ME 274 Lecture 12

Moving Reference Kinematics: 2D Part 2

Eugenio “Henny” Frias-Miranda

2/9/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 11 due tonight!!**
2. **Exam 1 Details on course website (https://www.purdue.edu/freeform/me274/exams-spring-2026/)**
3. **Lecture on Wednesday**
    - No HW due on Wednesday… This HW will be due on Friday.
4. **No lecture on Friday**

Goals: *Chapter 3 – Moving Reference Frame Kinematics*

1. Develop the **“moving reference frame”** velocity and acceleration kinematic equations

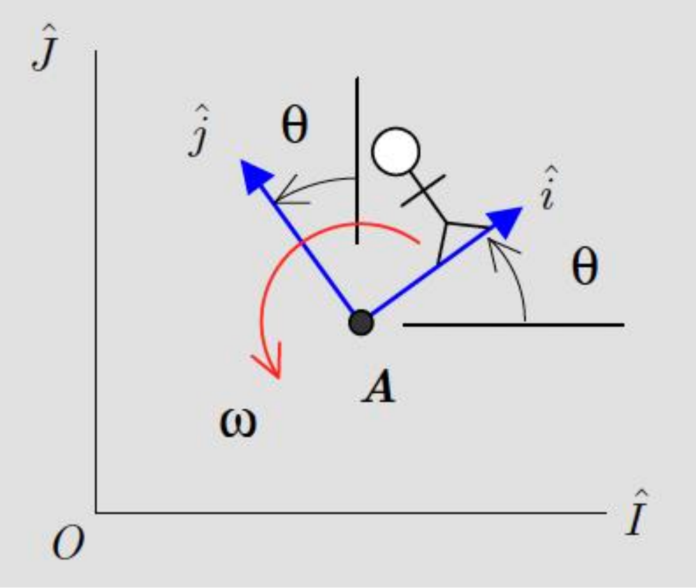
2. Apply these equations to the analysis of:
   a) More complicated **planar mechanisms**
   b) Problems in **3 dimensions**

[pg. 143 content]

What do each of the terms mean?

$$\boxed{\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}}$$

$$\boxed{\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right]}$$



- $\vec{v}_A$ and $\vec{v}_B$ are the velocities seen by a **fixed observer** [XYZ]
- $\vec{a}_A$ and $\vec{a}_B$ are the accelerations seen by **fixed observer** [XYZ]
- $\vec{\omega}$ is angular velocity of the **moving observer** [xyz]
- $\vec{\alpha}$ is angular acceleration of the **moving observer** [xyz]

How does the observer move?

- $\left(\vec{v}_{B/A}\right)_{rel}$ is the "velocity of point B as seen by the **moving observer** at A"
- $\left(\vec{a}_{B/A}\right)_{rel}$ is the "acceleration of point B as seen by the **moving observer** at A"
- $2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel}$ is known as the **"Coriolis" component** of acceleration.
  - Arises when observer has a non-zero angular velocity

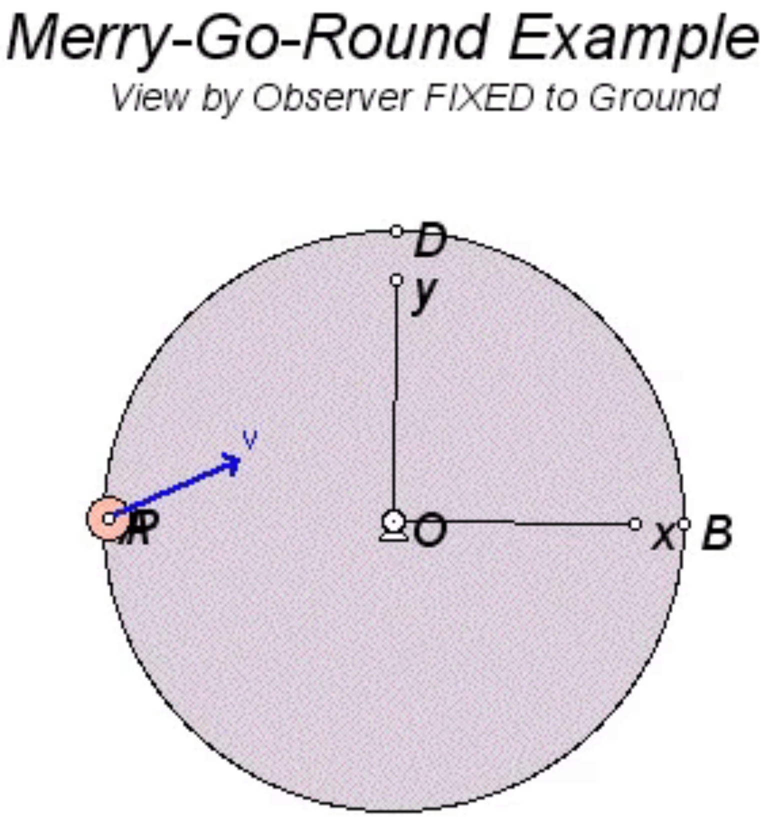
What does the observer see

[pg. 147 content]

