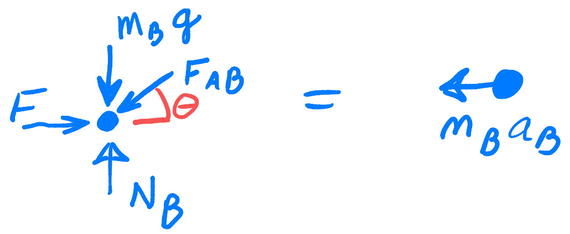


FBD

① Use θ for block A FBD



② Use θ for block B FBD



6 unknowns
6 eqns:

$F_{AB} = ?$

$N_A = ?$ $N_B = ?$

$a_A = ?$

$v_B = ?$ $a_B = ?$

Kinetics

③ ΣF in x & y of A

A: $\Sigma F_x = 0$

$\Rightarrow F_{AB}\cos\theta - N_A = 0$ (1)

$\Sigma F_y = m_A a_A$

$\Rightarrow F_{AB}\sin\theta - m_A g = m_A a_A$ (2)

④ ΣF in x & y of B

B: $\Sigma F_x = m_B a_B$

$\Rightarrow F_{AB}\cos\theta - F = m_B a_B$ (3)

$\Sigma F_y = 0$

$\Rightarrow N_B - m_B g - F_{AB}\sin\theta = 0$ (4)

4 eqns, 5 unkns

Kinematics

⑤ Method #1: Pythagoras for distance eqn

$s_A^2 + s_B^2 = L^2$ (5)

⑥ take derivative for $\vec{v}$

$\frac{d}{dt}(5) \Rightarrow 2 s_A v_A + 2 s_B v_B = 0$ (6)

5 eqns, 6 unkns...

⑦ take derivative for $\vec{a}$. 6th eqn!

$\frac{d}{dt}(6) \Rightarrow 2 v_A^2 + 2 s_A a_A + 2 v_B^2 + 2 s_B a_B = 0$ (7)

⑧ 6 eqns, 6 unkns. Solve.

Kinematics

⑤ Method 2: rigid body eqn

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$$

$$a_A \hat{j} = a_B \hat{i} + (\alpha \hat{k}) \times (L\cos\theta \hat{i} + L\sin\theta \hat{j}) - \omega^2 (L\cos\theta \hat{i} + L\sin\theta \hat{j})$$

$$= [a_B - L\alpha\sin\theta - L\omega^2\cos\theta]\hat{i} + [L\alpha\cos\theta - L\omega^2\sin\theta]\hat{j}$$

⑥ Separate $\hat{i}$ & $\hat{j}$

(3) $\hat{i}$: $0 = a_B - L\alpha\sin\theta - L\omega^2\cos\theta$

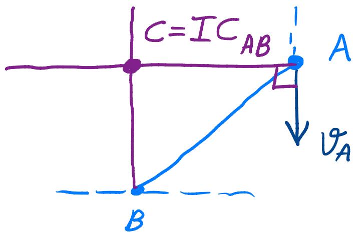
(4) $\hat{j}$: $a_A = L\alpha\cos\theta - L\omega^2\sin\theta$

⑦ Use IC's for (5)

$$v_A = \omega |\vec{r}_{A/C}| = \omega L\cos\theta$$

$$\hookrightarrow \vec{\omega} = \frac{-v_A}{L\cos\theta}\hat{k}$$

(5) $$\omega^2 = \left(\frac{v_A}{L\cos\theta}\right)^2$$



⑧ 4. Solve

$$(1), (2), (3), (4), (5) \Rightarrow a_A, a_B, T$$

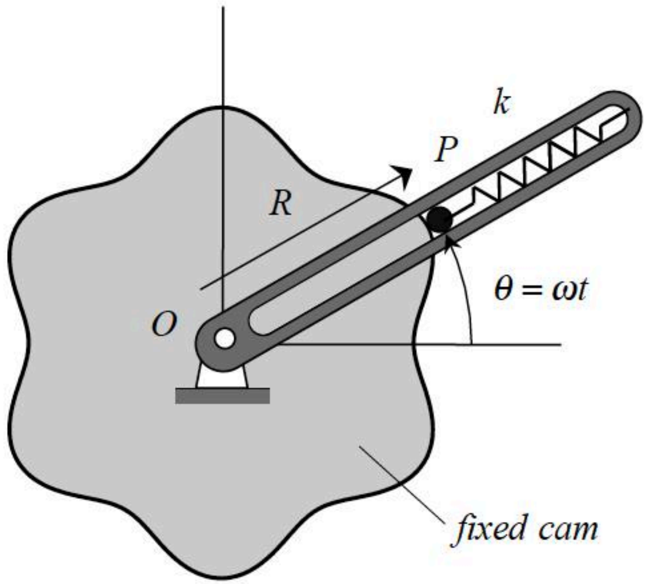
e.g. $\alpha = \frac{d\omega}{dt}$;

Note about α: we cant derivative bc we need ω = valid for all time. And IC is just one instant.

