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ME 274 Lecture 10

Planar kinematics: Rigid Bodies - Summary

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2/4/26

Housekeeping/Announcements

***Reminder for Henny to wear a mic during the lecture.

1. **HW 9 due tonight!!**
2. Exam 1 Details Below:

Midterm Exam 1

- Thursday, February 12, 8:00-9:30 PM
- Locations

– BHEE 129 (234): EF(109), LK (108) and 2 TAs

- Review sessions
  - Pi Tau Sigma: Thursday, February 5, 6:30-7:30 PM, WTHR 172 (WL live and Indy online)
  - ME 274 Instructor, CK: Tuesday, February 10, 7:00 PM, on Zoom for both WL and Indy: https://purdue-edu.zoom.us/j/92303777463?pwd=RGN5IQFIxdEcgIqOV28gGScUhm4fya.1 (video recording of the session will be posted on the Exams page of the course website)

Problem 1.1	joint kinematics
Problem 1.2	rigid body kinematics
Problem 1.3	conceptual on Chapters 1 and 2

EQUATIONS

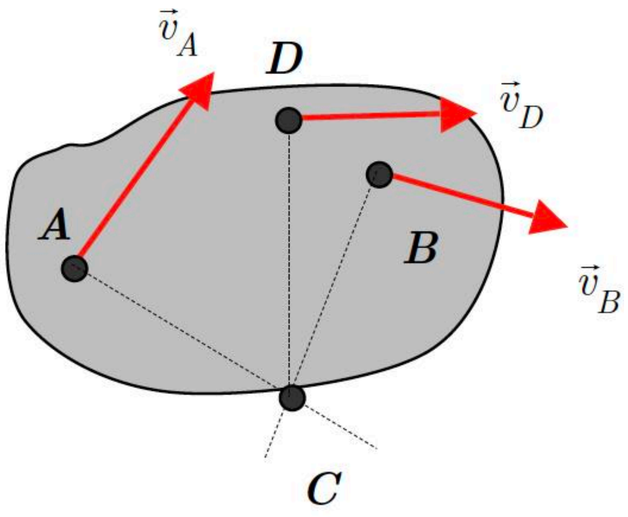
$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$$
$$= v\hat{e}_t$$
$$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$
$$= \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$
$$= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2\vec{r}_{B/A}$$

Last lecture… **Instant Centers**



1. Locate two points A and B on the rigid Body
   - Let A be a point for which you know:
     - **Magnitude**
     - **Direction**
   - Let B be a point for which you know:
     - **Direction**
2. Draw the directions of the velocity vectors
3. Draw the lines that are perpendicular to the velocity vectors
4. The intersection of the two perpendiculars is the instant center, C, of the body. For this we know that v_C = 0.
5. From this we can find:
   - **Angular velocity, w:**

$$\boxed{\omega = \frac{|\vec{v}_A|}{|\vec{r}_{A/C}|}}$$

   - **Direction of angular velocity from looking at the velocity vectors (ie CW or CCW)**
6. Velocity of any point D on the body is perpendicular to the line connecting C and D:

$$\boxed{|\vec{v}_D| = |\vec{\omega}| \; |\vec{r}_{D/C}|}$$

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Summary: Kinematic Analysis of Planar Mechanisms

Velocity and acceleration equations for ***planar motion of a rigid body:***

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

When in the same plane… :

$$\vec{a}_B = \vec{a}_A + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}] + \vec{\alpha} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}.$$

General steps to take to solve one of these problems:

1. Define your set of coordinate axes.
2. Write down a velocity/acceleration equation for each rigid link
3. Combine your equations from step 2.
4. Use the concept of instant centers to check the direction of the velocity and angular velocity vectors.

p.g 118 – worked out example

[pg. 117 content]