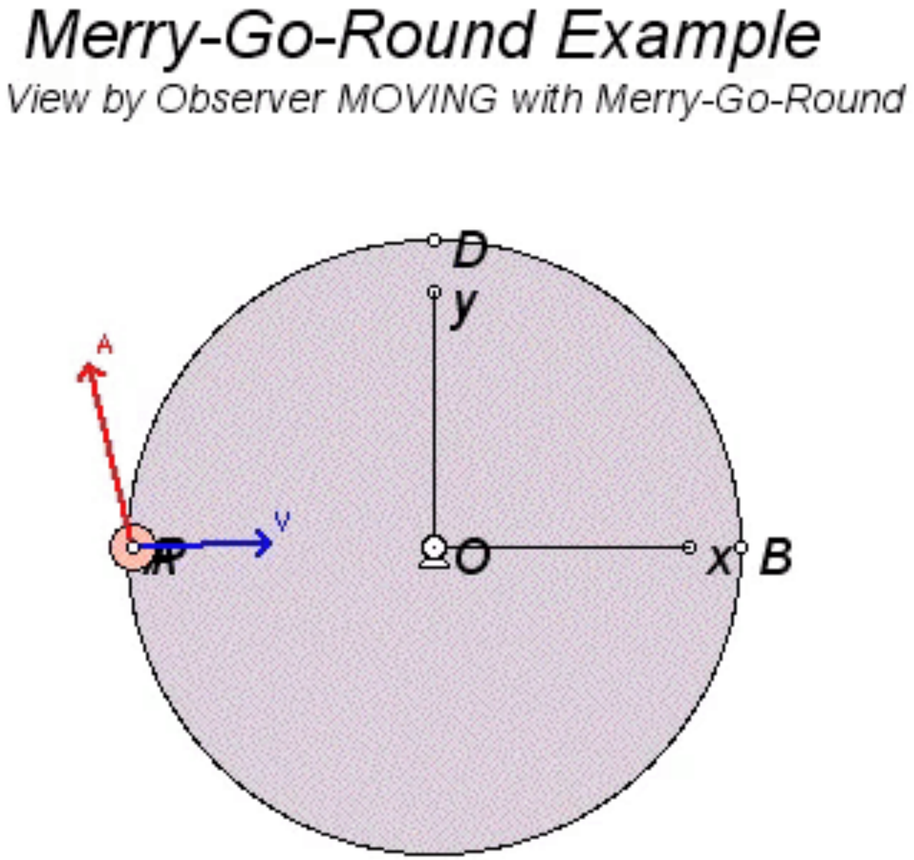
Merry-Go-Round Example: IRL Video

[Animation - THE MERRY-GO-ROUND AND THE CORIOLIS COMPONENT OF ACCELERATION]

Merry-Go-Round Example: Simulation

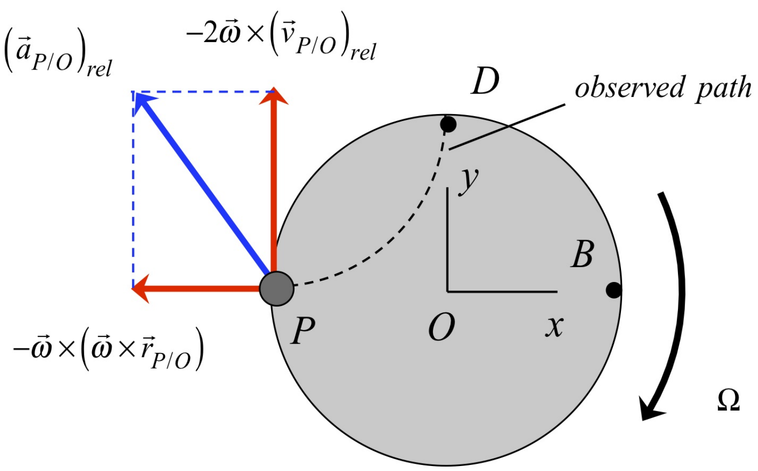


Stationary Observer



Rotating Observer

[Animation - THE MERRY-GO-ROUND AND THE CORIOLIS COMPONENT OF ACCELERATION]



$$\omega = -\Omega\,\hat{k} = \text{constant}\ (CW)$$

$$\alpha = 0$$

$$\left(\vec{a}_{P/O}\right)_{rel} = -2\vec{\omega}\times\left(\vec{v}_{P/O}\right)_{rel} - \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{P/O}\right)$$

Find: acceleration of P as seen by observer on merry-go-round. $(a_{P/O})_{rel}$?

