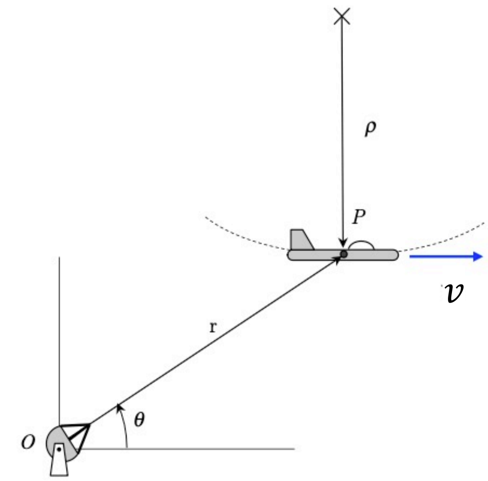
ME 274 – Spring 2026 - Quiz #3b **Name** SOLUTION

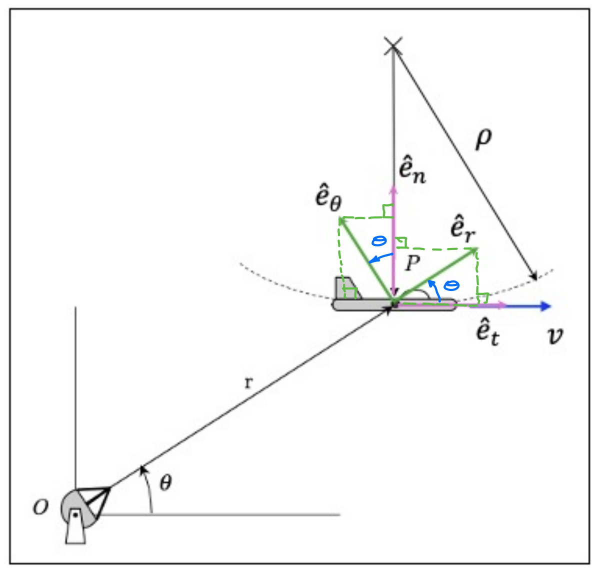


Given: At the instant when at the bottom of a loop, an airplane P is traveling horizontally with a *constant* speed v. At that position, the path of the path has a radius of curvature of ρ. A radar tracking P at point O sees the aircraft at a distance of r and at an angle of θ.

Find: At this instant: determine $\dot{r}$, $\dot{\theta}$, $\ddot{r}$ and $\ddot{\theta}$ in terms of v, ρ, r and θ.

Solution:

The path and polar unit vectors have been provided for you in the figure.



a) Write $\hat{e}_r$ and $\hat{e}_\theta$ in terms of $\hat{e}_t$ and $\hat{e}_n$:

$$\hat{e}_r = \underline{\cos\theta}\,\hat{e}_t + \underline{\sin\theta}\,\hat{e}_n$$

$$\hat{e}_\theta = \underline{-\sin\theta}\,\hat{e}_t + \underline{\cos\theta}\,\hat{e}_n$$

b) Find $\dot{r}$ and $\dot{\theta}$:

$$\dot{r} = \vec{V}\cdot\hat{e}_r = (V\hat{e}_t)\cdot(\cos\theta\,\hat{e}_t + \sin\theta\,\hat{e}_n) = V\cos\theta \longleftarrow \dot{r}$$

$$r\dot{\theta} = \vec{V}\cdot\hat{e}_\theta = (V\hat{e}_t)\cdot(-\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n) = -V\sin\theta$$

$$\hookrightarrow \dot{\theta} = -\frac{V}{r}\sin\theta \longleftarrow \dot{\theta}$$

c) Find $\ddot{r}$ and $\ddot{\theta}$:

$$\ddot{r} - r\dot{\theta}^2 = \bar{a}\cdot\hat{e}_r = \left(\frac{V^2}{\rho}\hat{e}_n\right)\cdot(\cos\theta\,\hat{e}_t + \sin\theta\,\hat{e}_n) = \frac{V^2}{\rho}\sin\theta$$

$$\hookrightarrow \ddot{r} = \frac{V^2}{\rho}\sin\theta + r\left(-\frac{V}{r}\sin\theta\right)^2 \longleftarrow \ddot{r}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \bar{a}\cdot\hat{e}_\theta = \left(\frac{V^2}{\rho}\hat{e}_n\right)\cdot(-\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n) = \frac{V^2}{\rho}\cos\theta$$

$$\hookrightarrow \ddot{\theta} = \frac{1}{r}\left[\frac{V^2}{\rho}\cos\theta - (2)(V\cos\theta)\left(-\frac{V}{r}\sin\theta\right)\right] \longleftarrow \ddot{\theta}$$