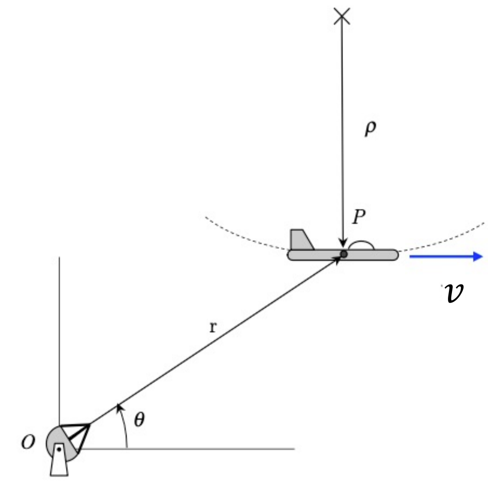
**_ME 274 – Spring 2026 - Quiz #3_** **Name** SOLUTION

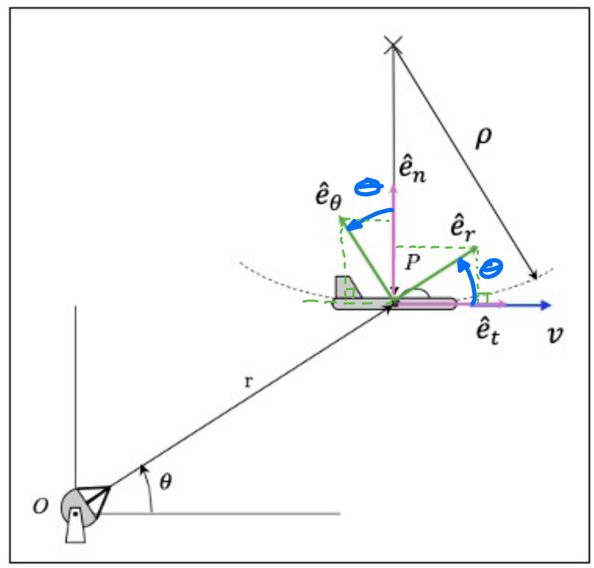
***Given***: At the instant when at the bottom of a loop, an airplane P is traveling horizontally with a speed of $v$ which is increasing at a rate of $\dot{v}$. At that position, the path of the path has a radius of curvature of $\rho$. A radar tracking P at point O sees the aircraft at a distance of $r$ and at an angle of $\theta$.




***Find***: At this instant: determine $\dot{r}$, $\dot{\theta}$, $\ddot{r}$ and $\ddot{\theta}$ in terms of $v$, $\dot{v}$, $\rho$ and $\theta$.

***Solution***:

The path and polar unit vectors have been provided for you in the figure.




a) Write $\hat{e}_r$ and $\hat{e}_\theta$ in terms of $\hat{e}_t$ and $\hat{e}_n$:

$\hat{e}_r =$ $\underline{\cos\theta}$ $\hat{e}_t +$ $\underline{\sin\theta}$ $\hat{e}_n$

$\hat{e}_\theta =$ $\underline{-\sin\theta}$ $\hat{e}_t +$ $\underline{\cos\theta}$ $\hat{e}_n$

b) Find $\dot{r}$ and $\dot{\theta}$:

$$\dot{r} = \vec{V}\cdot\hat{e}_r = (v\hat{e}_t)\cdot(\cos\theta\,\hat{e}_t + \sin\theta\,\hat{e}_n) = v\cos\theta$$

$$r\dot{\theta} = \vec{V}\cdot\hat{e}_\theta = (v\hat{e}_t)\cdot(-\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n) = -v\sin\theta$$

$$\hookrightarrow \dot{\theta} = -\frac{v}{r}\sin\theta$$

c) Find $\ddot{r}$ and $\ddot{\theta}$:

$$\ddot{r} - r\dot{\theta}^2 = \vec{a}\cdot\hat{e}_r = \left(\dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n\right)\cdot(\cos\theta\,\hat{e}_t + \sin\theta\,\hat{e}_n)$$

$$= \dot{v}\cos\theta + \frac{v^2}{\rho}\sin\theta \Rightarrow \ddot{r} = \dot{v}\cos\theta + \frac{v^2}{\rho}\sin\theta + r\left(\frac{v}{r}\sin\theta\right)^2$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \vec{a}\cdot\hat{e}_\theta = \left(\dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n\right)\cdot(-\sin\theta\,\hat{e}_t + \cos\theta\,\hat{e}_n)$$

$$= -\dot{v}\sin\theta + \frac{v^2}{\rho}\cos\theta$$

$$\hookrightarrow \ddot{\theta} = \frac{1}{r}\left[-\dot{v}\sin\theta + \frac{v^2}{\rho}\cos\theta + 2(v\cos\theta)\left(-\frac{v}{r}\sin\theta\right)\right.$$