Solution:

① Attach observer & xyz to merry-go-round

② Solve for $(a_{P/O})_{rel}$

no net force acting on ball

$$\cancel{a_P}^{\,0} = \cancel{a_O}^{\,0} + (a_{P/O})_{rel} + \cancel{\alpha}^{\,0} \times r_{P/O} + 2\omega\times(v_{P/O})_{rel} + \omega\times(\omega\times r_{P/O})$$

$$(a_{P/O})_{rel} = -2\omega\times(v_{P/O})_{rel} - \omega\times(\omega\times r_{P/O})$$

$$= -2(-\Omega\hat{k})\times(v_{rel}\,\hat{i}) - (-\Omega\hat{k})\times\left[(-\Omega\hat{k})\times(-r\,\hat{i})\right]$$

$$= -r\Omega^2\,\hat{i} + \underbrace{2\Omega v_{rel}}\,\hat{j}\ \Big\}$$ observed acceleration of the ball

- originates from Coriolis component of acceleration
- say that ball tends to curve to the left of desired path

Example 3.A.5

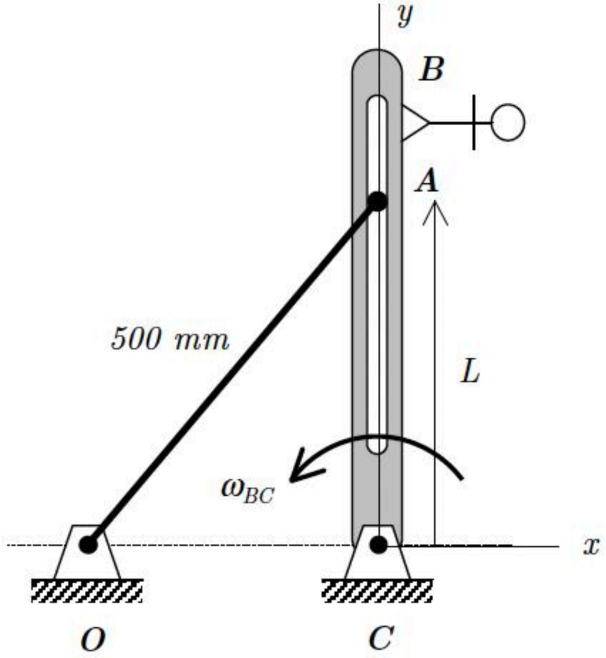
Given: The mechanism shown below is made up of links OA and BC. Links OA and BC are pinned to ground at points O and C, respectively. Pin A of link OA is able to slide within a slot that is cut in link BC, as shown. Pins O and C are on the same horizontal line. Let the xyz axes be attached to link BC. At the instant shown:

- Link BC is vertical with $L = 400$ mm; and
- Link BC is rotating counterclockwise at a rate of $\omega = 6$ rad/s.

Find: Determine:

(a) The angular velocity of link OA at the instant shown; and

(b) The value for $\dot{L}$ at the instant shown.



[pg. 154]

Example 3.A.5

p.154

Given: The mechanism shown below is made up of links OA and BC. Links OA and BC are pinned to ground at points O and C, respectively. Pin A of link OA is able to slide within a slot that is cut in link BC, as shown. Pins O and C are on the same horizontal line. Let the xyz axes be attached to link BC. At the instant shown:

- Link BC is vertical with $L = 400$ mm; and
- Link BC is rotating counterclockwise at a rate of $\omega = 6$ rad/s.

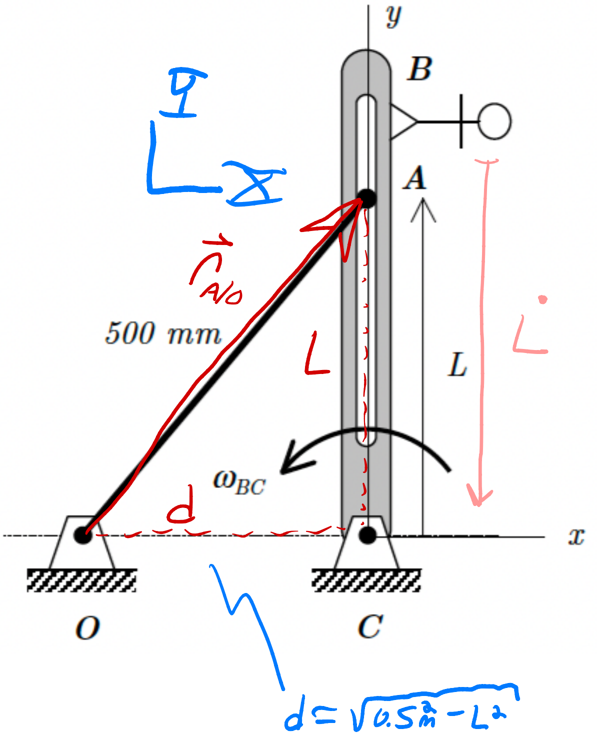
Find: Determine:

(a) The angular velocity of link OA at the instant shown; and ω_{OA}?

(b) The value for $\dot{L}$ at the instant shown. $\dot{L}$?

$L = 400\,mm$

$\omega = 6\,rad/s$



① Capital Letter Axes

② Link OA

$$\vec{v}_A = \vec{v}_O + \vec{\omega}_{OA} \times \vec{r}_{A/O}$$

fixed pt

$$= \vec{0} + \omega_{OA}\hat{K} \times (d\hat{I} + L\hat{J})$$

$$= d\omega_{OA}\hat{J} - \omega_{OA}L\hat{I}$$

③ Link CA

$$\vec{v}_A = \vec{v}_C + (\vec{v}_{A/C})_{rel} + \vec{\omega} \times \vec{r}_{A/C}$$