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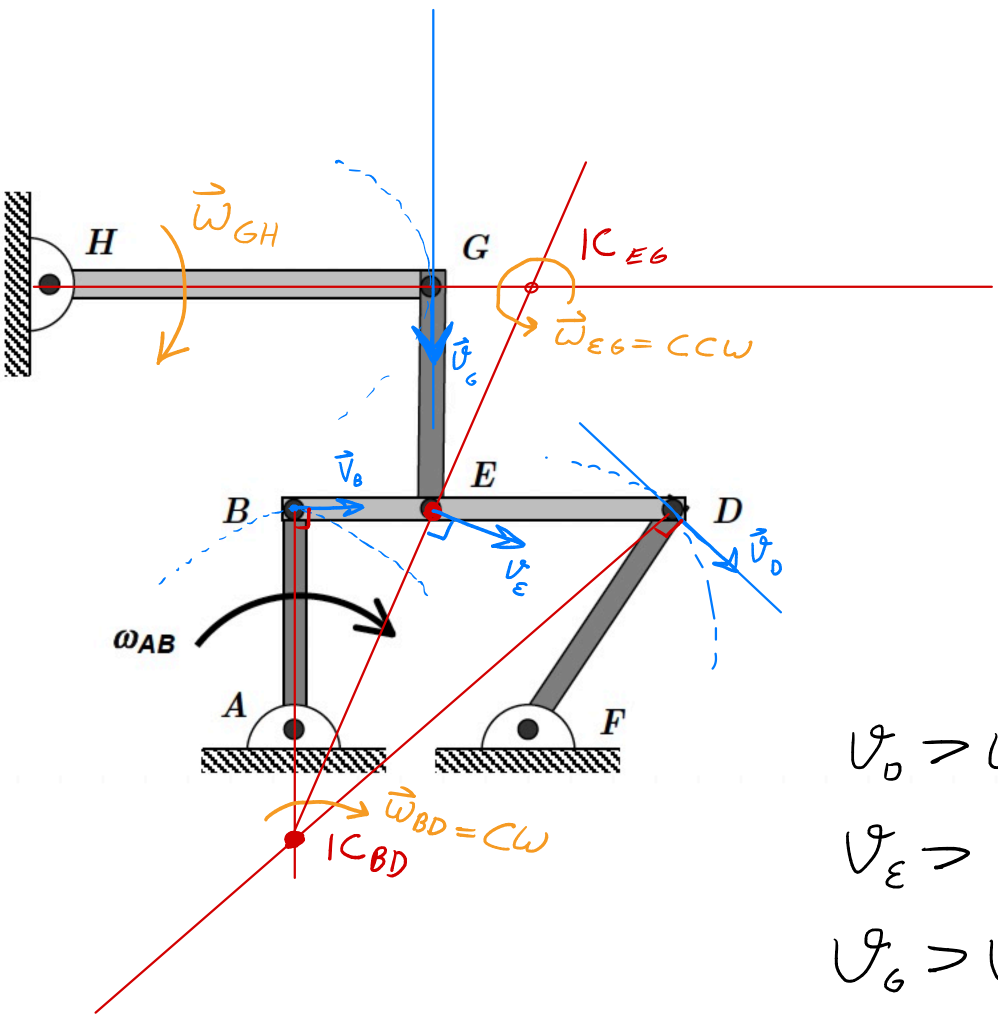
Link AB is rotating in the clockwise direction. What is the sense of rotation for the other links of this mechanism?



