ME 274 Lecture 17

Newton’s Laws Part 2

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2/21/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 16 (4.A and 4.B) due tomorrow!!**
2. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME

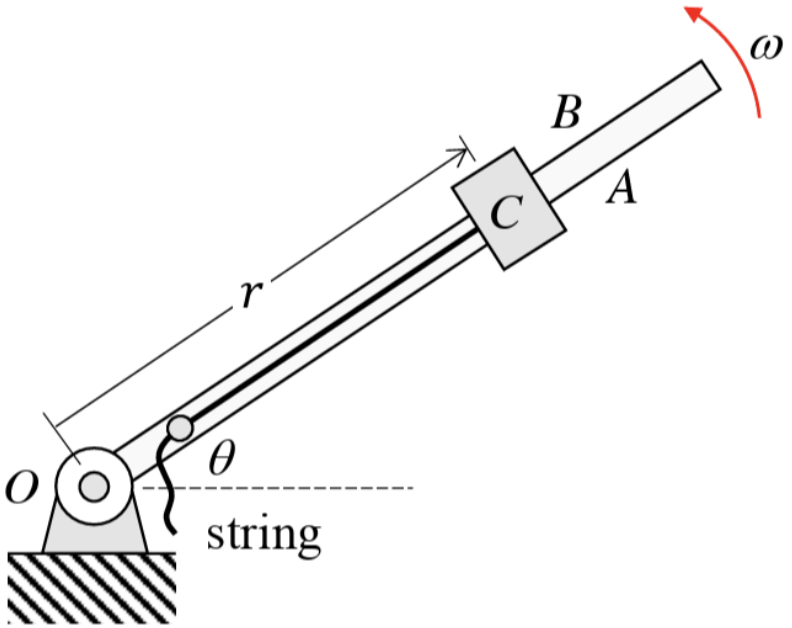
3 Thank you for fb :)

Hint:

Homework H.4.A

Given: An arm rotates about a vertical axis passing through O at a rate of ω with this rotation changing at a rate of $\dot{\omega}$. Block C, having a mass of m, slides smoothly over the arm at a rate of $\dot{r}$ with this sliding motion changing at a rate of $\ddot{r}$.

Find: At this instant, determine:
(a) The tension force in the cord;
(b) The normal force of the arm on block C;
(c) Which side of the arm (A or B) that the block makes contact.



Similar to 4A10 (today in class)

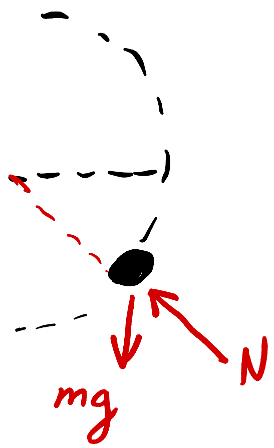
Use the following parameters in your analysis: m = 10 kg, ω = 5 rad/s, $\dot{\omega}$ = 2 rad/s^2, r = 0.3 m, $\dot{r}$ = -0.6 m/s and $\ddot{r}$ = 0 m/s^2.

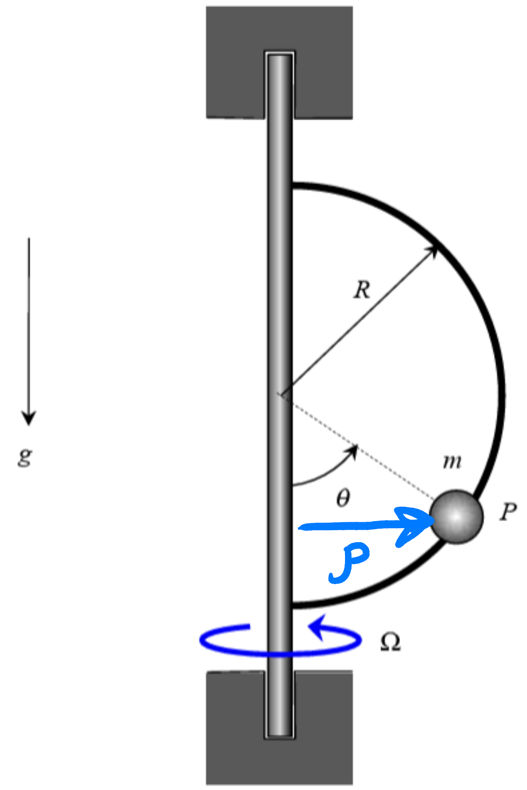
Hint:

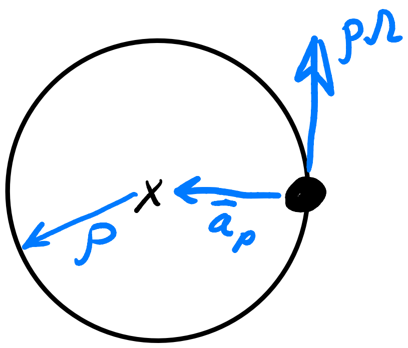
Homework 4.A.17

Given: A rigid semi-circular guide of radius R is attached to a vertical shaft, with the shaft rotating with a constant rate of Ω. Particle P (of mass m) is able to slide on the guide.