fixed pt

$$= \vec{0} + \dot{L}\hat{j} + \omega_{BC}\hat{k} \times L\hat{j}$$

$$= \dot{L}\hat{j} + -\omega_{BC}L\hat{i}$$

④ Equate CA & OA

$$\hat{i}:\; -\omega_{OA}L = -\omega_{BC}L$$

$$\Rightarrow \vec{\omega}_{OA} = \vec{\omega}_{BC} = \omega_{BC}\hat{k}$$

$$\hat{j}:\; d\omega_{OA} = \dot{L}$$

⑤ Solve for ω_{OA} & $\dot{L}$

Example 3.A.6

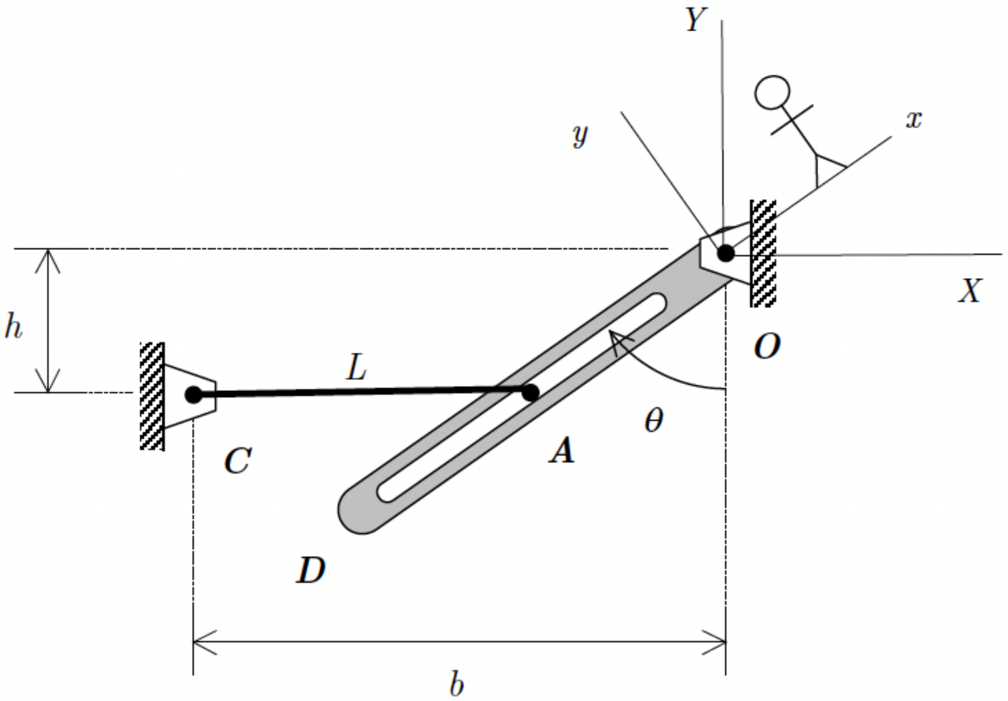
Given: Link OD is rotating clockwise at a constant rate of $\dot{\theta} = 2$ rad/s. When $\theta = 45°$, link CA is horizontal.

Find: Determine:

(a) The velocity of A when $\theta = 45°$; and

(b) The acceleration of A at the same position

Use the following parameters in your analysis: $L = 0.225$ m, $h = 0.225$ m and $b = 0.45$ m.



[pg. 155]

Example 3.A.6

p.155

Given: Link OD is rotating clockwise at a constant rate of $\dot{\theta} = 2$ rad/s. When $\theta = 45°$, link CA is horizontal.

Find: Determine:

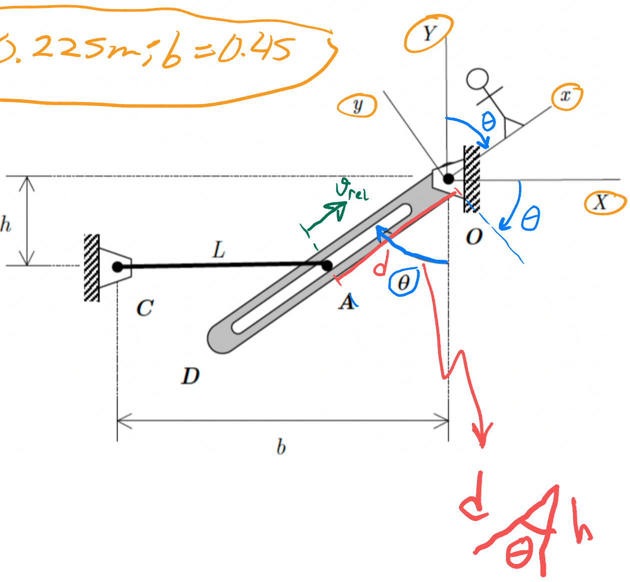
(a) The velocity of A when $\theta = 45°$; and $\vec{v}_A$?

(b) The acceleration of A at the same position $\vec{a}_A$?

Use the following parameters in your analysis: $L = 0.225$ m, $h = 0.225$ m and $b = 0.45$ m.