$v_D > v_B$

$v_E > v_B$

$v_G > v_E$

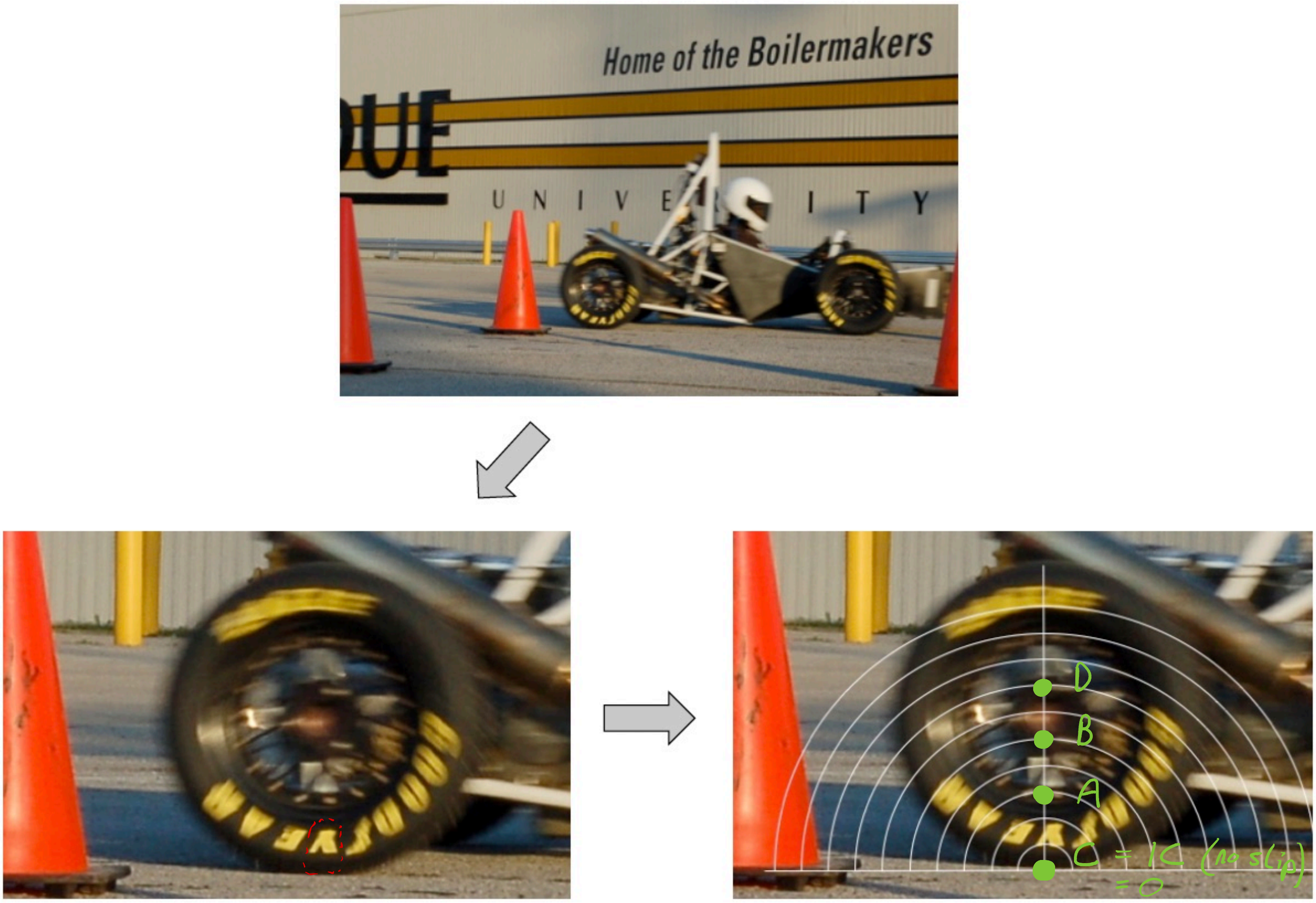
speed $\propto$ how far you are. ← conceptual question

is proportional to

Example 2.B.8 p. 116

Given: Shown below is a photo of the Purdue Formula SAE car on a test track. If you magnify the image of the right rear tire of the car, you see the lettering on the tire blurred in what might appear to be an arbitrary way, as shown in the lower left figure. If you draw semicircles centered on the point of contact of the tire with the roadway (as shown in the lower right photo), you observe that the blurriness of the tire lettering is aligned with these semicircles.

Find: Provide an explanation for this. [HINT: Consider the location of the instant center of the tire assuming that the tire does not slip on the roadway.]