Example 4.A.11

Given: A slotted arm rotates about a vertical shaft passing through O that is at the center of a FIXED cam, as shown in the figure. A particle P, having a mass of $m = 0.2$ kg moves within the slot in the arm and remains in contact with the surface of the cam under the action of a spring attached between P and the outer end of the arm. The shape of the cam is such that the radial distance from O to P is given by the equation $R = R_0 - R_1 \cos(6\theta)$ where $R_0 = 0.5$ m and $R_0 = 0.1$ m. The spring has a stiffness of $k = 500$ N/m and is compressed by an amount of $\Delta = 0.2$ m when $\theta = 0$. The arm rotates at a constant rate of $\omega = 10$ rad/s.

Find: Determine the force acting on P by the cam when P passes over the top of the lobe in the position shown.



The term "cam" in mechanisms originates from 18th-century Dutch *kam* (meaning "comb" or "cog of a wheel"). It entered

[pg. 204]

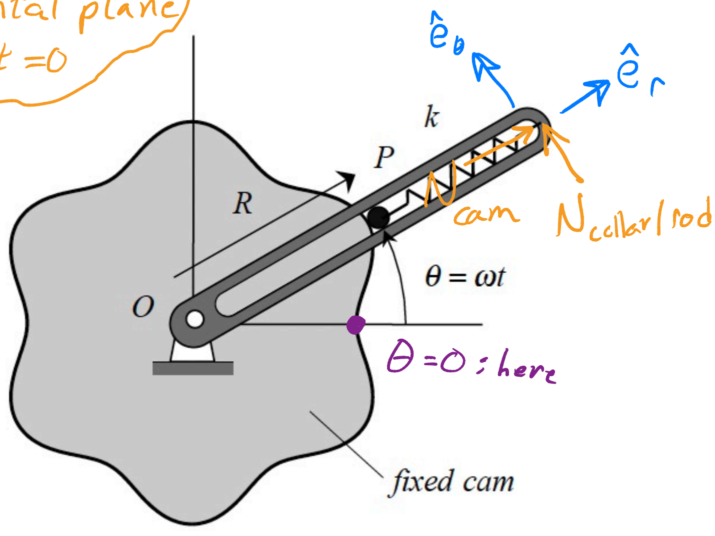
Example 4.A.11 p.204

Given: A slotted arm rotates about a vertical shaft passing through O that is at the center of a FIXED cam, as shown in the figure. A particle P, having a mass of $m = 0.2$ kg moves within the slot in the arm and remains in contact with the surface of the cam under the action of a spring attached between P and the outer end of the arm. The shape of the cam is such that the radial distance from O to P is given by the equation $R = R_0 - R_1 \cos(6\theta)$ where $R_0 = 0.5$ m and $R_1 = 0.1$ m. The spring has a stiffness of $k = 500$ N/m and is compressed by an amount of $\Delta = 0.2$ m when $\theta = 0$. The arm rotates at a constant rate of $\omega = 10$ rad/s.

Find: Determine the force acting on P by the cam when P passes over the top of the lobe in the position shown. N?

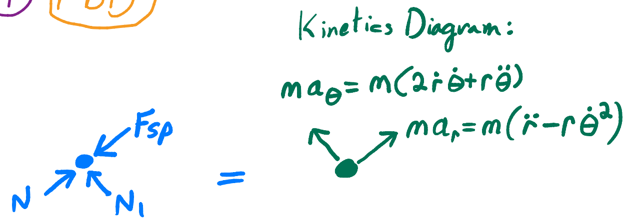
$m = 0.2$ kg
$R = R_0 - R_1\cos(6\theta)$
$R_0 = 0.5$ m
$R_1 = 0.1$ m
$K = 500$ N/m
$\Delta_0 = 0.2$ @ $\theta = 0$
$\omega = 10$ rad/s ; $\dot{\omega} = 0$

horizontal plane
Weight = 0



Top View:

① FBD



Kinetics

② ΣF in radial direction

$\Sigma F_r = ma_r$

$\Rightarrow N - F_{sp} = m(\ddot{r} - r\dot{\theta}^2)$

Kinematics

③ What's F_{sp}? Use Hook's law

$F_{sp} = k(\Delta)$: Hook's Law

④ What's Δ?

@ $\theta = 0 : \Delta_0$

@ $\theta : \Delta_0 + 2R_1$

⑤ Solve

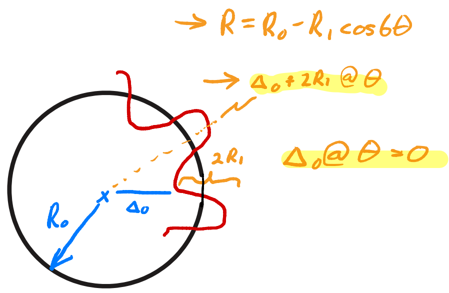
$N = k(\Delta_0 + 2R_1) + m(\ddot{r} - r\dot{\theta}^2)$ ($\dot{\theta} \to \omega$)

$r = R_0 - R_1 \cos 6\theta$

$\dot{r} = (R_1 \sin 6\theta)\, 6\dot{\theta}$ ($\dot{\theta} \to \omega$) ; $\frac{d}{dt}$

$\ddot{r} = R_1 \cos 6\theta\, (6\dot{\theta})^2 + R_1 \sin 6\theta\, (6\ddot{\theta})$ ($\dot{\theta} \to \omega$, const) ;

⑥ Solve for N



The term "cam" in mechanisms originates from 18th-century Dutch *kam* (meaning "comb" or "cog of a wheel"). It entered

Cam's are mostly used in automotive /mfg apps.

