ME 274 Lecture 16

Newton’s Laws Part 1

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2/20/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

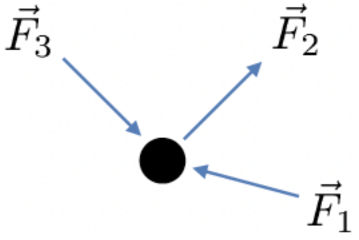
1. HW 15 (3.I and 3.J) due tonight!
2. **Midterm course evaluation due tonight!!!!!**
3. Office hours are changing to ME2008B…
    - Second floor of renovated side of ME
4. One HW problem due Monday is similar to p. 203 4.A.10
    - I will go over this problem on Monday

New chapter! Chapter 4: Particle Kinetics

- **Kinetics is the study of the relationship between *kinematics (position/vel/accel)* and force.**
- The principles of **kinetics** derived from the following two laws of Newton:

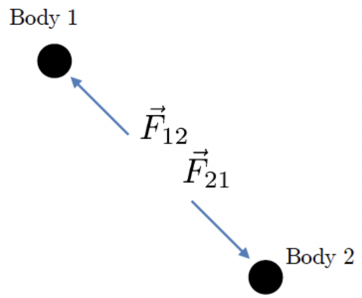
Newtons 2nd law:

$$\sum \vec{F} = m\vec{a}$$



Newtons 3rd law:

$$\vec{F}_{12} = -\vec{F}_{21}$$



- In the first part of this course, we developed sets of kinematic expressions using 3 different coordinate systems. For accelerations these were:

$$\textit{Cartesian description}: \vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$\textit{Path description}: \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\textit{Polar description}: \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

- We want to develop relationships between forces and accelerations of particles **in these three descriptions**

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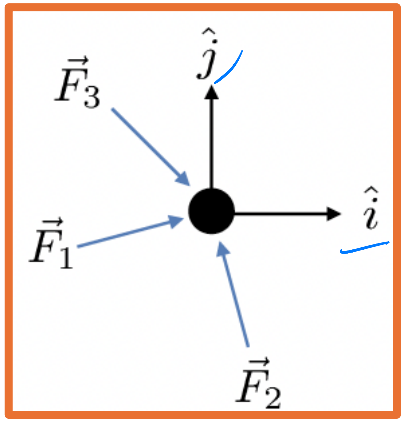
Chapter 4: Particle Kinetics [cont.]

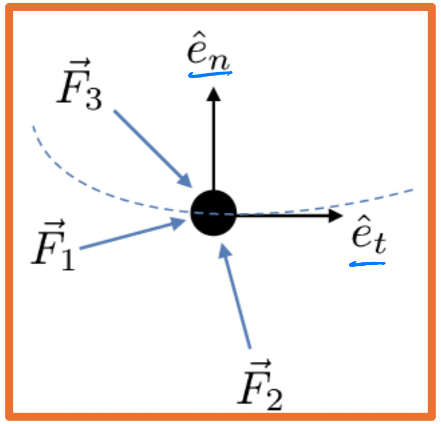
- ***Newton's second law in cartesian, path, and polar coordinates… :***
  - ***Cartesian coordinates:***

$$\sum \vec{F} = m\vec{a} \quad \Rightarrow \quad \sum F_x = m\ddot{x} \qquad \sum F_y = m\ddot{y}$$



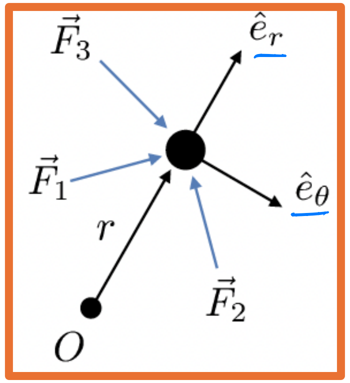
  - ***Path coordinates:***

$$\sum \vec{F} = m\vec{a} \quad \Rightarrow \quad \sum F_t = m\dot{v} \qquad \sum F_n = m\frac{v^2}{\rho}$$