Find: If the guide is smooth, determine the angle θ at which the particle will not slide on the guide.







Top View:

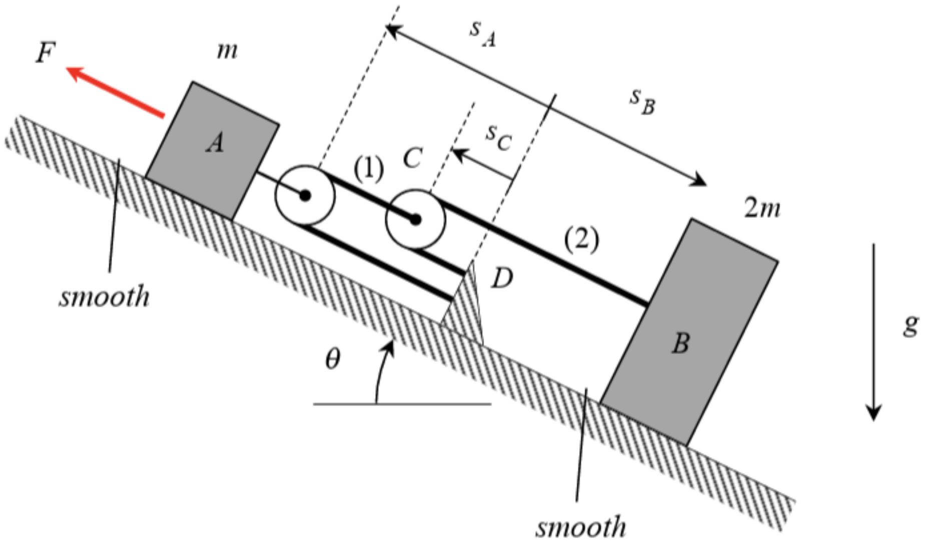
Please leave your answer in terms of, at most, g, m, R and Ω.

Hint: Next page similar

Homework H.4.C due Monday

Given: Blocks A and B (having masses of m and $2m$, respectively) are constrained to move along a smooth inclined surface. Cable (1) is connected to fixed ground at D and to the center of pulley C, as shown, with cable (1) being wrapped around a pulley connected to block A. A second cable (2) is connected between the fixed ground at D and block B. The pulleys are to be assumed to be of negligible mass, and the cables are assumed to be inextensible and not allowed to go slack. The sections of the cables not wrapped around pulleys are parallel to the incline on which blocks A and B move. A force F acts along the direction of the incline on block A.

Find: For this problem, determine the accelerations of blocks A and B.

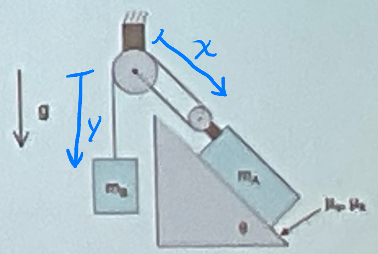


HW due
Wednesday
help:
2/23/26

Homework H.4.C

Given: The system shown below is released from rest in the configuration shown.

Find: Determine:
(a) The acceleration of block A;
(b) The acceleration of block B;
(c) The tension in the cable.

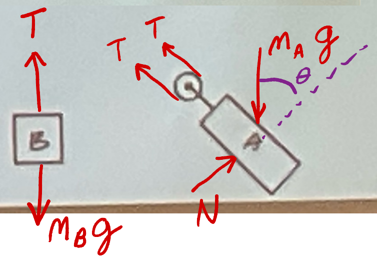


Assume pulleys to be ideal (negligible mass, & no friction)

smooth

Use the following parameters in your analysis: $m_A = 40$ kg, $m_B = 80$ kg, $\mu_s = \cancel{0.3}$ 0, $\mu_k = \cancel{0.5}$ 0 and $\theta = 60°$.