Good Year

$v_C < v_A < v_B < v_D$

Example 2.C.1

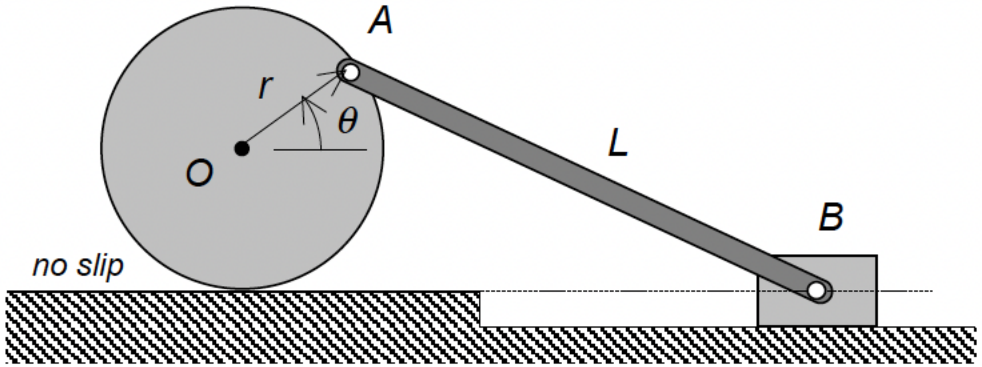
Given: The wheel rolls without slipping in such a way that slider B moves to the left with a constant speed of $v_B = 5$ ft/s.

Find: Determine:

(a) The angular velocity of the wheel when $\theta = 0$; and

(b) The angular acceleration of the wheel when $\theta = 0$.

Use the following parameters in your analysis: $L = 2$ ft and $r = 0.5$ ft.



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Example 2.C.1 p. 122

Given: The wheel rolls without slipping in such a way that slider B moves to the left with a constant speed of $v_B = 5$ ft/s.

Find: Determine: d = disk

(a) The angular velocity of the wheel when $\theta = 0$; and ω_{d_0}?

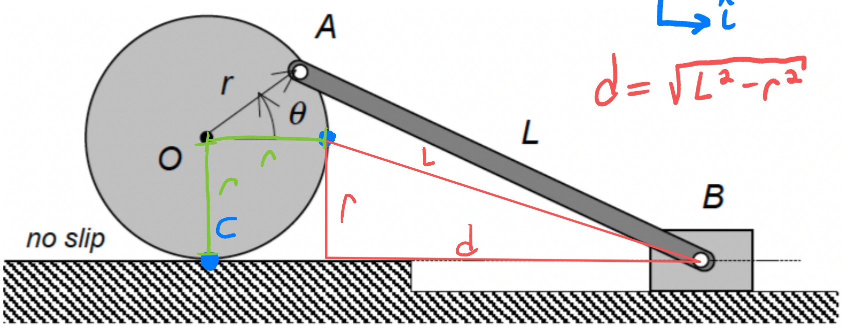
(b) The angular acceleration of the wheel when $\theta = 0$. α_{d_0}?

Use the following parameters in your analysis: $L = 2$ ft and $r = 0.5$ ft.

Given: $L = 2$ ft, $v_B = 5$ ft/s, $r = 0.5$ ft

$\hat{j}$, $\hat{i}$

$d = \sqrt{L^2 - r^2}$



⓪ Vector Approach? IC?

↳ Vector Approach

Chuck did this w/ IC... but we do vector

① Velocity

$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$

$= -v_B\hat{i} + \omega_{AB}\hat{k} \times (-d\hat{i} + r\hat{j})$

$= -v_B\hat{i} - \omega_{AB} d\hat{j} - \omega_{AB} r\hat{i}$

$\vec{v}_A = \vec{v}_c + \vec{\omega}_d \times \vec{r}_{A/c}$

$= \vec{0} + \omega_d\hat{k} \times (r\hat{i} + r\hat{j})$

$= \omega_d r\hat{j} - \omega_d r\hat{i}$

② Velocity split into $\hat{i}$ & $\hat{j}$

$\hat{i}: -v_B - \omega_{AB} r = -\omega_d r$

$\hat{j}: -\omega_{AB} d = \omega_d r$

③ Solve for $\vec{\omega}_d$ & $\vec{\omega}_{AB}$

(2 eqns, 2 unkns)

...

