Soln   Fall 2017 Exam 1 Prob 1

ME 274 Spring 2026
Quiz #3
Frias-Miranda

Name (print) ______________________
(Last) (First)

Given:
A particle P is traveling in the xy-plane on a path given by:

$$y = \sin\left(\frac{\pi}{2}x\right),$$

where **x** and **y** are given in meters, and in such a way that

$$\dot{x} = -4\ m/s \text{ and } \ddot{x} = 2\ m/s^2.$$

$\dot{y} = \frac{\pi}{2}\dot{x}\cos\frac{\pi}{2}x$

$\dot{y}(2) = \frac{\pi}{2}(-4)(-1) = 2\pi$

$\ddot{y} = \frac{\pi}{2}\left(\ddot{x}\cos\frac{\pi}{2}x - \dot{x}^2\frac{\pi}{2}\sin\frac{\pi}{2}x\right)$

$\ddot{y}(2) = \frac{\pi}{2}\left(-2 - \frac{16\pi}{2}0\right)$

$\ddot{y}(2) = -\pi$

Find: For the instant when $x = 2\ m$:

a) Determine the Cartesian components of the velocity and acceleration of P.
b) Make an accurate sketch of the velocity and acceleration vectors found in a), as well as the path unit vectors $\hat{e}_t$ and $\hat{e}_n$ on the figure provided below.
c) Determine the rate of change of speed and the radius of curvature for the path of P.
d) Is the speed of P increasing, decreasing or constant? Provide an explanation for your response.

a) ① velocity - find

$\vec{r}_P = x(t)\hat{\imath} + y_o(t)\hat{\jmath}$

$\vec{v}_P = \frac{dx}{dt}\hat{\imath} + \frac{dy_P}{dt}\hat{\jmath} \Rightarrow$ $\frac{dx}{dt} = -4\ m/s$

chain rule

$\frac{dy_P}{dt} = \cos\frac{\pi}{2}x \cdot \frac{\pi}{2}\dot{x}$ $\Rightarrow$ $\vec{v}_P = \dot{x}\hat{\imath} + \dot{x}\frac{\pi}{2}\cos\frac{\pi}{2}x$

$\Rightarrow$ $\vec{v}_P = -4\hat{\imath} + -2\pi\cos\pi\hat{\jmath}$

$\vec{v}_P = -4\hat{\imath} + 2\pi\hat{\jmath}\ m/s$

② acceleration - find

$\vec{a}(t) = \ddot{x}(t)\hat{\imath} + \ddot{y}(t)\hat{\jmath}$

$\Rightarrow \vec{a}(t) = \ddot{x}\hat{\imath} + \frac{d}{dt}\left(\dot{x}\frac{\pi}{2}\cos\frac{\pi}{2}x\right)\hat{\jmath}$

product rule

$\vec{a}(t) = \ddot{x}\hat{\imath} + \left(\ddot{x}\frac{\pi}{2}\cos\frac{\pi}{2}x - \dot{x}\frac{\pi}{2}\sin\left(\frac{\pi}{2}x\right)\cdot\dot{x}\frac{\pi}{2}\right)\hat{\jmath}$

$\vec{a}(t) = 2\hat{\imath} - \pi\hat{\jmath}\ m/s^2$

b)

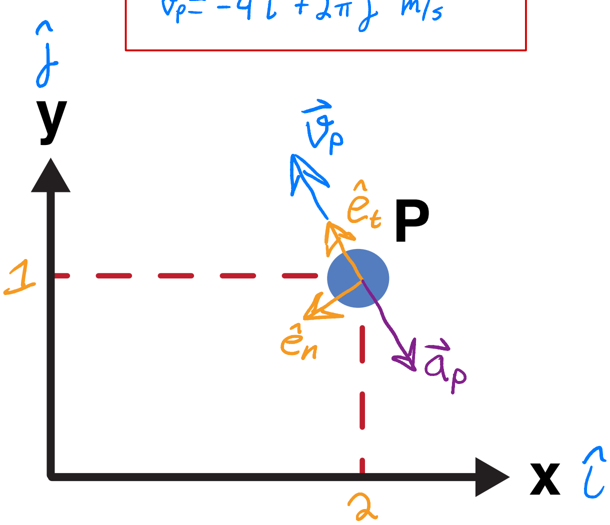

Figure 1: Please draw on this figure

③ Use results from ① & ②

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c)

④ Use eqn for $\dot{v}$ (rate of change of speed)

$$\dot{v} = \vec{a} \cdot \hat{e}_t$$

$$\hat{e}_t = \frac{\vec{v}}{|\vec{v}|} = \frac{-4\hat{i} + 2\pi\hat{j}}{\sqrt{16+4\pi^2}} = \frac{-2\hat{i} + \pi\hat{j}}{\sqrt{4+\pi^2}}$$

$$\dot{v} = (2\hat{i} - \pi\hat{j}) \cdot \frac{-2\hat{i} + \pi\hat{j}}{\sqrt{4+\pi^2}}$$

$$\Rightarrow \dot{v} = \frac{-4-\pi^2}{\sqrt{4+\pi^2}} = \frac{-(4+\pi^2)}{\sqrt{4+\pi^2}} = -\sqrt{4+\pi^2}$$

⑤ Use Path accel vector eqn.

$$|\vec{v}| = \sqrt{16+4\pi^2}$$
$$|\vec{a}| = \sqrt{4+\pi^2}$$

$$\vec{a} = \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\vec{a} = \sqrt{4+\pi^2}\,\hat{e}_t + \frac{16+4\pi^2}{\rho}\hat{e}_n$$

$$|\vec{a}|^2 = (4+\pi^2) + \left(\frac{16+4\pi^2}{\rho^2}\right)^2$$

$$\Rightarrow (4+\pi^2) - (4+\pi^2) = \frac{(16+4\pi^2)^2}{\rho^2}$$

$$0 = \frac{(16+4\pi^2)^2}{\rho^2} \Rightarrow \rho = \infty$$

since $= 0 \ldots$; ρ must go to ∞

if $\dot{v} < 0 \rightarrow$ decrease
if $\dot{v} > 0 \rightarrow$ increase

d) Speed at $x = 2m$ is dereasing because: $\dot{v} < 0$