Motivation / Real World Application:

Optimizing the Mechanics of a Variable-Stiffness Orthosis With Energy Recycling to Mitigate Foot Drop

Emily A. Bywater, *Graduate Student Member, IEEE*, Nikko Van Crey, *Member, IEEE*, and Elliott J. Rouse, *Senior Member, IEEE*

Design of a Quasi-Passive Ankle-Foot Orthosis with Customizable, Variable Stiffness

Nikko Van Crey, Marcos Cavallin, Max Shepherd, and Elliott J. Rouse

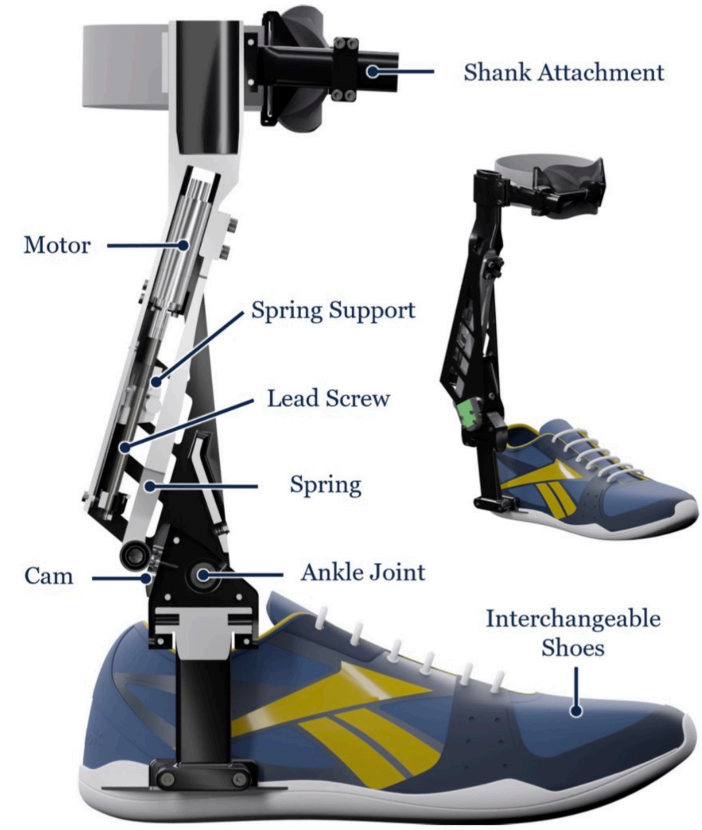


Fig. 1. The shape of the torque-angle relationship is governed by a cam-based transmission, and the average stiffness of the relationship can be adjusted by motorized reconfiguration of a spring support beneath a titanium leaf spring. The VSO has passive elastic mechanics in dorsiflexion/plantarflexion, and includes an unactuated ankle inversion/eversion joint. The RoM is independently adjustable in dorsiflexion and plantarflexion via set screws within the design, and the shin attachment is adjustable via clamping along a linear rail. Shoes of different sizes can also be quickly interchanged with two fasteners.

D. Cam Profile Derivation

1) Forward Model: In the forward model the torque-angle relationship is known and the cam profile is unknown (diagram shown in Fig. 3). The mathematics [27] are derived

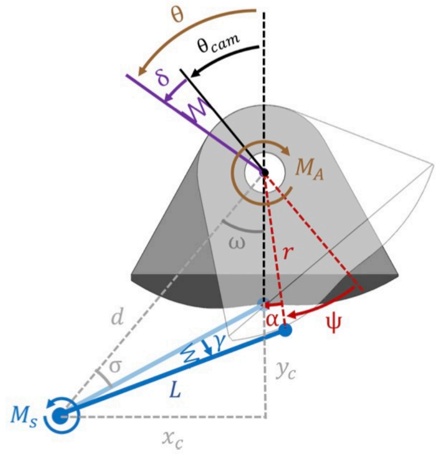


Fig. 3. The model has two series elastic elements that contribute to energy storage. The leaf spring is modeled as a rotary spring with a virtual pivot at the spring support location. The assembly compliance is modeled as a rotary spring at the ankle center with stiffness k_δ. The cam follower is simplified to a point and downward deflection of the cam follower γ creates a restoring moment in the leaf spring M_s. Deflection of the ankle θ is a combination of the cam deflection θ_{cam} and series compliance δ. This causes a moment at the ankle M_A, produced by the force between the cam follower and cam profile. The leaf spring is preloaded by a small angle γ_0 to prevent backlash. The cam is represented as a set of polar coordinates (r, ψ) and x_c, y_c, d, L, σ, ω, and α are all geometric parameters.

What's an Orthosis?