1 FBDs: one for each particle



2 Newton's 2nd law:

$$A: \sum F_x = -2T + m_A g \sin\theta = m_A a_A \quad (1)$$

$$B: \sum F_y = -T + m_B g = m_B a_B \quad (2)$$

Note: 2 eqns, 3 unkns

3 Kinematics:

L = length of cable

$$= y + c_1 + x + c_2$$

$$\frac{dL}{dt} = \dot{y} + 2\dot{x} = 0$$

$$\Rightarrow \dot{y} = -2\dot{x}$$

$$\frac{d\dot{L}}{dt} = \ddot{y} = -2\ddot{x} \Rightarrow a_B = -2a_A \quad (3)$$

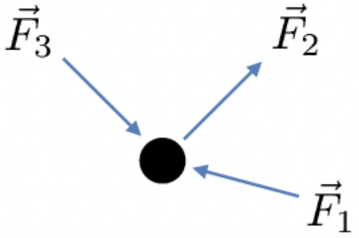
$\vec{a}_A = \ddot{x}$
$\vec{a}_B = \ddot{y}$

Chapter 4: Particle Kinetics

- **Kinetics is the study of the relationship between *kinematics (position/vel/accel)* and force.**
- The principles of **kinetics** derived from the following two laws of Newton:

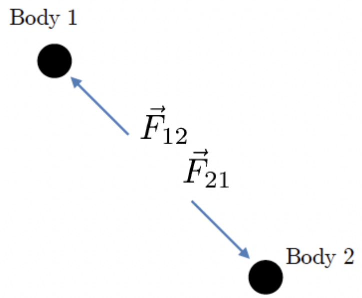
Newtons 2nd law:

$$\sum \vec{F} = m\vec{a}$$



Newtons 3rd law:

$$\vec{F}_{12} = -\vec{F}_{21}$$



- In the first part of this course, we developed sets of kinematic expressions using 3 different coordinate systems. For accelerations these were:

$$\textbf{\textit{Cartesian description}} : \vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$\textbf{\textit{Path description}} : \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\textbf{\textit{Polar description}} : \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

- We want to develop relationships between forces and accelerations of particles **in these three descriptions**

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Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - Newton/Euler
     - Work/Energy
     - Linear impulse/momentum
     - Angular impulse/momentum
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

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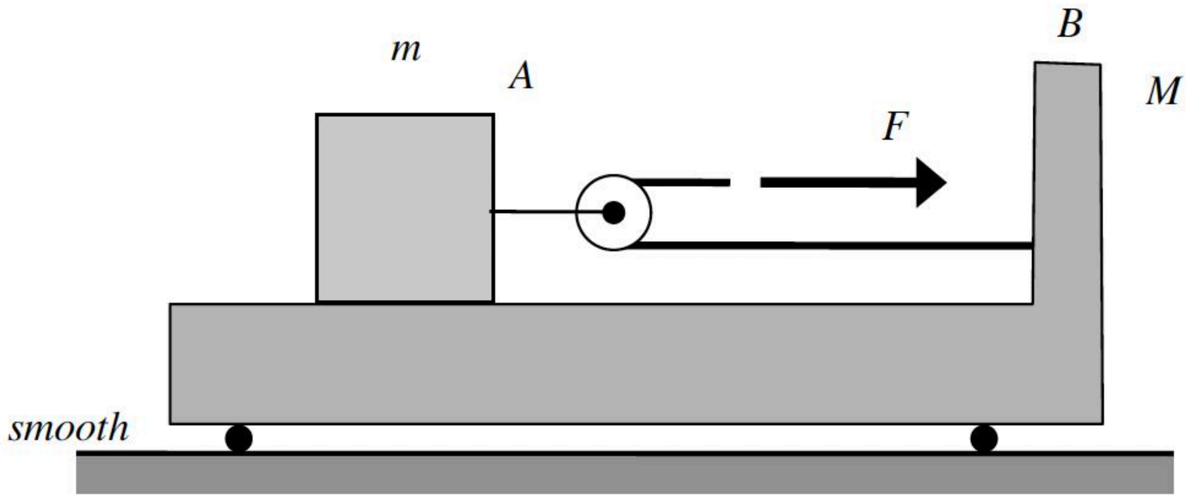
Example 4.A.6

Given: The system of blocks A and B shown below is initially at rest when a force F is applied to the free end of the cable.

Find: Determine the accelerations of blocks A and B: (a) if the surface between A and B is smooth, and (b) if the surface between A and B is rough with a coefficient of kinetic friction being μ_k, and where it is known that block A slips on block B.

Use the following parameters in your analysis: $F = 60$ N, $m = 20$ kg, $M = 100$ kg and $\mu_k = 0.5$.