  - ***Polar coordinates:***

$$\sum \vec{F} = m\vec{a} \quad \Rightarrow \quad \sum F_r = m(\ddot{r} - r\dot{\theta}^2) \qquad \sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$



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Kinetics: Four-step problem solving method

1. **FBDs:**
   - Draw appropriate FBD**(s).**
   - Choose your coordinate system.
2. **Kinetics:**
   - Choose what solution method for the particular problem at hand **(we will go over these in the coming days…)**:
     - *Newton/Euler*
     - *Work/Energy*
     - *Linear impulse/momentum*
     - *Angular impulse/momentum*
3. **Kinematics:**
   - Perform needed kinematic analysis (position/velocity/acceleration)
   - Equations from step 2 will guide you in deciding what kinematics are needed for the solution of the problem
4. **Solve:**
   - Count the number of equations/unknowns. *If you do not have enough equations to solve for unknowns:*
     a) Draw more FBDs
     b) You will need to do more kinematic analysis

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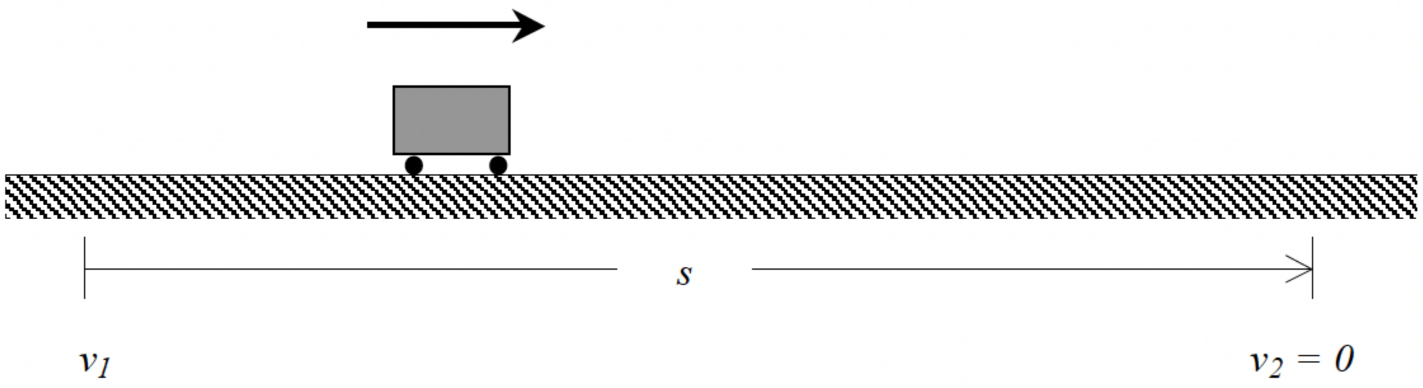
Example 4.A.1

Given: As a car of mass m brakes with a constant braking force F_f, the speed of the car drops from v_1 to a speed of $v_2 = 0$ in a distance of s and in a time t.

Find: Determine:
(a) The distance s; and
(b) The time t.

Use the following parameters in your analysis: $m = 2000$ kg, $v_1 = 80$ m/s and $F_f = 10{,}000$ N.



[pg. 194]

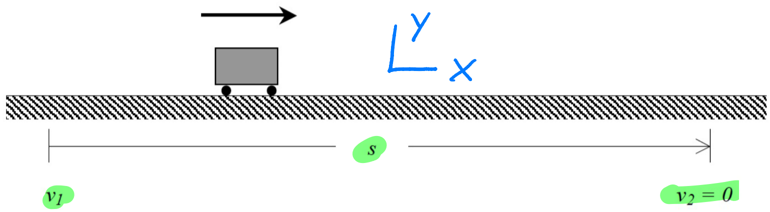
Example 4.A.1

Given: As a car of mass m brakes with a constant braking force F_f, the speed of the car drops from v_1 to a speed of $v_2 = 0$ in a distance of s and in a time t.

Find: Determine:
(a) The distance s; and — stopping s?
(b) The time t. — t?