$\boxed{\theta = 45° ;\ \dot{\theta} = 2\,\text{rad/s} ;\ \ddot{\theta} = 0 ;\ L = 0.225\text{m} ;\ h = 0.225\text{m} ; b = 0.45}$

① Both Axes Defined by the problem



Velocity

② Link CA, on same rigid body

$\vec{v}_A = \vec{v}_c + \vec{\omega}_{AC} \times \vec{r}_{A/c}$

$= \vec{0} + \omega_{AC}\hat{K} \times L\hat{I}$

fixed

$= \omega_{AC} L \hat{J}$

③ Points O & A. Different rigid bodies

$\vec{v}_A = \vec{v}_o + (\vec{v}_{A/o})_{rel} + \vec{\omega} \times \vec{r}_{A/o}$

$= \vec{0} + v_{rel}\hat{i} + -\dot{\theta}\hat{k} \times -d\hat{i}$

fixed CW

$= v_{rel}\hat{i} + d\dot{\theta}\hat{j}$

$\cos\theta = \frac{h}{d}$

$\Rightarrow d = \frac{h}{\cos\theta}$

④ trig to convert $XYZ \Rightarrow xyz$: $\hat{J} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

⑤ Combine: CA . O&A. 2 eqns 2 unkns

$\hat{i}: \omega_{AC} L \cos\theta = v_{rel}$

$\hat{j}: \omega_{AC} L \sin\theta = d\dot{\theta}$

⑥ Solve for v_{rel} & ω_{AC}. Then plug back solve for $\vec{v}_A$

$> \begin{matrix} v_{rel} ✓ \\ \omega_{AC} ✓ \end{matrix} \Rightarrow \vec{v}_A$

Part (a)

Acceleration

$\boxed{\vec{\omega} = -\dot{\theta}\hat{k} ;\ \vec{\alpha} = \vec{0}\hat{k} ;\ (\vec{v}_{A/o})_{rel} = \dot{d}\hat{i} ;\ (\vec{a}_{A/o})_{rel} = \ddot{d}\hat{i}}$

⑦ Link CA, on same rigid body

$\vec{a}_A = \vec{a}_c + \vec{\alpha}_{AC} \times \vec{r}_{A/c} - \omega_{AC}^2 r_{A/c}$

$= \vec{0} + \alpha_{AC}\hat{K} \times L\hat{I} - \omega_{AC}^2 L\hat{I}$

fixed

$= \alpha_{AC} L\hat{J} - \omega_{AC}^2 L\hat{I}$

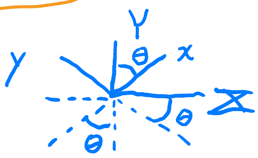
⑧ Points O & A. Different rigid bodies

$\vec{a}_A = \vec{a}_o + (\vec{a}_{A/o})_{rel} + \vec{\alpha} \times \vec{r}_{A/o} + 2\vec{\omega} \times (\vec{v}_{A/o})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/o})$

$= \vec{0} + a_{rel}\hat{i} + \vec{0} + 2(-\dot{\theta}\hat{k}) \times v_{rel}\hat{i} + (-\dot{\theta}\hat{k}) \times \underbrace{(-\dot{\theta}\hat{k} \times -d\hat{i})}_{\dot{\theta}d\hat{j}}$

$\vec{a}_A = a_{rel}\hat{i} - 2\dot{\theta}v_{rel}\hat{j} + \dot{\theta}^2 d\hat{i}$

⑨ trig to convert $XYZ \Rightarrow xyz$:



$\hat{I} = \sin\theta\,\hat{i} - \cos\theta\,\hat{j}$

$\hat{J} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

⑩ Combine: CA . O&A. 2 eqns 2 unkns

$\hat{i}: \alpha_{AC} L \cos\theta - \omega_{AC}^2 L \sin\theta = a_{rel} + \dot{\theta}^2 d$

$\hat{j}: \alpha_{AC} L \sin\theta + \omega_{AC}^2 L \cos\theta = -2\dot{\theta}v_{rel}$

⑪ Solve for $\vec{\alpha}_{AC}$ & a_{rel}. Then plug back solve for $\vec{a}_A$

$\vec{\alpha}_{AC} =$ ___

$a_{rel} =$ ___

$\Rightarrow \vec{a}_A =$ ___ Part (b)

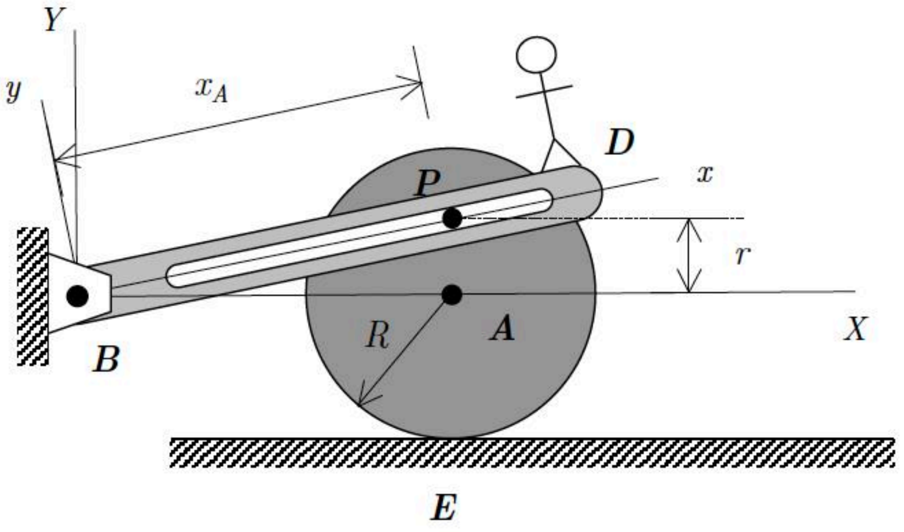
Example 3.A.7

Given: The disk rolls without slipping to the right with a constant angular speed of ω_d. At the instant shown, pin P is directly above the center A of the disk.