$\vec{\omega}_d = ?$

$\vec{\omega}_{AB} = ?$

④ Acceleration

$\vec{a}_A = \cancel{\vec{a}_B} + \vec{\alpha}_{AB} \times \vec{r}_{A/B} - \omega_{AB}^2 \vec{r}_{A/B}$

$= \alpha_{AB}\hat{k} \times (-d\hat{i} + r\hat{j}) - \omega_{AB}^2(-d\hat{i} + r\hat{j})$

$= -d\alpha_{AB}\hat{j} - r\alpha_{AB}\hat{i} + \omega_{AB}^2 d\hat{i} - \omega_{AB}^2 r\hat{j}$

⑤ Point A & C. C bc it'll be useful in future

$\vec{a}_A = \vec{a}_c + \vec{\alpha}_d \times \vec{r}_{A/c} - \omega_d^2 \vec{r}_{A/c}$

No slip.. so $\neq 0$. top of p. 92

$= a_c\hat{j} + \alpha_d\hat{k} \times (r\hat{i} + r\hat{j}) - \omega_d^2(r\hat{i} + r\hat{j})$

$= a_c\hat{j} + \alpha_d r\hat{j} - \alpha_d r\hat{i} - \omega_d^2 r\hat{i} - \omega_d^2 r\hat{j}$ ←

⑥ Since 3 unknowns & 2 eqns we use point O

$\vec{a}_o = \vec{a}_c + \vec{\alpha}_d \times \vec{r}_{o/c} - \omega_d^2 \vec{r}_{o/c}$

$a_o\hat{i} = a_c\hat{j} + \alpha_d\hat{k} \times r\hat{j} - \omega_d^2 r\hat{j}$

$= a_c\hat{j} - \alpha_d r\hat{i} - \omega_d^2 r\hat{j}$

⑦ Use $\hat{j}$ to solve for a_c

$\hat{i}: a_o = -\alpha_d r$

$\hat{j}: 0 = a_c - \omega_d^2 r \Rightarrow a_c$

⑧ Knowing a_c we now have 2 eqns & 2 unknowns so we can solve for $\vec{\alpha}_d$ & $\vec{\alpha}_{AB}$

$a_A = a_A$

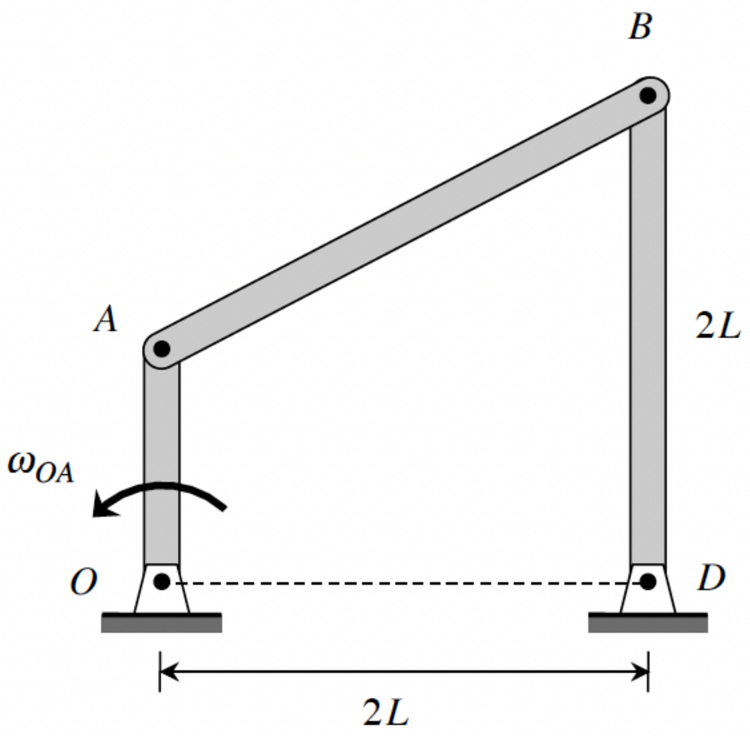
$-d\alpha_{AB}\hat{j} - r\alpha_{AB}\hat{i} + \omega_{AB}^2 d\hat{i} - \omega_{AB}^2 r\hat{j} = a_c\hat{j} + \alpha_d r\hat{j} - \alpha_d r\hat{i} - \omega_d^2 r\hat{i} - \omega_d^2 r\hat{j}$