Use the following parameters in your analysis: $m = 2000$ kg, $v_1 = 80$ m/s and $F_f = 10{,}000$ N.

$m = 2000\,\text{Kg}\ ;\ v_1 = 80\,\text{m/s}$
$F_f = 10000\,\text{N}\ ;\ v_2 = 0$



① "ΣF FBD": F_f ← • (mg ↓, N ↑) = "ma FBD": • → ma

Kinetics

② Forces on x. Kinetics. Newton's second law

$\Sigma F_x = ma$

$\Rightarrow -F_f = ma$

③ Kinematics

For s:

$-F_f = ma$
$= m\dfrac{dv}{dt}$
$= m\dfrac{dv}{ds}\dfrac{ds}{dt}$; chain rule
$= mv\dfrac{dv}{ds}$; $\dfrac{ds}{dt} = v$

For t:

$-F_f = ma$
$= m\dfrac{dv}{dt}$

Solve

④ Find Stop distance, s

$\Rightarrow \int_0^s -F_f\,ds = \int_{v_1}^{0} mv\,dv$

$\Rightarrow -F_f s = -\frac{1}{2}mv_1^2$

$\Rightarrow s = \dfrac{mv_1^2}{2F_f}$

⑤ Find Stop time, t

$\Rightarrow \int_0^t -F_f\,dt = \int_{v_1}^{v_2=0} m\,dv$

$\Rightarrow -F_f t = -mv_1$

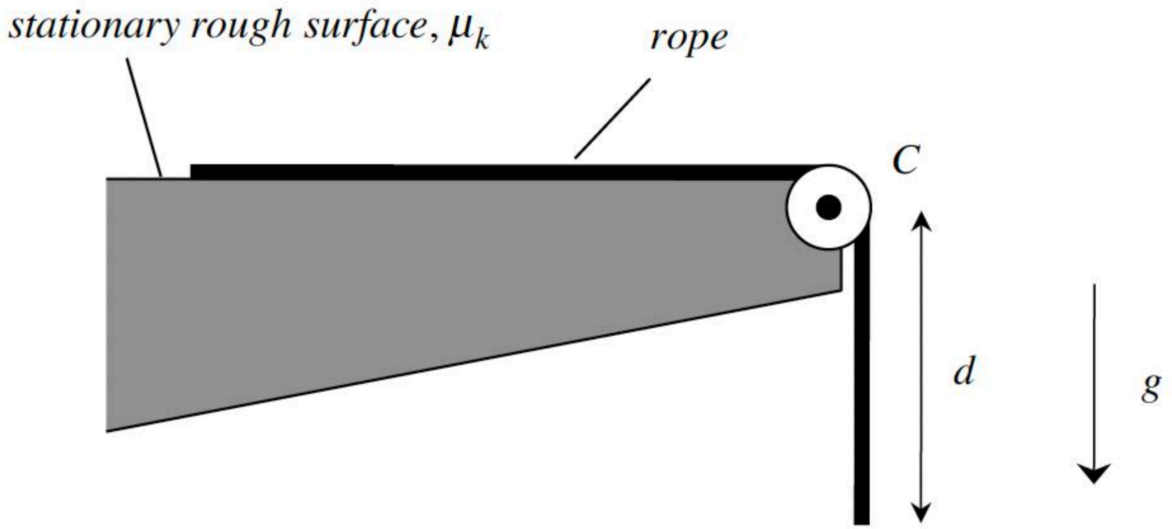
$\Rightarrow t = \dfrac{mv_1}{F_f}$

Example 4.A.2

Given: A rope of length L and mass per length ρ is initially at rest on a rough, horizontal surface (with coefficient of kinetic friction μ_k) and with a portion of its length d hanging over a pulley on the right end of the resting surface. Ignore the mass of the pulley.

Find: Determine the speed of the rope when the left end of the rope reaches the pulley. Assume that the rope remains taut.

Use the following relationship in your analysis: $d = 0.2L$.