Find: Determine:

(a) The angular acceleration of the disk; and
(b) The acceleration of P as seen by an observer on arm BD.

Use the following parameters in your analysis: $\omega_d = 20$ rad/s (clockwise), $x_A = 0.48$ m, $r = 0.14$ m and $R = 0.2$ m.



[pg. 156]

Example 3.A.7

p.156

(one of the longer problems this semester...)

Given: The disk rolls without slipping to the right with a constant angular speed of ω_d. At the instant shown, pin P is directly above the center A of the disk.

Find: Determine:

(a) (a) The angular acceleration of the disk; and α_d ?

(b) (b) The acceleration of P as seen by an observer on arm BD. $(\vec{a}_{P/B})_{rel} = ?$

Use the following parameters in your analysis: $\omega_d = 20$ rad/s (clockwise), $x_A = 0.48$ m, $r = 0.14$ m and $R = 0.2$ m.

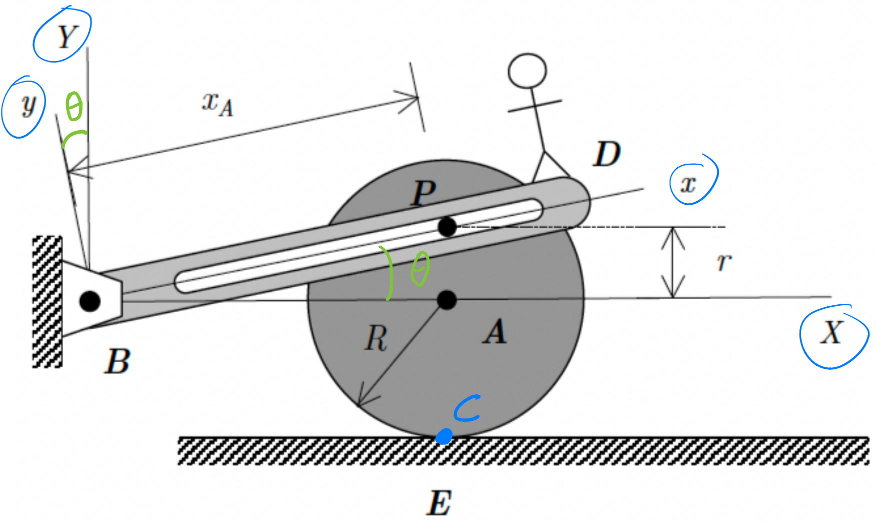
$W_d = 20 \text{rad/s}\ (CW);\ \vec{\alpha}_d = \vec{0}$

$x_A = 0.48m\ ;\ \vec{v}_A = \text{const}\ ;\ \vec{a}_A = \vec{0}$

$r = 0.14m$

$R = 0.2m$

① Both Axes Defined by the problem



$\sin\theta = \dfrac{r}{x_A}$

$\Rightarrow \theta$

Velocity

② CP. Same Rigid Body

$\vec{v}_P = \vec{v}_C + \vec{\omega}_d \times \vec{r}_{P/C}$

IC; =0

$= \vec{0} + -\omega_d \hat{K} \times (R+r)\hat{J}$

$= +\omega_d (R+r)\hat{I}$

③ BP. Diff rigid body

$\vec{v}_P = \vec{v}_B + (\vec{v}_{P/B})_{rel} + \vec{\omega} \times \vec{r}_{P/B}$

$= \vec{0} + \dot{x}_A \hat{i} + \omega \hat{k} \times x_A \hat{i}$

$= \dot{x}_A \hat{i} + \omega x_A \hat{j}$

④ trig

$\hat{i} = \cos\theta \hat{I} + \sin\theta \hat{J}$

$\hat{j} = -\sin\theta \hat{I} + \cos\theta \hat{J}$

⑤ Solve for $\vec{\omega}$, $\dot{x}_A$

$\hat{I}:\ \omega_d (R+r) = \dot{x}_A \cos\theta + \omega x_A (-\sin\theta)$

$\hat{J}:\ 0 = \dot{x}_A \sin\theta + \omega x_A \cos\theta$

$> \begin{matrix} \vec{\omega} \\ \dot{x}_A \end{matrix}$

Acceleration

⑥ AP. Same rigid body

$\vec{a}_P = \cancel{\vec{a}_A} + \vec{\alpha}_d \times \vec{r}_{P/A} - \omega_d^2 \vec{r}_{P/A}$

$= -\omega_d^2 r \hat{J}$

⑦ BP. Diff rigid body

$\vec{a}_P = \vec{a}_B + (\vec{a}_{P/B})_{rel} + \vec{\alpha} \times \vec{r}_{P/B} + 2\vec{\omega} \times (\vec{v}_{P/B})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/B})$