[pg. 199]

Example 4.A.6 p.199

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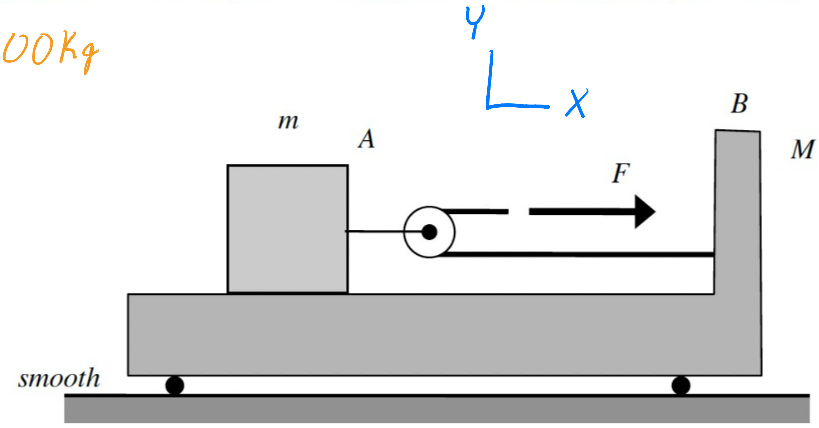
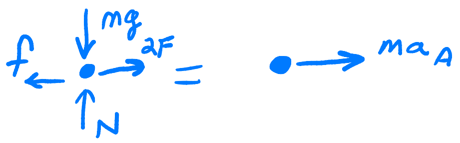
Use the following parameters in your analysis: $F = 60$ N, $m = 20$ kg, $M = 100$ kg and $\mu_k = 0.5$.

$\mu_k = 0.5$; $F = 60N$; $m = 20\,Kg$; $M = 100\,Kg$

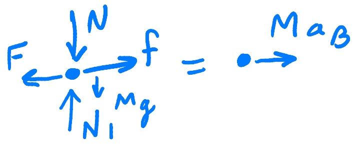


FBD

1



2



Kinetics

3

(a) $\Sigma F_x = m a_x$

$\Rightarrow 2F = m a_A \quad (1)$

$\Rightarrow \vec{a}_A = \frac{2F}{m}\hat{i}$

$\Rightarrow -F = M a_B \quad (2)$

$\Rightarrow \vec{a}_B = \frac{-F}{M}\hat{i}$

4

(b) $\Sigma F_x = m a_x$

$\Rightarrow 2F - f = m a_A \quad (3)$

$\Rightarrow f - F = M a_B \quad (4)$

$f = \mu_k N \quad (5)$

$\Sigma F_y = 0$

$\Rightarrow N - mg = 0$

$\Rightarrow N = mg \; (6)$

Kinematics

5

$(5) \Rightarrow f = \mu_k m g$

$(3) \Rightarrow 2F - \mu_k m g = m a_A$

$\Rightarrow \vec{a}_A = \frac{2F - \mu_k m g}{m}\hat{i}$

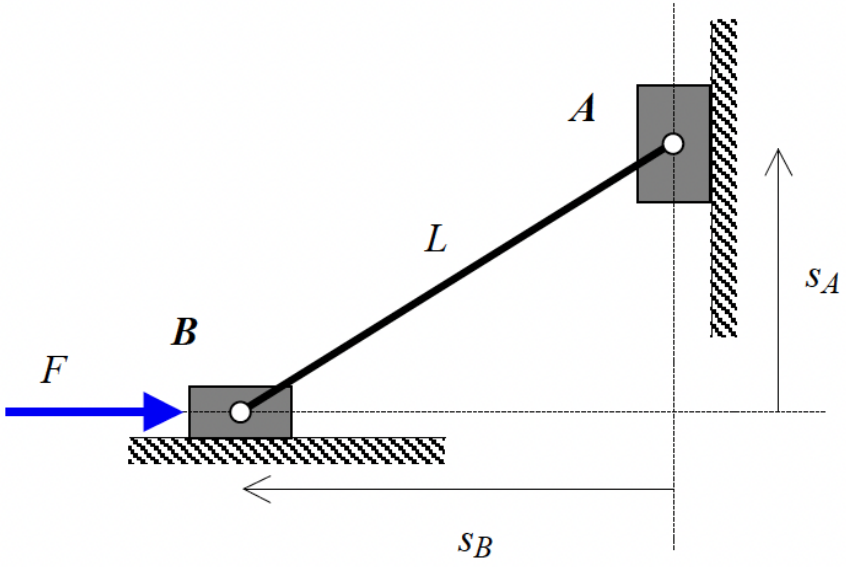
$(4) \Rightarrow \mu_k m g - F = M a_B$

$\Rightarrow \vec{a}_B = \frac{\mu_k m g - F}{M}\hat{i}$

Example 4.A.8

Given: Blocks A and B (having masses of 10 kg and 5 kg, respectively) are constrained to move along smooth, vertical and horizontal guides, as shown in the figure. A and B are connected by a lightweight rod of length $L = 2.5$ m. A force of $F = 50$ N acts to the right on block B. At the position where $s_A = 1.5$ m, A is moving downward with a speed of 4 m/s.

Find: Determine:
(a) The acceleration of A and B at this instant; and
(b) The force in rod AB at this instant.