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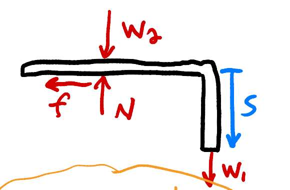
Find: Determine the speed of the rope when the left end of the rope reaches the pulley. Assume that the rope remains taut. v ?

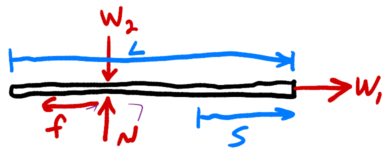
Use the following relationship in your analysis: $d = 0.2L$.

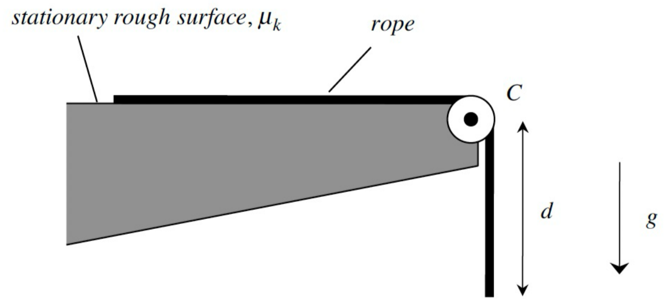
① FBD

Vertical FBD:



Horizontal FBD:





FYI: $m = \rho L$ (density, Length)

② Define variables

$W_1 = \rho s \cdot g$

$W_2 = \rho (L-s) \cdot g$

$N = W_2 = \rho(L-s) \cdot g$

$f = \mu_k N = \mu_k \cdot \rho (L-s) \cdot g$

Kinetics:

③ Sum of forces in

$$\Sigma F_s = W_1 - f = m\ddot{s}$$

$$(\rho s \cdot g) - [\mu_k \cdot \rho(L-s) \cdot g] = m\ddot{s}$$

Kinematics

④

Chain rule:

$$\ddot{s} = \frac{d\dot{s}}{dt} = \frac{d\dot{s}}{ds}\frac{ds}{dt} = \dot{s}\frac{d\dot{s}}{ds} \; ; \; \dot{s} = v$$
$$= v\frac{dv}{ds}$$

Solve

⑤ Solve

$$(\rho s \cdot g) - [\mu_k \cdot \rho(L-s) \cdot g] = \rho L v \frac{dv}{ds} \quad ; \; m = \rho L$$

$$-\mu_k \frac{g}{L} + \frac{(1+\mu_k)g}{L} s = v\frac{dv}{ds} \quad ;$$

$$\int_d^L \left[-\mu_k \frac{g}{L} + \frac{(1+\mu_k)g}{L} s\right] ds = \int_0^v v\, dv \quad ; \text{ separation of vars}$$

$$\left[-\mu_k \frac{g}{L} s + \frac{(1+\mu_k)g}{2L} s^2\right]_d^L = \frac{1}{2}v^2$$

$$\left[-\mu_k \frac{g}{L}(L-d) + \frac{1}{2}\frac{(1+\mu_k)g}{L}(L^2-d^2)\right] = \frac{1}{2}v^2$$

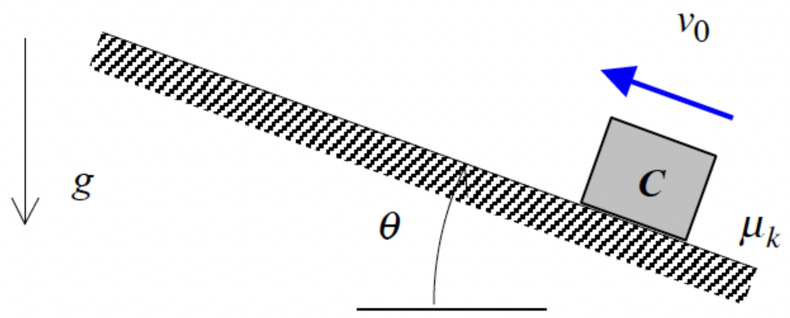
$$\longrightarrow v = \sqrt{-2\mu_k(L-d) + \frac{(1+\mu_k)g}{L}(L^2-d^2)} \longleftarrow v \text{ @ } s = L$$

Example 4.A.3

Given: Crate C of weight W is sliding with a speed of v_0 up a rough incline (with a coefficient of kinetic friction of μ_k).