$= \vec{0} + \ddot{x}_A \hat{i} + \alpha \hat{k} \times x_A \hat{i} + 2\omega \hat{k} \times \dot{x}_A \hat{i} + \omega \hat{k} \times (\omega \hat{k} \times x_A \hat{i})$

$= \ddot{x}_A \hat{i} + \alpha x_A \hat{j} + 2\omega \dot{x}_A \hat{j} - x_A \omega^2 \hat{i}$

⑧ Trig

$\hat{J} = \sin\theta \hat{i} + \cos\theta \hat{j}$

⑨ Solve for $\vec{\alpha}$ & $\ddot{x}_A$

$\hat{i}:\ -\omega_d^2 r \sin\theta = \ddot{x}_A - x_A \omega^2$

$\hat{j}:\ -\omega_d^2 r \cos\theta = \alpha x_A + 2\omega \dot{x}_A$

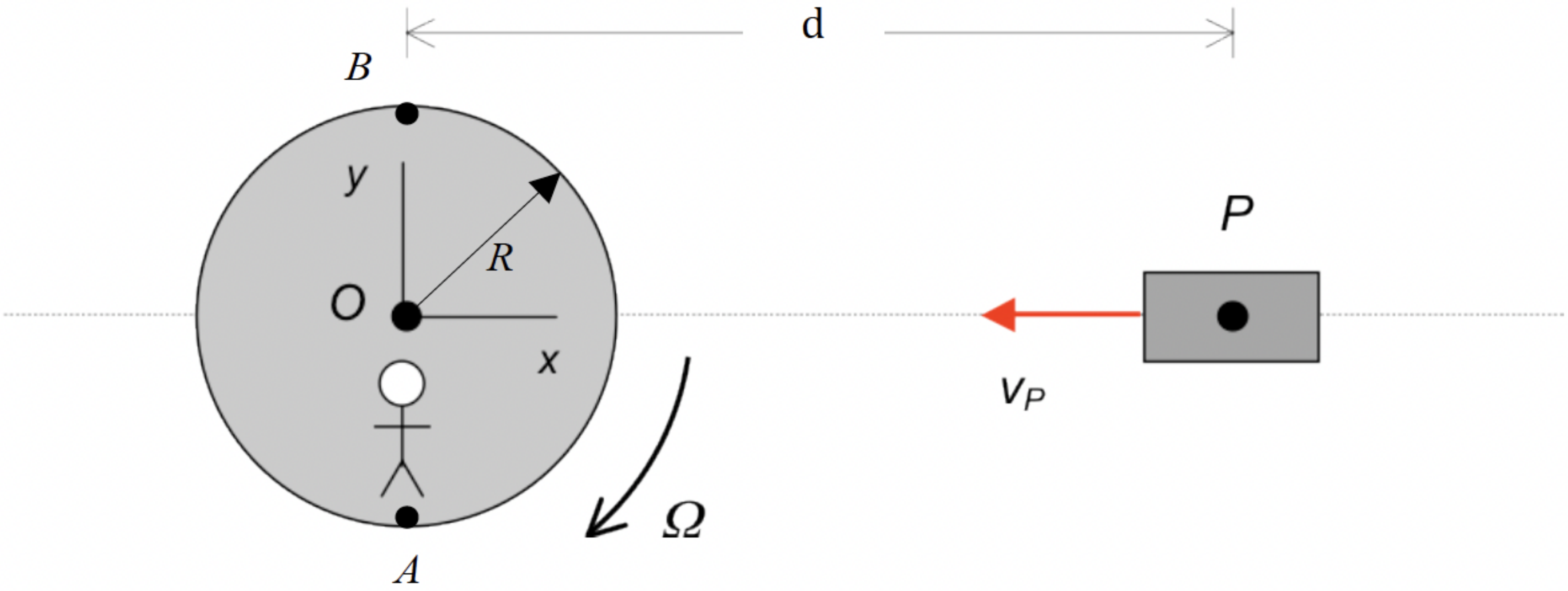
$> \begin{matrix} \alpha \ (a) \\ \ddot{x}_A \end{matrix}$

⑩ Plug back in using $\ddot{x}_A$

$\Rightarrow (\vec{a}_{P/B})_{rel}$ (b)

Question C3.2

A disk is pinned to ground at its center O with the disk rotating clockwise at a constant rate of $\Omega = 3$ rad/s. Block P is traveling to the left along a straight path toward O with a constant speed of $v_P = 20$ ft/s. Determine the acceleration of P as seen by an observer on the disk when P is at a distance of 50 ft from O.