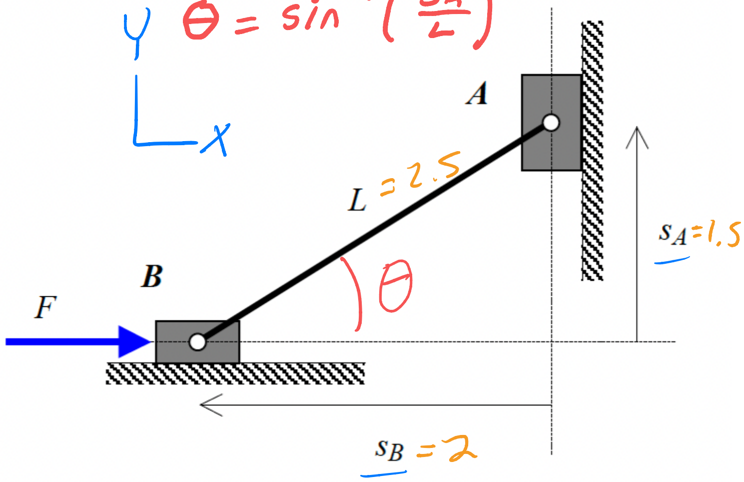


[pg. 201]

Example 4.A.8 p.201 Similar to a hw problem

Given: Blocks A and B (having masses of 10 kg and 5 kg, respectively) are constrained to move along smooth, vertical and horizontal guides, as shown in the figure. A and B are connected by a lightweight rod of length $L = 2.5$ m. A force of $F = 50$ N acts to the right on block B. At the position where $s_A = 1.5$ m, A is moving downward with a speed of 4 m/s.

Find: Determine:
(a) The acceleration of A and B at this instant; and
(b) The force in rod AB at this instant.



$m_A = 10\,kg$; $m_B = 5\,kg$
$L = 2.5\,m$; $F = 50\,N$;
$s_A = 1.5\,m$; $v_A = -4\,m/s$
$s_B = \text{trig...} = 2\,m$

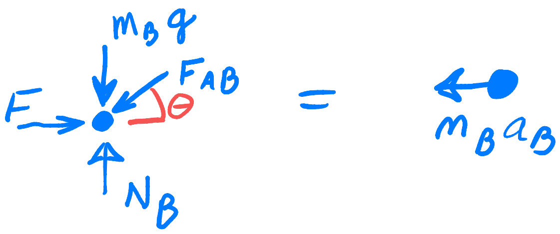
FBD

(1) Use θ for block A FBD

A:

(2) Use θ for block B FBD

B:



6 unknowns
6 eqns:
$F_{AB} = ?$
$N_A = ?$ $N_B = ?$
$a_A = ?$
$v_B = ?$ $a_B = ?$

Kinetics

(3) ΣF in x & y of A

A: $\Sigma F_x = 0$
$\Rightarrow F_{AB}\cos\theta - N_A = 0$ (1)

$\Sigma F_y = m_A a_A$
$\Rightarrow F_{AB}\sin\theta - m_A g = m_A a_A$ (2)

(4) ΣF in x & y of B

B: $\Sigma F_x = m_B a_B$
$\Rightarrow F_{AB}\cos\theta - F = m_B a_B$ (3)

$\Sigma F_y = 0$
$\Rightarrow N_B - m_B g - F_{AB}\sin\theta = 0$ (4)

4 eqns 5 unkns

Kinematics

(5) Method #1: Pythagoras for distance eqn

$s_A^2 + s_B^2 = L^2$ (5)

(6) take derivative for $\vec{v}$

$\frac{d}{dt}(5) \Rightarrow 2 s_A v_A + 2 s_B v_B = 0$ (6)

5 eqns 6 unkns...

(7) take derivative for $\vec{a}$. 6th eqn!

$\frac{d}{dt}(6) \Rightarrow 2 v_A^2 + 2 s_A a_A + 2 v_B^2 + 2 s_B a_B = 0$ (7)

(8) 6 eqns, 6 unkns. Solve.

Kinematics

⑤ Method 2: rigid body eqn

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$$

$$a_A \hat{j} = a_B \hat{i} + (\alpha \hat{k}) \times (L\cos\theta \hat{i} + L\sin\theta \hat{j}) - \omega^2 (L\cos\theta \hat{i} + L\sin\theta \hat{j})$$

$$= [a_B - L\alpha \sin\theta - L\omega^2 \cos\theta] \hat{i} + [L\alpha\cos\theta - L\omega^2 \sin\theta] \hat{j}$$

⑥ Separate $\hat{i}$ & $\hat{j}$

(3) $\hat{i}$: $0 = a_B - L\alpha\sin\theta - L\omega^2\cos\theta$

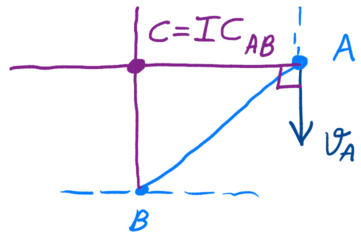
(4) $\hat{j}$: $a_A = L\alpha\cos\theta - L\omega^2\sin\theta$