Example 2.C.2

Given: Link OA, of length L, in the mechanism shown below has a constant counterclockwise angular velocity of ω_{OA} as it moves into a vertical position. At this same instant, link BD is also vertically oriented, where pins O and D lie along the same horizontal line.

Find: Determine, at this instant in time:
(a) The angular velocity of link AB;
(b) The angular velocity of link BD;
(c) The angular acceleration of link AB; and
(d) The angular acceleration of link BD

Use the following parameter in your analysis: $\omega_{OA} = 0.5$ rad/s.



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Example 2.C.2 p. 123 Similar to Friday due HW

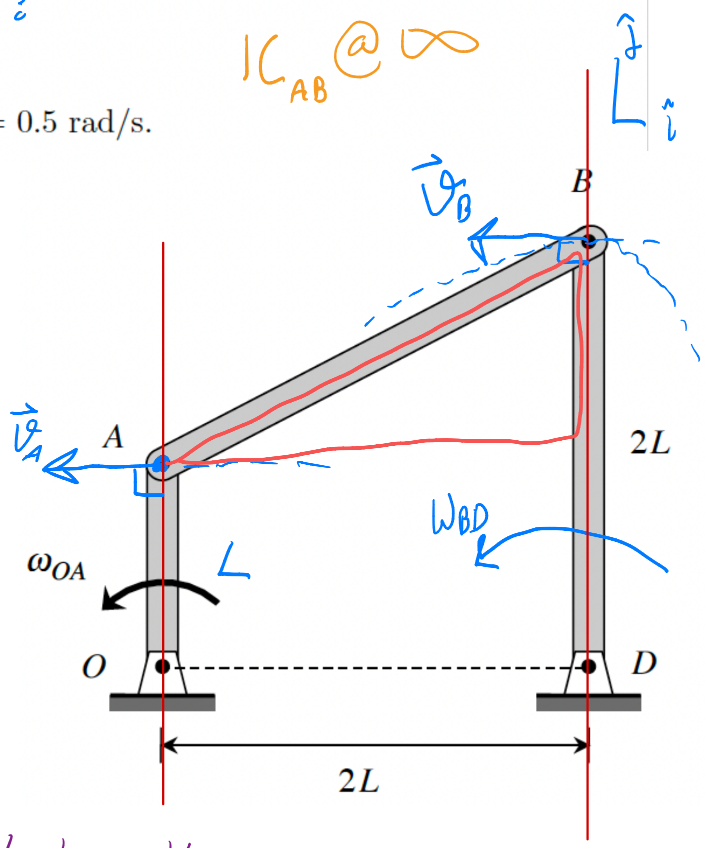
Given: Link OA, of length L, in the mechanism shown below has a constant counterclockwise angular velocity of ω_{OA} as it moves into a vertical position. At this same instant, link BD is also vertically oriented, where pins O and D lie along the same horizontal line.

Find: Determine, at this instant in time:

(a) The angular velocity of link AB; ω_{AB}?
(b) The angular velocity of link BD; ω_{BD}?
(c) The angular acceleration of link AB; and α_{AB}?
(d) The angular acceleration of link BD α_{BD}?