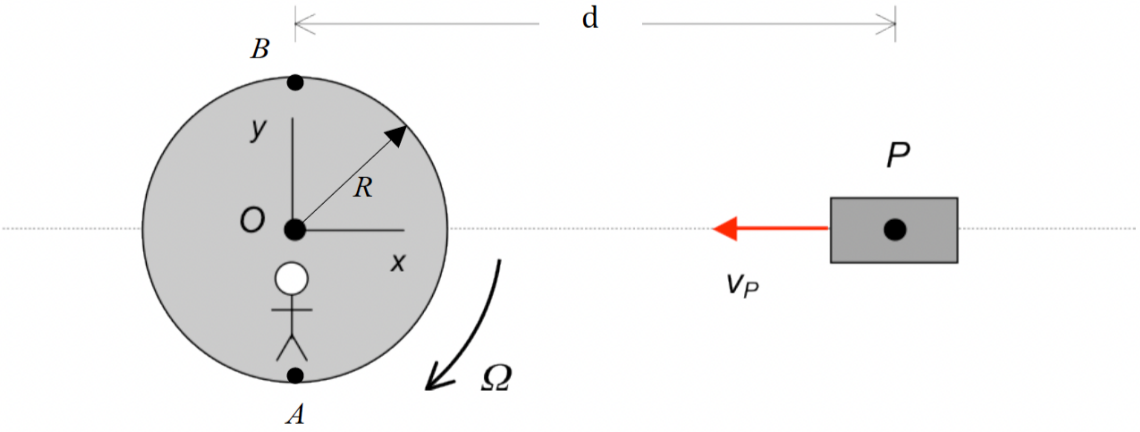


[pg. 174]

Question C3.2

p.174

A disk is pinned to ground at its center O with the disk rotating clockwise at a constant rate of $\Omega = 3$ rad/s. Block P is traveling to the left along a straight path toward O with a constant speed of $v_P = 20$ ft/s. Determine the acceleration of P as seen by an observer on the disk when P is at a distance of 50 ft from O.



Suppose observer is on disk at point O.

$$(\vec{a}_{P/O})_{rel} = \underset{0}{\cancel{\vec{a}_P}} - \underset{0}{\cancel{\vec{a}_O}} - \underset{0}{\cancel{\vec{\alpha}}} \times \vec{r}_{P/O} - 2\vec{\omega} \times (\vec{v}_{P/O})_{rel} - \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/O})$$

with $\vec{\omega} = -\Omega \hat{k}$

$$(\vec{v}_{P/O})_{rel} = \boxed{\vec{v}_P} - \overset{0}{\cancel{\vec{v}_O}} - \vec{\omega} \times \vec{r}_{P/O}$$

$$\hookrightarrow -v_P \hat{i}$$

$$\vec{r}_{P/O} = d\hat{i}$$

[Same final answer if observer is at A or B or any point on the rotating disk.]

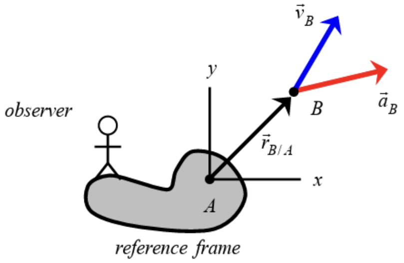
[pg. 145]

Summary: 2D Moving Reference Frame Kinematics 2

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point B.

$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha}\times\vec{r}_{B/A} + 2\vec{\omega}\times\left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{B/A}\right)$$



Lec 12 Short Feedback Form:

APPLICATION: Using 2D MRF equations in solving problems in the kinematics of mechanisms.

AP (rigid body):

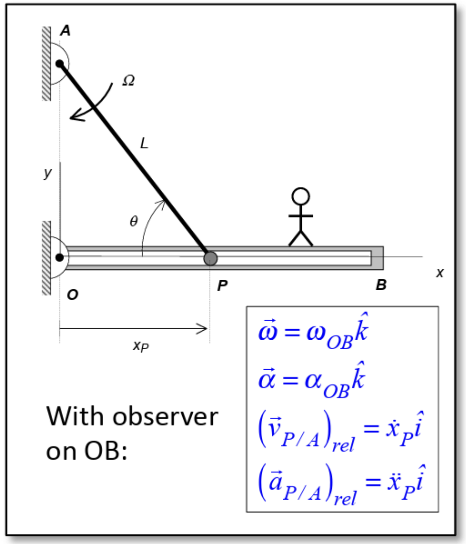
$$\vec{v}_P = \left(-\Omega\hat{k}\right)\times\vec{r}_{P/A}$$

$$\vec{a}_P = \left(-\dot{\Omega}\hat{k}\right)\times\vec{r}_{P/A} + \left(-\Omega\hat{k}\right)\times\left[\left(-\Omega\hat{k}\right)\times\vec{r}_{P/A}\right]$$

OP (*not* a rigid body):

$$\vec{v}_P = \dot{x}_P\hat{i} + \left(\omega_{OB}\hat{k}\right)\times\vec{r}_{P/A}$$

$$\vec{a}_P = \ddot{x}_P\hat{i} + \left(\alpha_{OB}\hat{k}\right)\times\vec{r}_{P/A} + 2\left(\omega_{OB}\hat{k}\right)\times\left(\dot{x}_P\hat{i}\right) + \left(\omega_{OB}\hat{k}\right)\times\left[\left(\omega_{OB}\hat{k}\right)\times\vec{r}_{P/A}\right]$$



With observer on OB:

$$\vec{\omega} = \omega_{OB}\hat{k}$$
$$\vec{\alpha} = \alpha_{OB}\hat{k}$$
$$\left(\vec{v}_{P/A}\right)_{rel} = \dot{x}_P\hat{i}$$
$$\left(\vec{a}_{P/A}\right)_{rel} = \ddot{x}_P\hat{i}$$