Example 4.A.12

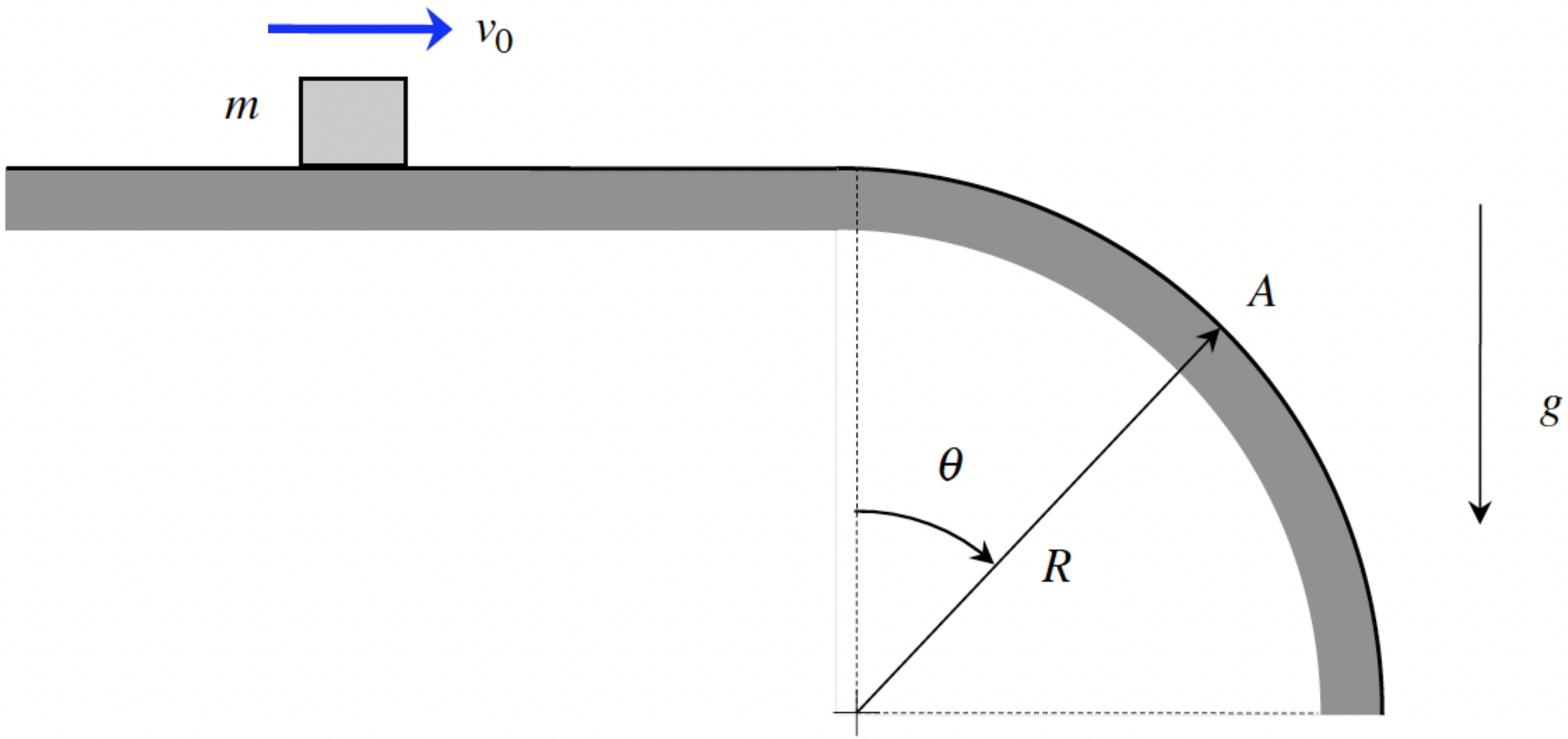
Given: A particle of mass m moves along the smooth path shown.

Find:

(a) Determine the speed v_0 such that the particle loses contact with the semi-circular hill at $\theta = 53.13°$.

(b) Using the speed results from above, determine the normal contact force acting on the block by the hill when $\theta = 36.87°$.

Use the following parameter in your analysis: $m = 10$ kg and $R = 2.5$ m.



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Example 4.A.12 p.205

Given: A particle of mass m moves along the smooth path shown.

Find:

(a) Determine the speed v_0 such that the particle loses contact with the semi-circular hill at $\theta_{CR} = 53.13°$. v_0 ?

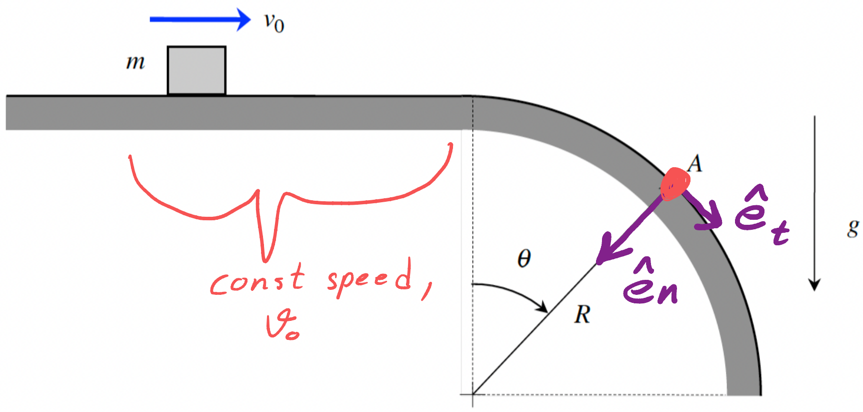
(b) Using the speed results from above, determine the normal contact force acting on the block by the hill when $\theta = 36.87°$. N ?