Use the following parameter in your analysis: $\omega_{OA} = 0.5$ rad/s.

IC_{AB} @ ∞



Given:

$\omega_{OA} = 0.5$ rad/s, $\alpha_{OA} = 0$

Velocity:

⓪ Vector Approach? IC?
↳ IC.

① IC Approach for velocity

IC is @ ∞ $\therefore$ $\vec{\omega}_{AB} = \vec{0}$

② velocity eqn for AB

$\Rightarrow \vec{v}_B = \vec{v}_A + \cancel{\vec{\omega}_{AB}}^{0} \times \vec{r}_{B/A}$

$\vec{v}_A = -L\omega_{OA}\hat{i} = \vec{v}_B$

③ relate $\vec{v}$ to $\vec{\omega}$

$\omega_{BD} = \frac{|\vec{v}_B|}{|\vec{r}_{D/B}|}$ ~pg. 106

$= \frac{v_B}{2L}$

$= \frac{L\omega_{OA}}{2L}$

$= \frac{\omega_{OA}}{2}$

$\vec{\omega}_{BD} = \frac{\omega_{OA}}{2}\hat{k}$

Acceleration: Vector approach... eqns of all 3 links

④ $\overline{OA}$. This gives us $\vec{a}_A$. which we will use later.

$\vec{a}_A = \cancel{\vec{a}_O}^{0 = \text{fixed pt}} + \cancel{\vec{\alpha}_{AO}}^{0} \times \vec{r}_{A/O} - \omega_{AO}^2 \vec{r}_{A/O}$

$= -\omega_{AO}^2 L\hat{j}$

⑤ AB

$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \cancel{\omega_{BA}^2 \vec{r}_{B/A}}$

$= -\omega_{AO}^2 L\hat{j} + \alpha_{AB}\hat{k} \times (2L\hat{i} + L\hat{j})$

$= -\omega_{AO}^2 L\hat{j} + 2L\alpha_{AB}\hat{j} - \alpha_{AB}L\hat{i}$

⑥ DB

$\vec{a}_B = \vec{a}_D + \vec{\alpha}_{BD} \times \vec{r}_{B/D} - \omega_{BD}^2 \vec{r}_{B/D}$

$= \vec{0} + \alpha_{BD}\hat{k} \times 2L\hat{j} - \left(\frac{\omega_{OA}}{2}\right)^2 2L\hat{j}$

$= -2L\alpha_{BD}\hat{i} - \frac{\omega_{OA}^2 L}{2}\hat{j}$

⑦ Combine/Equate: AB & DB. $\vec{a}_B = \vec{a}_B$.

$\hat{i}: -\alpha_{AB}L = -2L\alpha_{BD}$

$\hat{j}: -\omega_{AO}^2 L + 2L\alpha_{AB} = -\frac{\omega_{OA}^2 L}{2}$

⑧ Solve, 2 Eqn 2 unkns. $\vec{\alpha}_{BD}$. $\vec{\alpha}_{AB}$.

Summary: Rigid Body Kinematics 4

PROBLEM: Two points A and B on the same rigid body undergoing planar motion.

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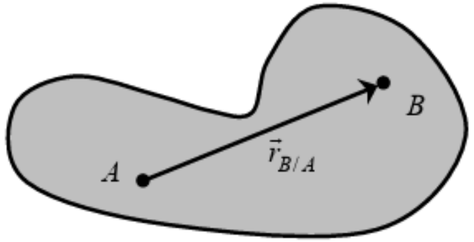
$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$



Lec 10 Short Feedback Form:

KINEMATICS OF MECHANISMS

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- Write down the rigid body velocity equation for each link in a mechanism.
- Combine together the velocity equations and solve for two unknowns from the x- and y-components of this combined equation.
- Repeat for acceleration.
- You generally need to solve the velocity equations *BEFORE* you can solve the acceleration equations – WHY?