Find: Determine the time t after which the crate comes to rest.

Use the following parameters in your analysis: $W = 100$ lb, $\theta = 36.87°$, $v_0 = 40$ ft/s and $\mu_k = 0.25$.



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Example 4.A.3

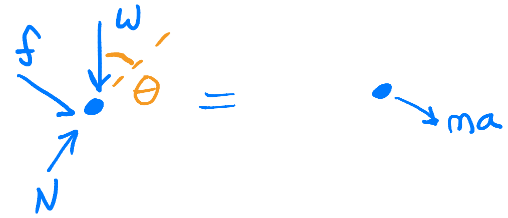
Given: Crate C of weight W is sliding with a speed of v_0 up a rough incline (with a coefficient of kinetic friction of μ_k).

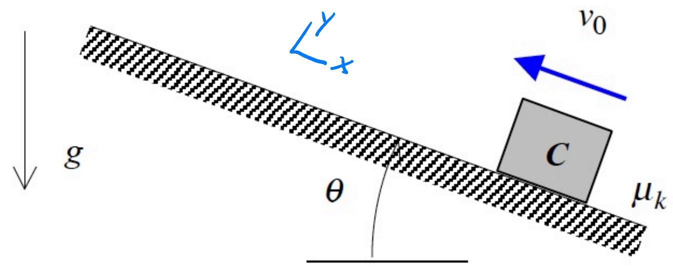
Find: Determine the time t after which the crate comes to rest. t?

Use the following parameters in your analysis: $W = 100$ lb, $\theta = 36.87°$, $v_0 = 40$ ft/s and $\mu_k = 0.25$.

① FBD

"ΣF FBD:" "ma FBD":





② Horizontal Forces

$\Sigma F_x = ma$

$\Rightarrow f + W\sin\theta = ma \quad (1)$

③ Vertical Forces

$\Sigma F_y = 0$

$\Rightarrow N - W\cos\theta = 0 \quad (2)$

$f = \mu_k N \quad (3)$

④ Use eqns (2) & (3). Solve for N & f

$(2) \Rightarrow N = W\cos\theta$

$(3) \Rightarrow f = \mu_k W\cos\theta$

⑤ w/ N & f, use eqn (1). Solve for t

$(1) \Rightarrow \mu_k W\cos\theta + W\sin\theta = ma = \frac{W}{g}a$

$\Rightarrow \mu_k g\cos\theta + g\sin\theta = a = \frac{dv}{dt}$

$\Rightarrow \int_0^t (\mu_k g\cos\theta + g\sin\theta)\,dt = \int_{-v_0}^0 dv$; separate variables, Integrate dt, dv

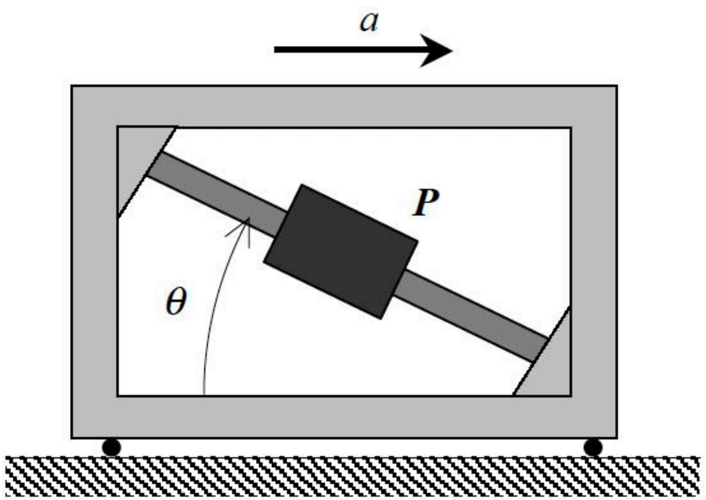
$\Rightarrow (\mu_k g\cos\theta + g\sin\theta)t = [v]_{-v_0}^0 = v_0$; Solve for t

$$t = \frac{v_0}{\mu_k g\cos\theta + g\sin\theta}$$

Example 4.A.4

Given: A collar P of mass m is free to slide along a smooth rod that is mounted at angle of $\theta = 36.87°$ in a frame. The frame is constrained to move along a horizontal surface, as shown, with a constant acceleration of a.