Use the following parameter in your analysis: $m = 10$ kg and $R = 2.5$ m.

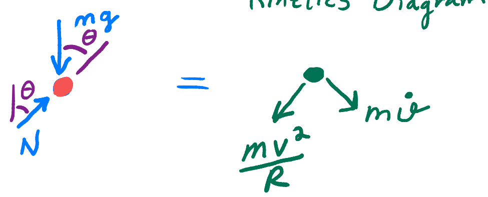
$\theta_{CR} = 53.13°$; $\theta = 36.87°$

$m = 10\,Kg$; $R = 2.5\,m$

$g = 9.8\,m/s^2$



① FBD



Kinetics & Kinematics

② Newton. ΣF in 't'

$$\Sigma F_t = m\dot{v}$$

$$\Rightarrow mg\sin\theta = m\dot{v} \quad (1)$$

③ Newton. ΣF in 'n'

$$\Sigma F_n = m\frac{v^2}{R}$$

$$\Rightarrow mg\cos\theta - N = m\frac{v^2}{R} \quad (2)$$

Aside: Where's Kinetics/Kinematics TLDR:

→ Kinetics: $\Sigma F_{t/n} = m a_{t/n}$

→ Kinematics: $a_n = \frac{v^2}{R}$ $\quad a_t = \frac{dv}{dt}$

Solve

④ Use (2) solve for v_{CR} (critical speed, particle loses contact). Make $N=0$ (no contact). We will use later in step ⑥.

$(2) \Rightarrow N = 0$

$\Rightarrow g\cos\theta = \frac{v^2}{R}$

$(a) \Rightarrow v_{cr} = \sqrt{gR\cos\theta_{cr}}$

⑤ a) Above doesn't tell us v_0 bc (1) $\dot{v} = g\sin\theta$.
b) Use chain rule w/ eqn (1) & c) relate w/ arc length.

(1) a) $\dot{v} = g\sin\theta$

b)

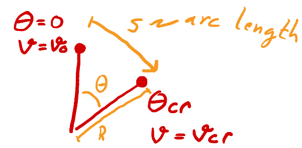
$$\frac{dv}{dt} = g\sin\theta$$

$$\Rightarrow \frac{dv}{d\theta}\frac{d\theta}{dt} = g\sin\theta$$

$$\Rightarrow \frac{dv}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt} = g\sin\theta$$

(chain rule)

c) $\Rightarrow \frac{dv}{d\theta}\frac{1}{R}v = g\sin\theta$; relate w/ arc length



$s = R\theta$; arc length

$ds = R\,d\theta$

$\frac{d\theta}{ds} = \frac{1}{R}$

$$\Rightarrow \int_{v_0}^{v_{cr}} \frac{1}{R}v\,dv = \int_0^{\theta_{cr}} g\sin\theta\,d\theta \text{ ; separate vars, integrate}$$

⑥ Solve for v_0

$$\Rightarrow \frac{1}{2R}\left[v_{cr}^2 - v_0^2\right] = \left[-g\cos\theta\right]_0^{\theta_{cr}}$$

$\Rightarrow v_0$

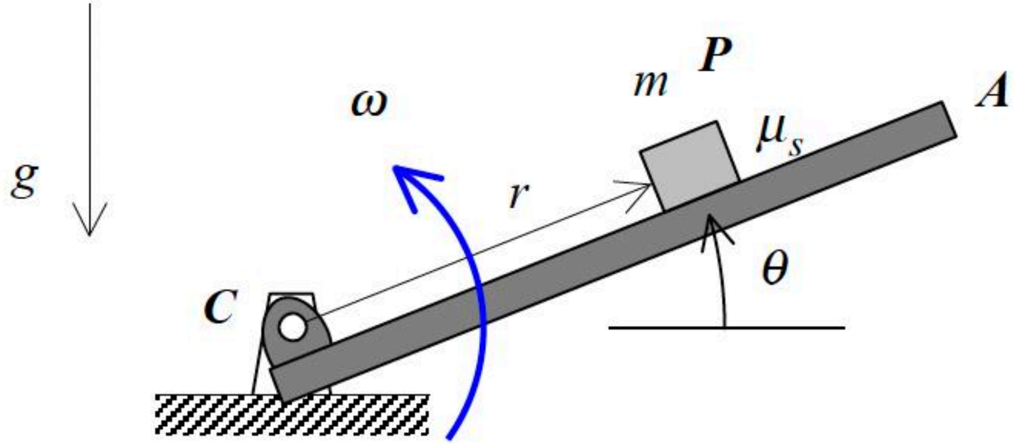
$\Rightarrow N$

Example 4.A.13

Given: Arm CA is rotating in a vertical plane with a constant rate of ω. When CA is horizontal, particle P (having a mass of m) is stationary with respect to CA. However, P is known to start slipping with respect to CA when CA has reached an angle of θ_{slip}. The coefficient of static friction between P and the arm is μ_S.

Find: The value for the coefficient of static friction μ_S between P and CA.

Use the following parameters in your analysis: $\omega = 2$ rad/s, $r = 3$ ft and $\theta_{slip} = 53.13°$.