⑦ Use IC's for (5)

$$v_A = \omega |\vec{r}_{A/C}| = \omega L\cos\theta$$

$$\hookrightarrow \vec{\omega} = \frac{-v_A}{L\cos\theta} \hat{k}$$

(5) $$\omega^2 = \left(\frac{v_A}{L\cos\theta}\right)^2$$



⑧ 4. Solve

$$(1), (2), (3), (4), (5) \Rightarrow a_A, a_B, T$$

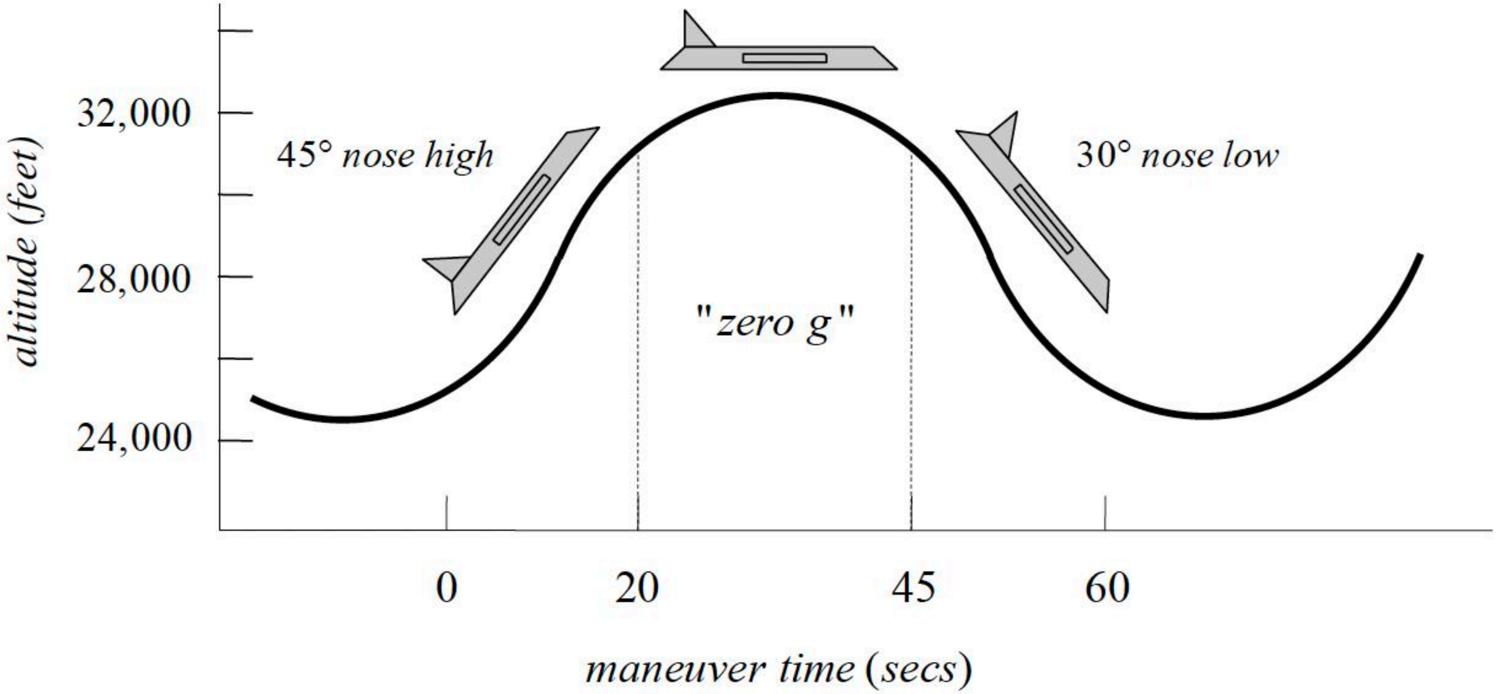
$$\text{eg. } \alpha = \frac{d\omega}{dt};$$

Note about α we cant derivative bc we need ω = valid for all time. And IC is just one Instant.

Example 4.A.9

Given: Shown below is the trajectory taken by an aircraft that is used to simulate weightlessness by its passengers over a short range of motion of the path (the data is from the Zero Gravity Corporation website: http://www.gozerog.com/). From this figure it is seen that the passenger feels weightless over a period of about 25 s near the maximum height of the trajectory. The aircraft is traveling 230 m/s at the maximum height (where the gravitational acceleration is known to be 9.79 m/s^2).

Find: Determine the radius of curvature of the path at that point. Also, identify the shape of the trajectory near the maximum height?

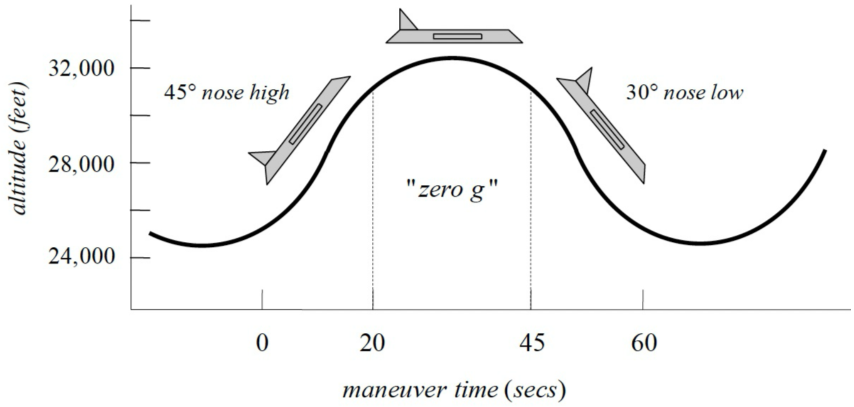


[pg. 202]

Example 4.A.9 p.202

Given: Shown below is the trajectory taken by an aircraft that is used to simulate weightlessness by its passengers over a short range of motion of the path (the data is from the Zero Gravity Corporation website: http://www.gozerog.com/). From this figure it is seen that the passenger feels weightless over a period of about 25 s near the maximum height of the trajectory. The aircraft is traveling 230 m/s at the maximum height (where the gravitational acceleration is known to be 9.79 m/s^2).

Find: Determine the radius of curvature of the path at that point. Also, identify the shape of the trajectory near the maximum height? (a) ρ? (b)



$g = 9.79\ m/s^2$;
$v = 230\ m/s$;
$N = 0$

Kinetics & Kinematics

FBD

① FBD. Use path coord.

$$mg\downarrow,\ N\uparrow \quad = \quad m\dot{v}\rightarrow,\ \frac{mv^2}{\rho}\downarrow$$

② When they say "zero g" they mean "N=0"

$$\Sigma F_n = m\frac{v^2}{\rho}$$

$$\Rightarrow mg - \cancel{N} = m\frac{v^2}{\rho}$$

$$\Rightarrow g = \frac{v^2}{\rho}$$

$$\Rightarrow \rho = \frac{v^2}{g} \quad (a)$$

If $\rho = \frac{v^2}{g} \Rightarrow N = 0$ (weightless)

$\rho < \frac{v^2}{g} \Rightarrow N < 0$

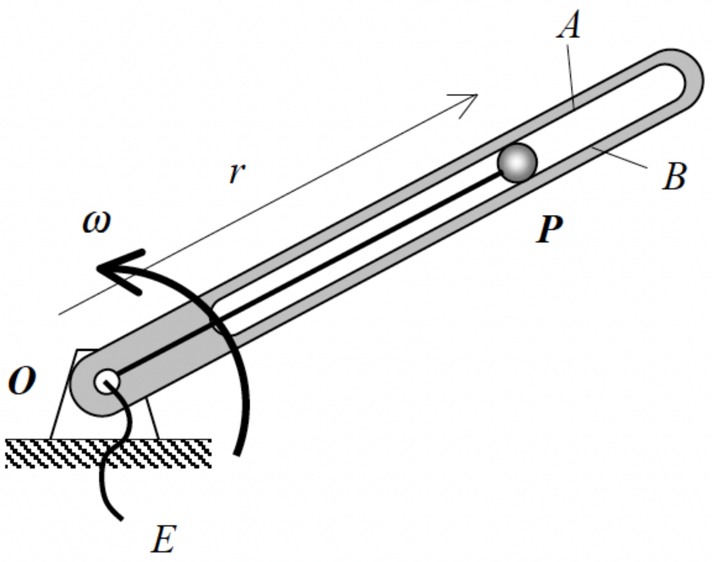
(b) parabola. A plane follows a parabolic trajectory under gravity alone (similar to throwing an object). Since everything falls @ same speed you experience weightlessness

Example 4.A.10

Given: Particle P (weighing W) is able to slide within a straight slot cut into an arm. The arm is rotating within a horizontal plane about end O at a constant rate of ω. The slider is being pulled toward O at a constant rate of $\dot{r}$.

Find: Determine:
(a) The tension in the cord;
(b) The normal contact force of the slot on P; and
(c) Which side of the slot (A or B) is P in contact with?

Use the following parameters in your analysis: W = 5 lb, ω = 5 rad/s, r = 0.75 ft and $\dot{r}$ = -0.5 ft/s.



[pg. 203]

Example 4.A.10 p.203

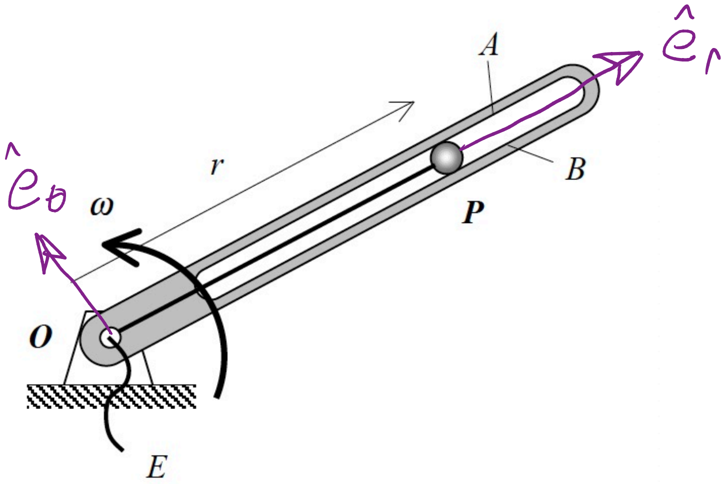
Given: Particle P (weighing W) is able to slide within a straight slot cut into an arm. The arm is rotating within a horizontal plane about end O at a constant rate of ω. The slider is being pulled toward O at a constant rate of $\dot{r}$.

Find: Determine:

(a) The tension in the cord; T?
(b) The normal contact force of the slot on P; and N?
(c) Which side of the slot (A or B) is P in contact with?

Use the following parameters in your analysis: W = 5 lb, ω = 5 rad/s, r = 0.75 ft and $\dot{r}$ = -0.5 ft/s.

$W = 5\,lb$;
$r = 0.75\,ft$ $\dot{r} = -0.5\,ft/s$; $\ddot{r} = 0$
$\omega = 5\,rad/s$; $\dot{\omega} = 0$
$= \dot{\theta}$ $= \ddot{\theta}$



FBD

① FBD. Use polar.

$m a_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$

$m a_r = m(\ddot{r} - r\dot{\theta}^2)$

Kinetics & Kinematics

② ΣF in r direction

$\Sigma F_r = m a_r$

$\Rightarrow -T = m(\cancel{\ddot{r}}^{\,const} - r\cancel{\dot{\theta}}^{\,\omega\,2})$

$= m(-r\omega^2)$ (1)

③ ΣF in θ direction

$\Sigma F_\theta = m a_\theta$

$\Rightarrow N = m(2\dot{r}\cancel{\dot{\theta}}^{\,\omega} + r\cancel{\ddot{\theta}}^{\,const})$

$= 2m\dot{r}\omega$ (2)

④ Solve

(a) (1) $\Rightarrow T = m r \omega^2$

(b) (2) $\Rightarrow N = 2m\dot{r}\omega$ ⟵

(c)

To determine (c), whether or not the sign of N is "+" or "-". If "+", then its in direction drawn (in contact w/ B). And vice versa.

Summary: Newton's Laws 2

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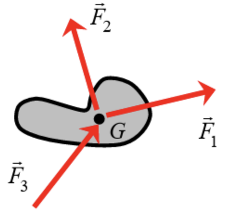
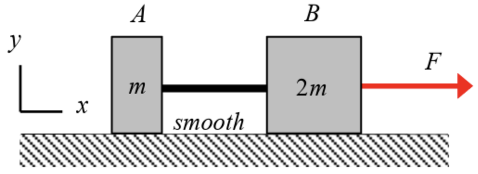
FUNDAMENTAL equation: For a set of forces acting concurrently at the center of mass G of a body, we have Newton's 2nd Law:

$$\sum \vec{F} = m\vec{a}_G$$

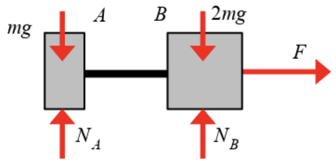


DRAWING THE RIGHT FBD: Be aware that, for good or bad, "internal forces" of an FBD will not appear in Newton's 2nd Law equation. Consider the system shown with a force *F* acting on block B.



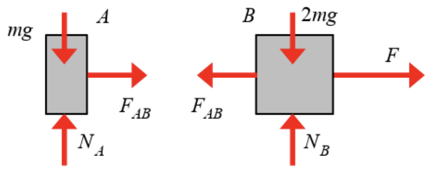
Lec 17 Short Feedback Form:

Question 1: What is the acceleration of block B?



$$\sum F_x = F = 3ma \;\;\Rightarrow\;\; a = \frac{F}{3m}$$

Question 2: What is the force carried by member AB?



$$B:\;\sum F_x = F - F_{AB} = 2ma \;\;\Rightarrow\;\; F_{AB} = F - 2ma = \frac{F}{3}$$