Find: Determine the value of a that is required for the collar to not slide along the rod as the frame accelerates to the right.



[pg. 197]

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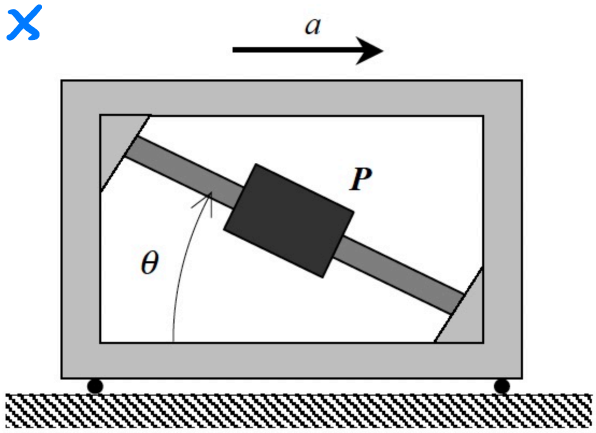
Find: Determine the value of a that is required for the collar to not slide along the rod as the frame accelerates to the right.

a?

$\theta = 36.87$



① FBD

"ΣF FBD:" "ma FBD":

mg, N, θ = ma

make 'ma' purely to the right

② Forces on x

$\Sigma F_x = ma$

$\Rightarrow N \sin\theta = ma \quad (1)$

③ Forces on y

$\Sigma F_y = 0$

$\Rightarrow N\cos\theta - mg = 0 \quad (2)$

④ Solve Eqn (2) for N. Then, Solve Eqn (1) (w/ (2)) for a

$(2) \Rightarrow N = \frac{mg}{\cos\theta}$

$(1) \Rightarrow \frac{mg\sin\theta}{\cos\theta} = ma$

$\Rightarrow g\tan\theta = a$

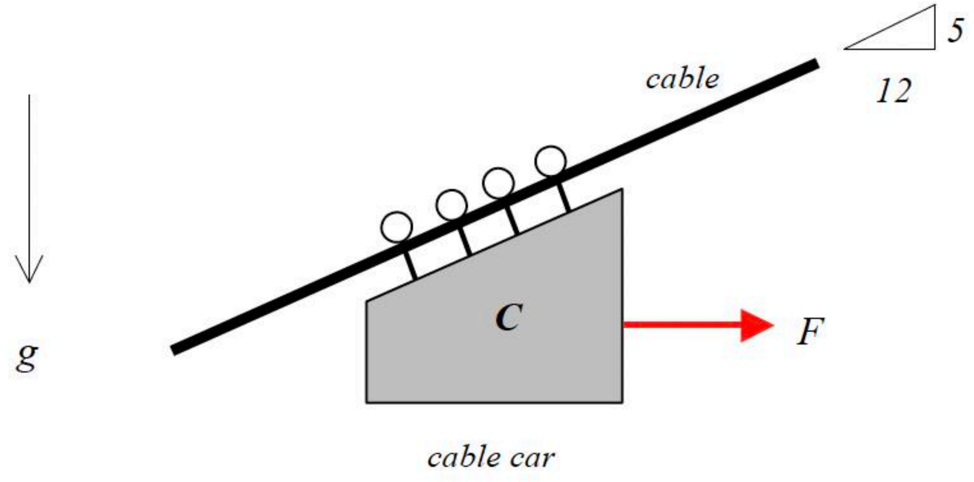
Example 4.A.5

Given: The cable car C (of mass m = 200 kg) is pulled up a taut cable with a horizontal force F = 2400 N.