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Example 4.A.13 p.206

Given: Arm CA is rotating in a vertical plane with a constant rate of ω. When CA is horizontal, particle P (having a mass of m) is stationary with respect to CA. However, P is known to start slipping with respect to CA when CA has reached an angle of θ_{slip}. The coefficient of static friction between P and the arm is μ_S.

Find: The value for the coefficient of static friction μ_S between P and CA. μ_s?

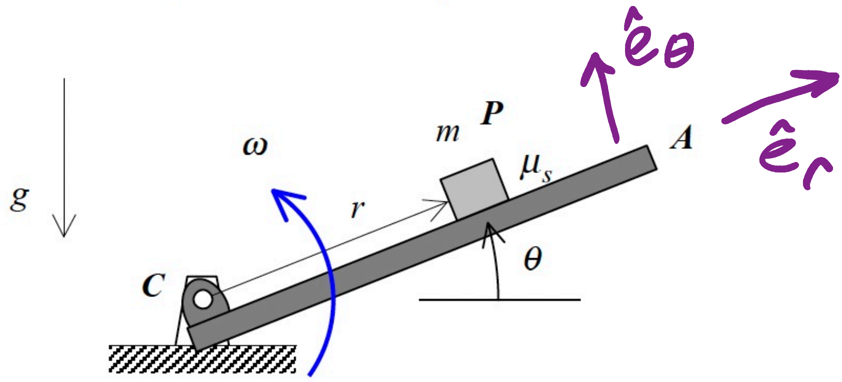
Use the following parameters in your analysis: $\omega = 2$ rad/s, $r = 3$ ft and $\theta_{slip} = 53.13°$.

@ instant slip not occuring

$r = 3ft$; $\dot{r} = 0$; $\ddot{r} = 0$

$g = 9.8 m/s^2$

$\theta_{slip} = 53.13°$; $\omega = 2 rad/s = \dot{\theta}$; $\ddot{\theta} = 0$



① FBD

Kinetics diagram:

$m a_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$

$m a_r = m(\ddot{r} - r\dot{\theta}^2)$

θ, mg, f, N =

Kinetics & Kinematics

② ΣF in radial direction

$\Sigma F_r = m a_r$

$\Rightarrow f - mg\sin\theta = m(\ddot{r} - r\dot{\theta}^2)$ (1) [$\ddot{r} = 0$, $\dot{\theta} = \omega$]

③ ΣF in θ direction

$\Sigma F_\theta = m a_\theta$

$\Rightarrow N - mg\cos\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$ (2) [$\dot{r} = 0$, $\dot{\theta} = \omega$, $\ddot{\theta} = 0$]

④ friction force

$f = \mu_s N$ (3)

⑤ Solve for μ_s

(2) $\Rightarrow N = mg\cos\theta$; solve for N

(3) $\Rightarrow f = \mu_s mg\cos\theta$; plug in N

(1) $\Rightarrow \mu_s mg\cos\theta - mg\sin\theta = -mr\omega^2$

$\Rightarrow \mu_s g\cos\theta_{cr} - g\sin\theta_{cr} = -r\omega^2$

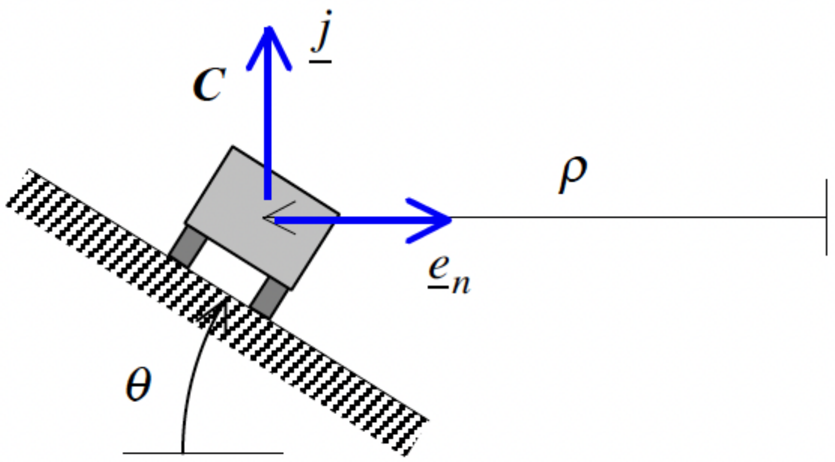
$\Rightarrow \mu_s$

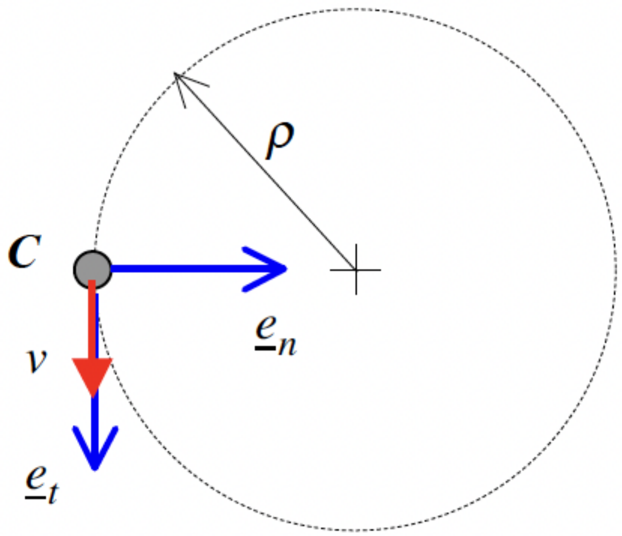
Example 4.A.14

Given: Car C is traveling with a speed of v on a roadway with a banked turn.