Find: Determine:

(a) The acceleration of the cable car; and

(b) The total force acting on the cable by the cable car.



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Example 4.A.5

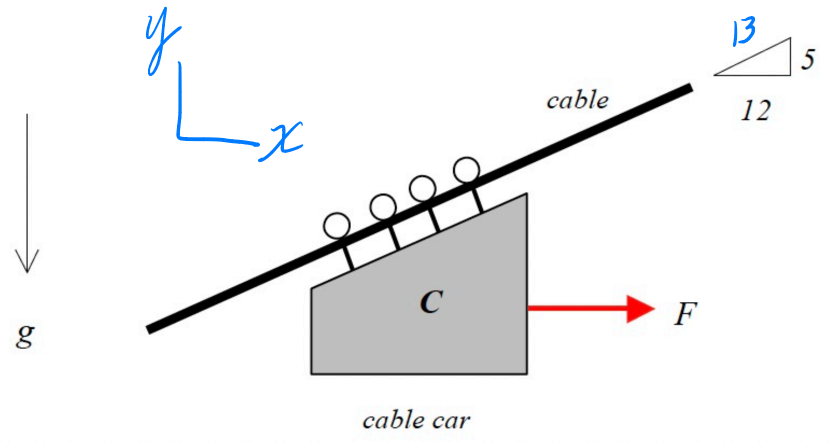
Given: The cable car C (of mass m = 200 kg) is pulled up a taut cable with a horizontal force F = 2400 N.

Find: Determine:

(a) The acceleration of the cable car; and a?

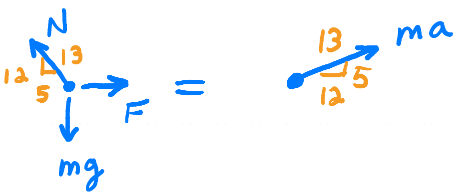
(b) The total force acting on the cable by the cable car. N_c?

$m = 200\,kg$
$F = 2400\,N$



① FBD

"ΣF FBD": "ma FBD":



② Horizontal Forces

$\Sigma F_x = m a_x$

$\Rightarrow F - \frac{5}{13} N = \frac{12}{13} m a \quad (1)$

③ Vertical Forces

$\Sigma F_y = m a_y$

$\Rightarrow \frac{12}{13} N - mg = \frac{5}{13} m a \quad (2)$

Kinematics

④ Find Normal force exerted on cable

$\vec{N}_c = -\vec{N}$

$= -N\left(-\frac{5}{13}\hat{\imath} + \frac{12}{13}\hat{\jmath}\right) \quad (3)$

⑤ Project value of $\vec{a}$

$\vec{a} = a\left(\frac{12}{13}\hat{\imath} + \frac{5}{13}\hat{\jmath}\right) \quad (4)$

Solve

⑥ Solve for N & a. Using (1) & (2)

⑦ Solve for $\vec{N}_c$ & $\vec{a}$. Using (3) & (4)

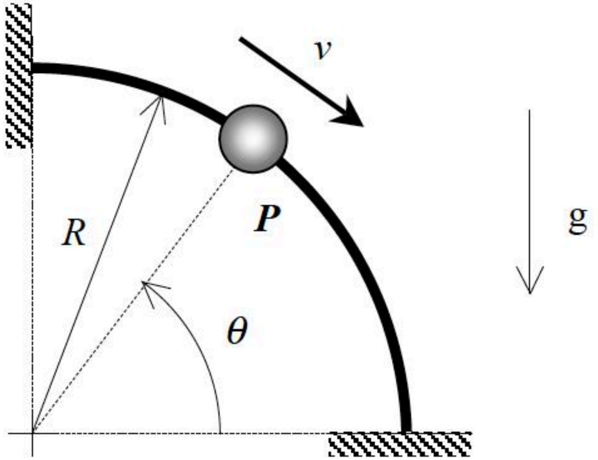
Example 4.A.7

Given: Particle P, having a weight of $W = 5$ lb, slides along a smooth, curved rod where $R = 3$ ft. At the position where $\theta = 53.13°$, the speed of P is known to be 20 ft/s.