Find: Determine the speed v of the car for which no friction force is required to keep the car on the roadway.

Use the following parameters in your anlaysis: $\rho = 500$ m and $\theta = 36.87°$.





top view

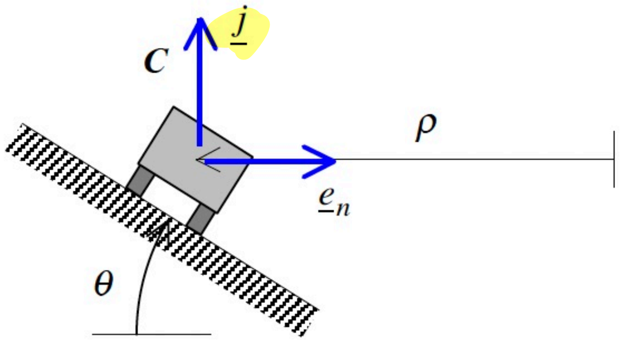
[pg. 207]

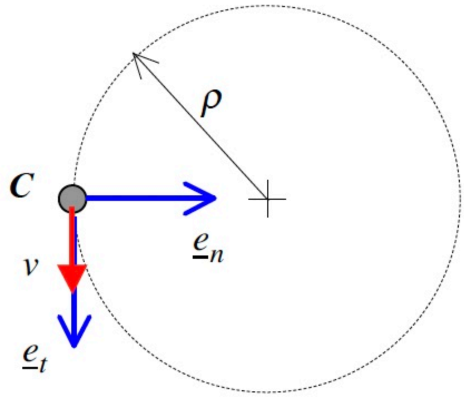
Example 4.A.14 p.207

Given: Car C is traveling with a speed of v on a roadway with a banked turn.

Find: Determine the speed v of the car for which no friction force is required to keep the car on the roadway. v?

Use the following parameters in your anlaysis: $\rho = 500$ m and $\theta = 36.87°$.





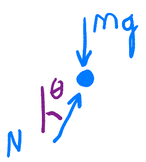
top view

$\rho = 500\ m$; $\theta = 36.87°$; $g = 9.8\ m/s^2$

① FBD

Kinetics Diagram:

nothing on vertical bc no slip



$=$

$\bullet \rightarrow m a_n = m \frac{v^2}{\rho}$

Kinetics & Kinematics

② ΣF in normal direction

$$\Sigma F_n = m a_n$$

$$\Rightarrow N \sin\theta = m \frac{v^2}{\rho} \quad (1)$$

③ ΣF in y direction. Problem recommends this.

$$\Sigma F_y = 0 \quad ; 0 \text{ bc car doesnt slip}$$

$$\Rightarrow N \cos\theta - mg = 0 \quad (2)$$

Solve

④ Use (2), solve for N

$$(2) \Rightarrow N = \frac{mg}{\cos\theta}$$

⑤ Plug in (2) onto (1)

$$(1) \Rightarrow \frac{mg \sin\theta}{\cos\theta} = m \frac{v^2}{\rho}$$

$$\Rightarrow g \tan\theta = \frac{v^2}{\rho}$$

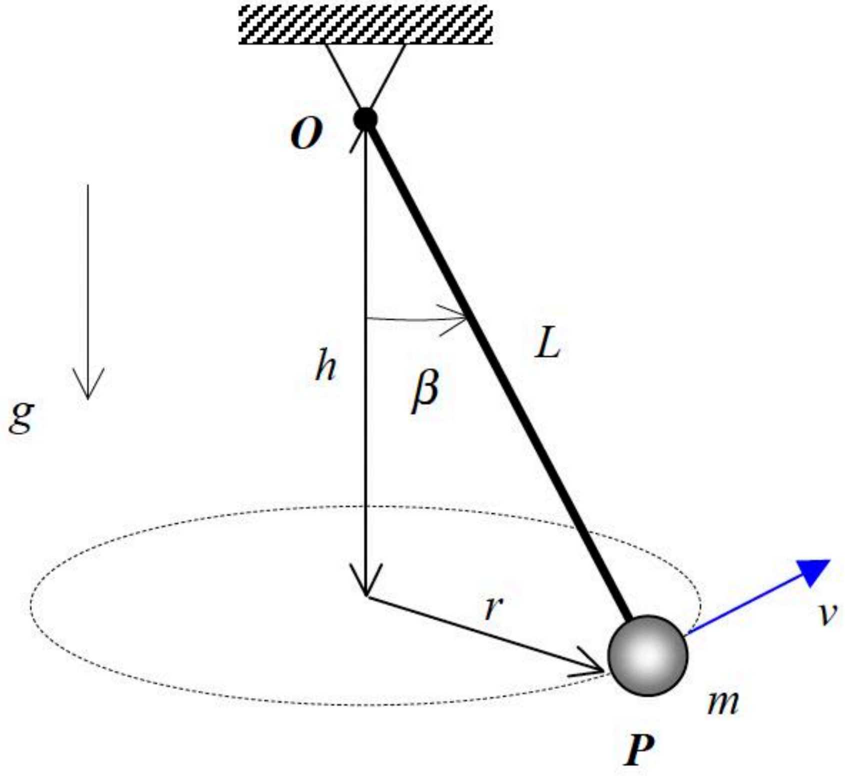
$$\Rightarrow v$$

Fun fact: Rhoads said this a common problem used in race car/track design.

Example 4.A.15

Given: Particle P is attached to a fixed support point O with a cable of length L. P is moving with a speed of v on a horizontal circular path with a radius of r.

Find: Find the angle β.

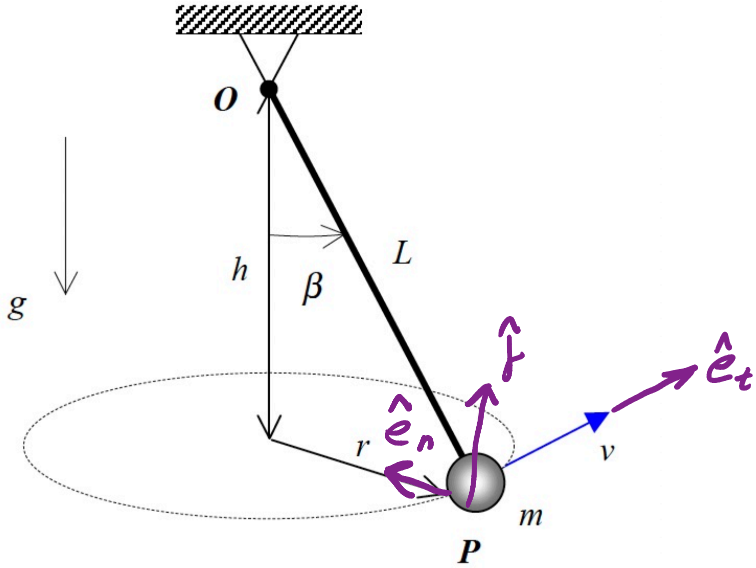


[pg. 208]

Example 4.A.15 p. 208

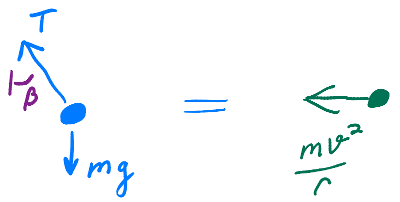
Given: Particle P is attached to a fixed support point O with a cable of length L. P is moving with a speed of v on a horizontal circular path with a radius of r.

Find: Find the angle β. β?



① FBD in "$\hat{e}_n$ & $\hat{j}$ plane"

Kinetics Diagram:



Kinetics & Kinematics

② ΣF in normal direction

$$\Sigma F_n = m(a_n)$$

$$\Rightarrow T\sin\beta = m\frac{v^2}{r} \quad (1)$$

③ ΣF in y direction

$\Sigma F_y = 0$; 0 bc look @ Kinetics diagram

$$\Rightarrow T\cos\beta - mg = 0 \quad (2)$$

④ Solve

$(2) \Rightarrow T = \frac{mg}{\cos\beta}$; Solve for Tension, using (2)

⑤ Plug in (2) onto (1). Solve for β

$$(1) \Rightarrow \frac{mg\sin\beta}{\cos\beta} = m\frac{v^2}{r}$$

$$g\tan\beta = \frac{v^2}{r}$$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$; answer

Summary: Newton's Laws 2

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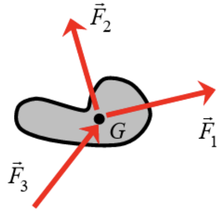
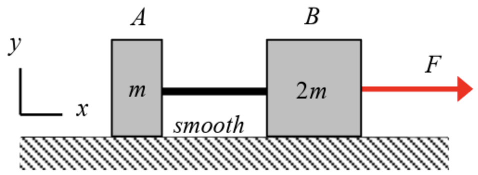
FUNDAMENTAL equation: For a set of forces acting concurrently at the center of mass G of a body, we have Newton's 2nd Law:

$$\sum \vec{F} = m\vec{a}_G$$

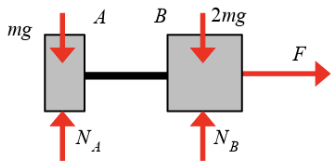


Lec 18 Short Feedback Form:

DRAWING THE RIGHT FBD: Be aware that, for good or bad, "internal forces" of an FBD will not appear in Newton's 2nd Law equation. Consider the system shown with a force *F* acting on block B.

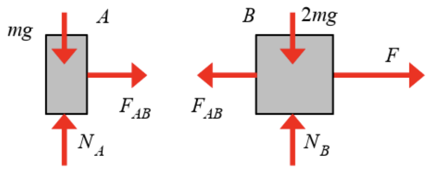


Question 1: What is the acceleration of block B?



$$\sum F_x = F = 3ma \quad \Rightarrow \quad a = \frac{F}{3m}$$

Question 2: What is the force carried by member AB?



$$B: \; \sum F_x = F - F_{AB} = 2ma \quad \Rightarrow \quad F_{AB} = F - 2ma = \frac{F}{3}$$