Find: Determine:

(a) The normal force acting on P by the rod at the instant shown; and

(b) The rate of change of speed of P at the instant shown.



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Example 4.A.7 p.200

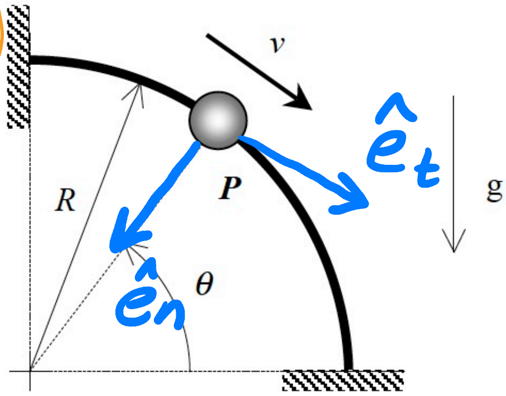
Given: Particle P, having a weight of $W = 5$ lb, slides along a smooth, curved rod where $R = 3$ ft. At the position where $\theta = 53.13°$, the speed of P is known to be 20 ft/s.

Find: Determine:

(a) The normal force acting on P by the rod at the instant shown; and N?

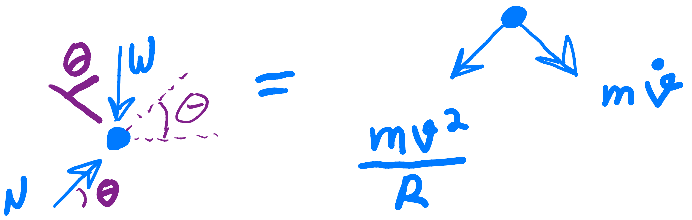
(b) The rate of change of speed of P at the instant shown. $\dot{v}$?
(path description)

$W = 5\,lb$; $R = 3\,ft$; $\theta = 53.13°$; $v = 20\,ft/s$



① FBD.

"ΣF FBD:" "ma FBD": (in line w/ axes)



Kinetics & Kinematics

② Aside... Kinematics

$a_t = \dot{v} = ?$

$a_n = \frac{v^2}{\rho} = \frac{v^2}{R}$

③ Sum of forces in 't' direction

$\Sigma F_t = m\dot{v}$

$\Rightarrow W\cos\theta = m\dot{v}$

$= \frac{W\dot{v}}{g}$ (1)

④ Sum of forces in 'n' direction

$\Sigma F_n = m\frac{v^2}{R}$

$\Rightarrow -N + W\sin\theta = \frac{mv^2}{R}$

$= \frac{Wv^2}{gR}$ (2)

⑤ Use (1) & (2) to solve for N & $\dot{v}$

Summary: Newton's Laws 1

FUNDAMENTAL equation: For a set of forces acting concurrently at the center of mass G of a body, we have Newton's 2nd Law:

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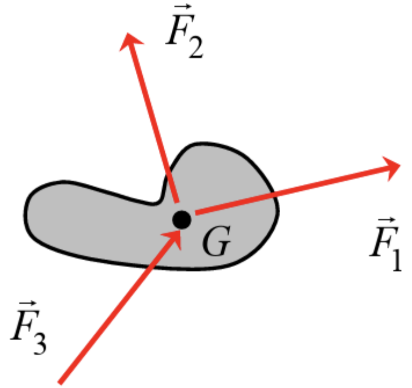
$$\sum \vec{F} = m\vec{a}_G$$



SOLUTION PROCESS:

1. Draw free body diagram (FBD) for the bodies of interest.
2. Write down the vector components of Newton's 2nd Law for each FBD.
3. Write down the appropriate kinematics (acceleration) equations for the right-hand side of the equations.
4. Count the number of equations available from above, and count the number of unknowns. If you have enough equations, solve for the desired unknowns. If you do not have enough equations, then you have either missed needed information from kinematics or you need to draw more/different FBDs.

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Lec 16 Short Feedback Form: