ME 274: Basic Mechanics II

Week 1 *– Monday, January 12*

Particle kinematics: Cartesian description

Instructor: Manuel Salmerón

Today’s Agenda

1. Course Logistics
2. Introduction
3. Motivating Example
4. Kinematic Equations for Cartesian Coordinates
5. Examples
6. Summary and Closure

1. Course Logistics

Instructor:	Manuel Salmerón
E-mail:	salmeron@purdue.edu
Office hours:	MW, 9:30 – 10:30 AM, ME 2008

1. Course Logistics

Important points:

- Lecture book: *Dynamics – A Lecturebook*, Krousgrill and Rhoads (University Bookstore)
- Website: https://www.purdue.edu/freeform/me274/
- Discussion threads: log in, get approved, and comment (bonus points!)
- Quizzes: unannounced, need to be in person
- Attendance: not required, but expected; taken through brief activities
- Homework: assignments to be submitted via Gradescope. See syllabus for submission requirements and format.

1. Course Logistics

Grading:

Homework and quizzes*:	**25%**
Midterm and final exams:	**75%**
Bonus points**:	**??%**

* The share for homework assignments and quizzes will be decided based on the number of quizzes at the end of the semester.

** There will be bonus points for commenting in the website’s blog AND for attending class. Attendance will be taken through activities.

Homework + Quizzes + Bonus Points $\leq$ 25%

2. Introduction

Dynamics studies the motion of particles and bodies, and the forces causing such motion. It can be divided into:

Kinematics: describes motion — **How things move?**

and

Kinetics: studies forces causing motion — **Why things move?**

2. Introduction

Kinematic Equations

Let $s(t)$ be the position of a particle at time t. Thus, the velocity, $v(t)$, and acceleration, $a(t)$, are given by…

- Navigate to pollev.com/manuelsalmeron386 (or scan the QR code)
- To get credit, enter your first and last names when asked

2. Introduction

<u>Kinematic Equations</u>

Let $s(t)$ be the position of a particle at time t. Thus, the velocity, $v(t)$, and acceleration, $a(t)$, are given by…

$$v(t) = \frac{ds}{dt} \qquad \text{and} \qquad a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

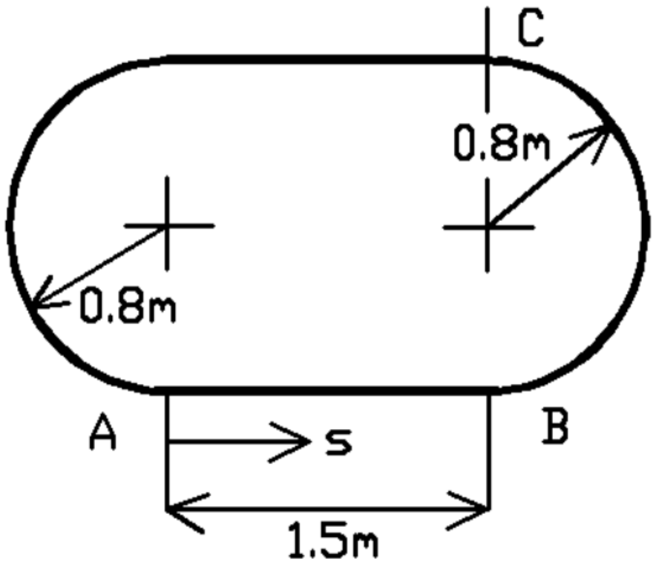
3. Motivating Example

The figure represents the track of a toy train. The train starts from point A and moves according to the expression $s(t) = 0.1t^2$, where $s(t)$, measured in meters, is the distance of the train from point A along the track.

Taking point A as the origin, determine:

a) the position and velocity of the train at $t = 2$ sec;

b) its position and velocity at $t = 5$ sec.



3. Motivating Example

Given: the expression for the position, $s(t) = 0.1t^2$, and the starting point A

Find:

a) the position, $s(t)$, and velocity, $v(t)$, of the train at $t = 2$ sec

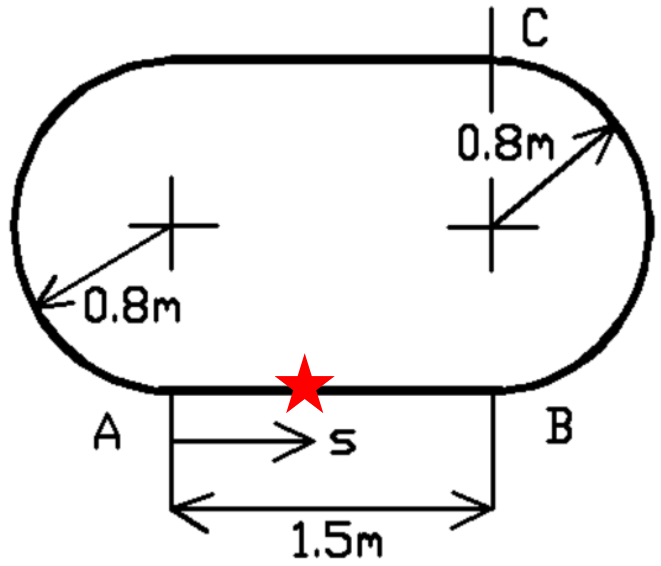
Solution:

a) From the kinematic relationship $v(t) = ds/dt$, we have:

$$v(t) = \frac{d(0.1t^2)}{dt} = 0.2t$$

Thus,

$$s(t = 2) = 0.4 \text{ m and } v(t = 2) = 0.4 \text{ m/s}$$



3. Motivating Example

Given: the expression for the position, $s(t) = 0.1t^2$, and the starting point A

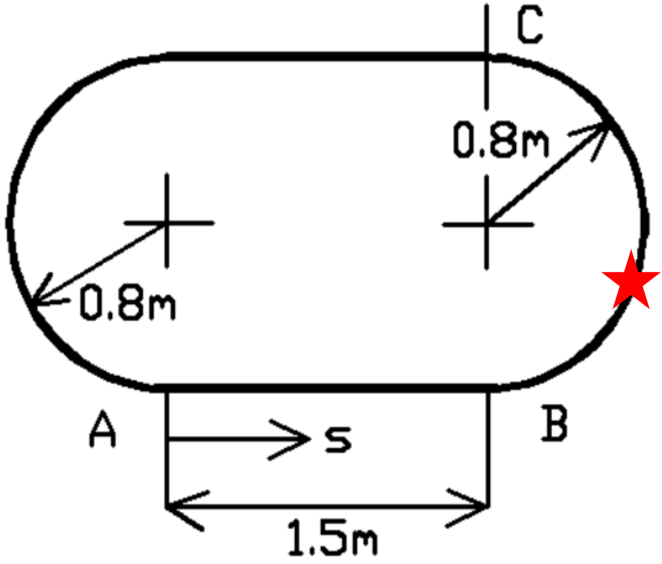
Find:

a) the position and velocity of the train at $t = 5$ sec;

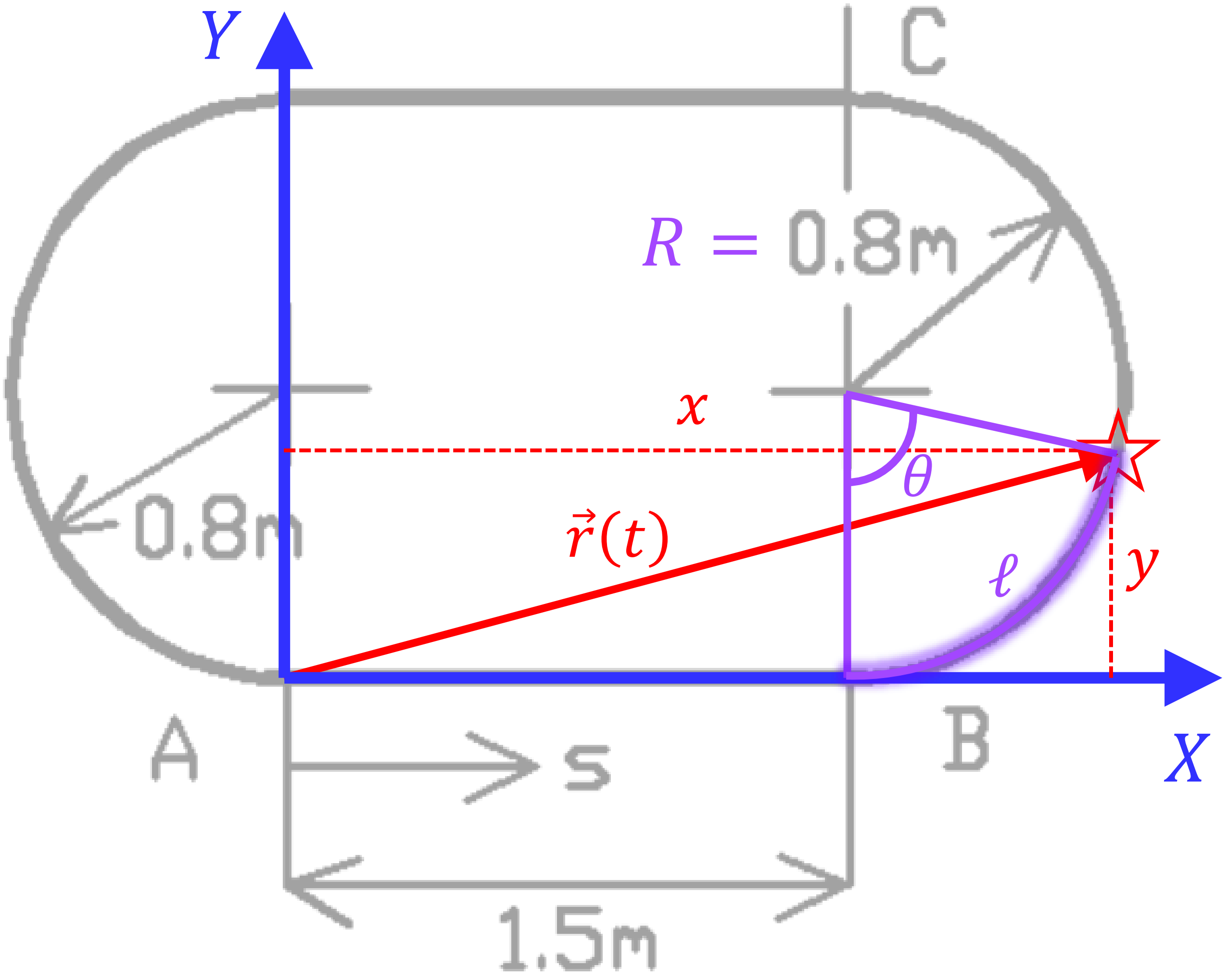
Solution:

b) For $t = 5$ sec: $s(t = 5) = 2.5 \text{ m} > 1.5 \text{ m}$

No longer rectilinear motion!



3. Motivating Example



1. Define reference frame
2. Define a position vector, $\vec{r}(t)$
3. Get the coordinates of $\vec{r}(t)$

$$x = 1.5 + R\sin\theta$$

$$y = R - R\cos\theta$$

HINT: the arc length ℓ of a circle of radius R with central angle θ is given by

$$\ell = R\theta$$

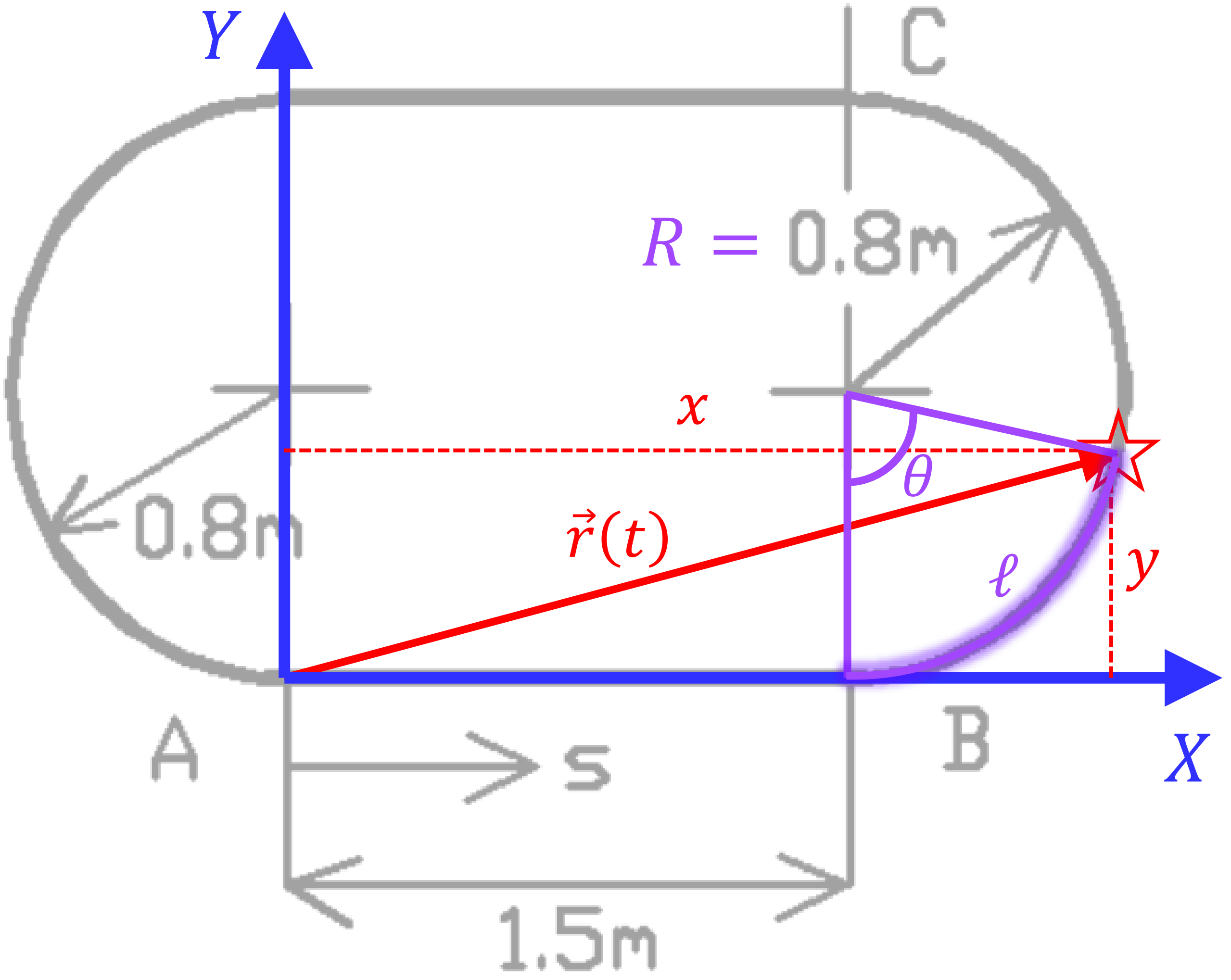
In our problem, $\ell = 1$ m and $R = 0.8$ m:

$$\theta = \frac{\ell}{R} = 1.25 \text{ rad}$$

Thus:

$$\vec{r}(t = 5) = (x\hat{\imath} + y\hat{\jmath}) \text{ m}$$

3. Motivating Example



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In our problem, $\ell = 1$ m and $R = 0.8$ m:

$$\theta = \frac{\ell}{R} = 1.25 \text{ rad}$$

Thus:

$$\vec{r}(t = 5) = (2.26\hat{\imath} + 0.548\hat{\jmath}) \text{ m}$$

3. Motivating Example

Now, for the velocity…

From a), we know $v(t) = 0.2t$:

$$v(t = 5) = 0.2(5) = 1 \text{ m/s}$$

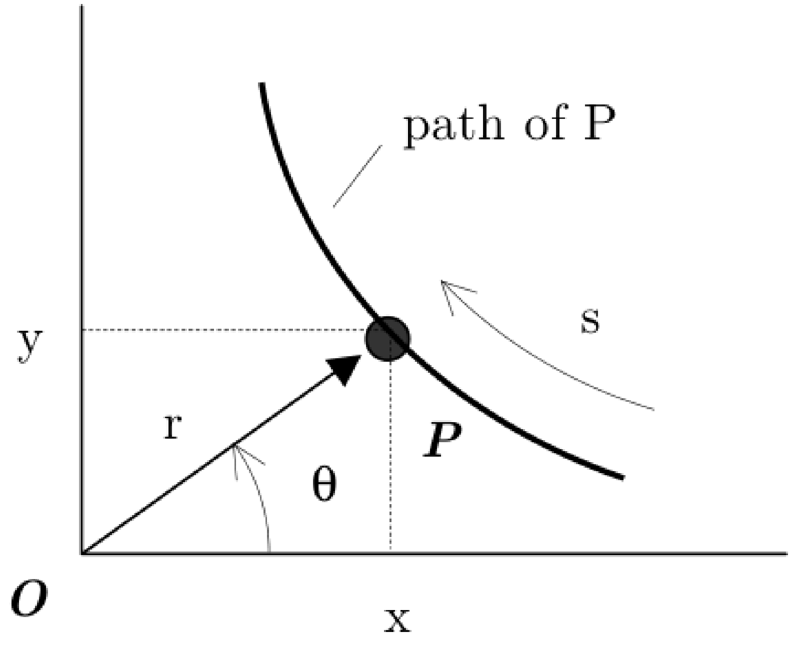
And the direction? – wait for next lecture

4. Kinematic Relations for Cartesian Coordinates

Scalar kinematic relationships do not suffice when we move from rectilinear to curvilinear motion

The following descriptions are more useful:

- Cartesian – path of P in terms of x and y
- Path – position r in terms of a distance s
- Polar – position in terms of r and θ



4. Kinematic Relations for Cartesian Coordinates

Irrespective of the description chosen, the kinematic relationships hold:

$$\vec{v}(t) = \frac{d\vec{r}}{dt} \qquad \text{and} \qquad \vec{a}(t) = \frac{d^2\vec{r}}{dt^2}$$

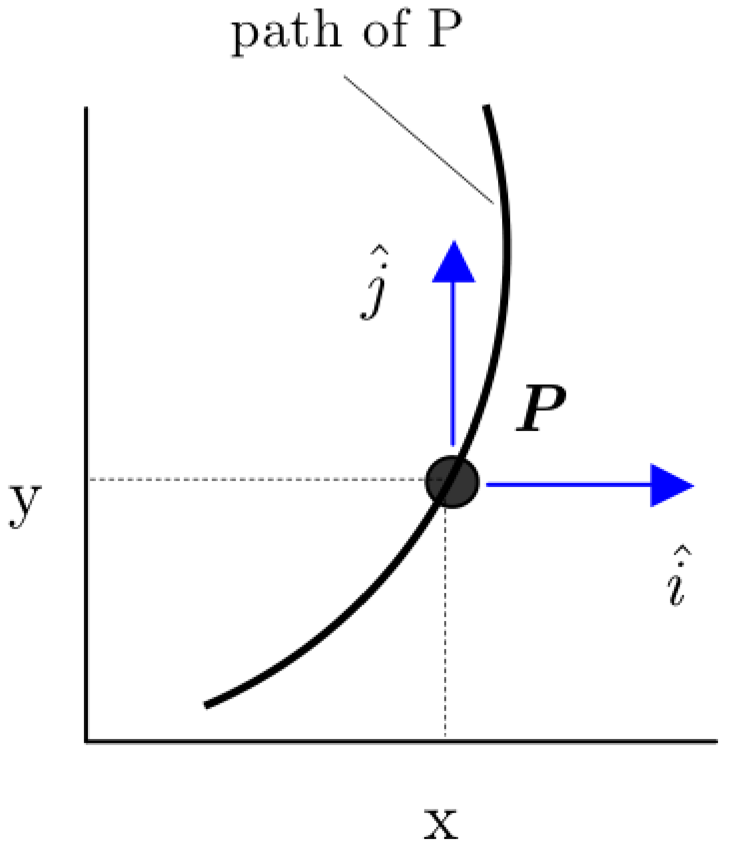
For cartesian coordinates, the position is given by:

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

Thus:

$$\vec{v} = \left(\frac{dx}{dt}\right)\hat{\imath} + \left(\frac{dy}{dt}\right)\hat{\jmath} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$$

$$\vec{a} = \left(\frac{d^2x}{dt^2}\right)\hat{\imath} + \left(\frac{d^2y}{dt^2}\right)\hat{\jmath} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$



5. Example

A point moves along a trajectory whose equation is

$$y = x^3$$

according to the law

$$x = 2t$$

where both x and y are in inches and t is in seconds. What are the velocity and the acceleration when $t = 1$ sec?

5. Example

Given: the equation of the trajectory, $y = x^3$, and the law $x = 2t$

Find: the velocity, $\vec{v}$, and the acceleration, $\vec{a}$

Solution: we know the kinematic equations:

$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} \qquad \text{and} \qquad \vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

$\dot{x} =$

$\ddot{x} =$

$\dot{y} =$

$\ddot{y} =$

5. Example

Given: the equation of the trajectory, $y = x^3$, and the law $x = 2t$

Find: the velocity, $\vec{v}$, and the acceleration, $\vec{a}$

Solution: we know the kinematic equations:

$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} \qquad \text{and} \qquad \vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

$\dot{x} = 2$

$\ddot{x} = 0$

$$\dot{y} = \frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dx}\dot{x} = (3x^2)(2) = (3 \cdot 4t^2)(2) = 24t^2$$

$$\ddot{y} = \frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d\dot{y}}{dt} = \frac{d}{dt}(24t^2) = 48t$$

Thus, at $t = 1$ sec:

$$\vec{v} = \dot{x}(1)\hat{\imath} + \dot{y}(1)\hat{\jmath}$$

$$\vec{a} = \ddot{x}(1)\hat{\imath} + \ddot{y}(1)\hat{\jmath}$$

5. Example

Given: the equation of the trajectory, $y = x^3$, and the law $x = 2t$

Find: the velocity, $\vec{v}$, and the acceleration, $\vec{a}$

Solution: we know the kinematic equations:

$$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} \qquad \text{and} \qquad \vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

$\dot{x} = 2$

$\ddot{x} = 0$

$$\dot{y} = \frac{dy}{dx}\frac{dx}{dt} = \frac{dy}{dx}\dot{x} = (3x^2)(2) = (3 \cdot 4t^2)(2) = 24t^2$$

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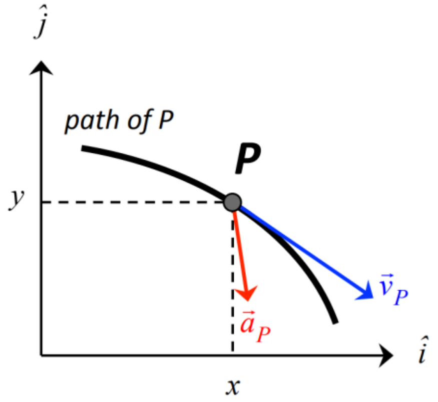
Thus, at $t = 1$ sec:

$$\vec{v} = 2\hat{\imath} + 24\hat{\jmath}$$

$$\vec{a} = 0\hat{\imath} + 48\hat{\jmath}$$

6. Summary and Closure

1. *PROBLEM:* Describe the motion of a point P in Cartesian coordinates
2. *FUNDAMENTAL EQUATIONS:*



$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$: velocity of the point P

$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$: acceleration of the point P

3. *CHAIN RULE OF DIFFERENTIATION:* If y is given in terms of x (instead of time t)…

$$\dot{y} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \dot{x}\frac{dy}{dx}$$

4. *COMMENT:* The Cartesian description is easy to use, but not as useful as other descriptions. More in upcoming lectures…

ME 274: Basic Mechanics II

Week 1 *– Wednesday, January 14*

Particle kinematics: Path description

Instructor: Manuel Salmerón

Today’s Agenda

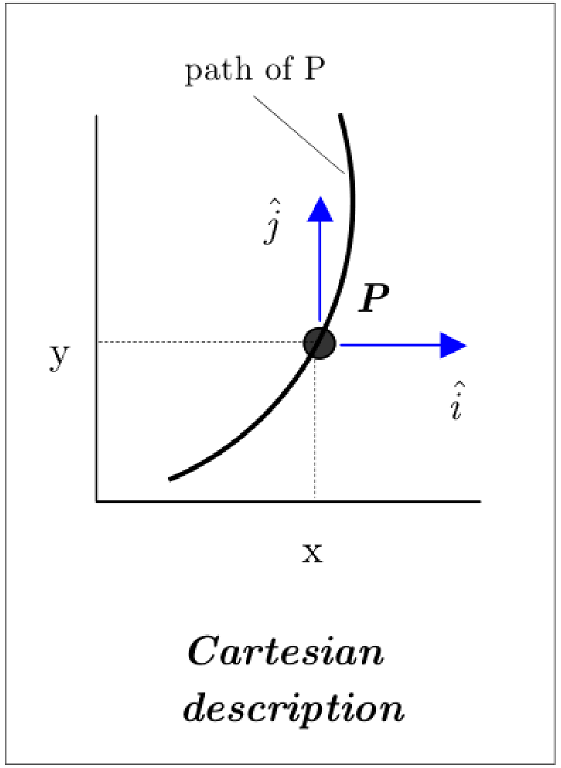
1. Recap: Cartesian coordinates
2. Path Kinematics
3. Example(s)
4. Summary
5. Homework Questions

1. Recap: Cartesian Coordinates

Kinematic Equations for Cartesian Coordinates:

$$\vec{v}(t) = \dot{x}\hat{\imath} + y\hat{\jmath}$$

$$\vec{a}(t) = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$



What if y (or x) is **not** an EXPLICIT function of t?

Chain Rule! If $y = f(x)$:

$$\dot{y} = \frac{dy}{dt} = \frac{df}{dx}\frac{dx}{dt} = \dot{x}\frac{df}{dx}$$

2. Path Kinematics

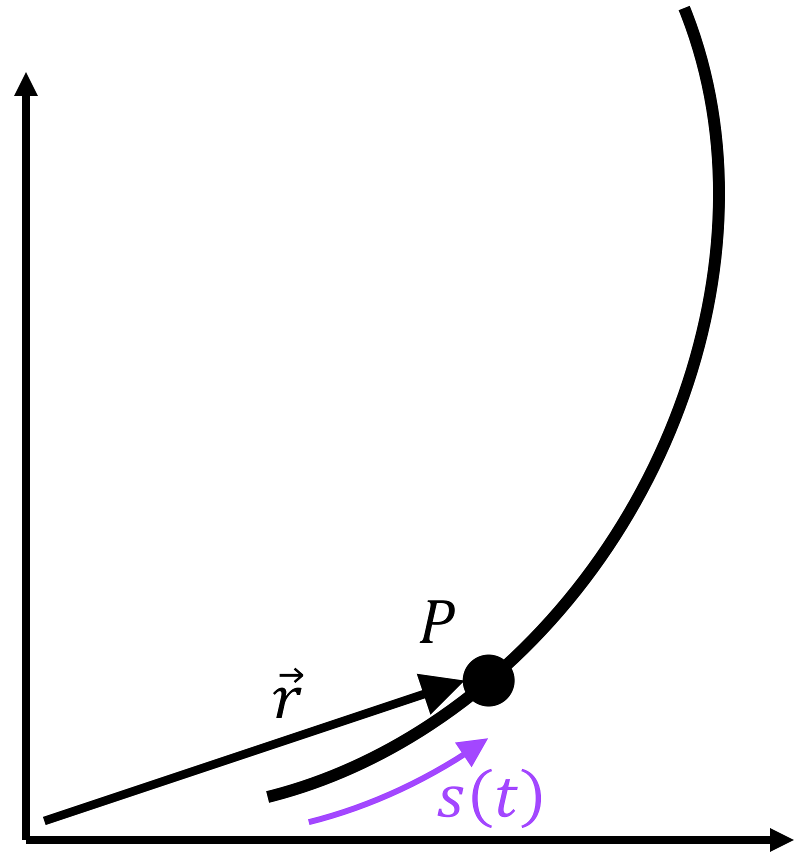
Position: $\vec{r}(t) = f\big(s(t)\big)$

Velocity: $\vec{v}(t) = \dfrac{d\vec{r}}{dt}$

$$\vec{v}(t) = \frac{d\vec{r}}{ds}\frac{ds}{dt} = v\,\frac{d\vec{r}}{ds}$$

v : speed

$d\vec{r}/ds$: direction



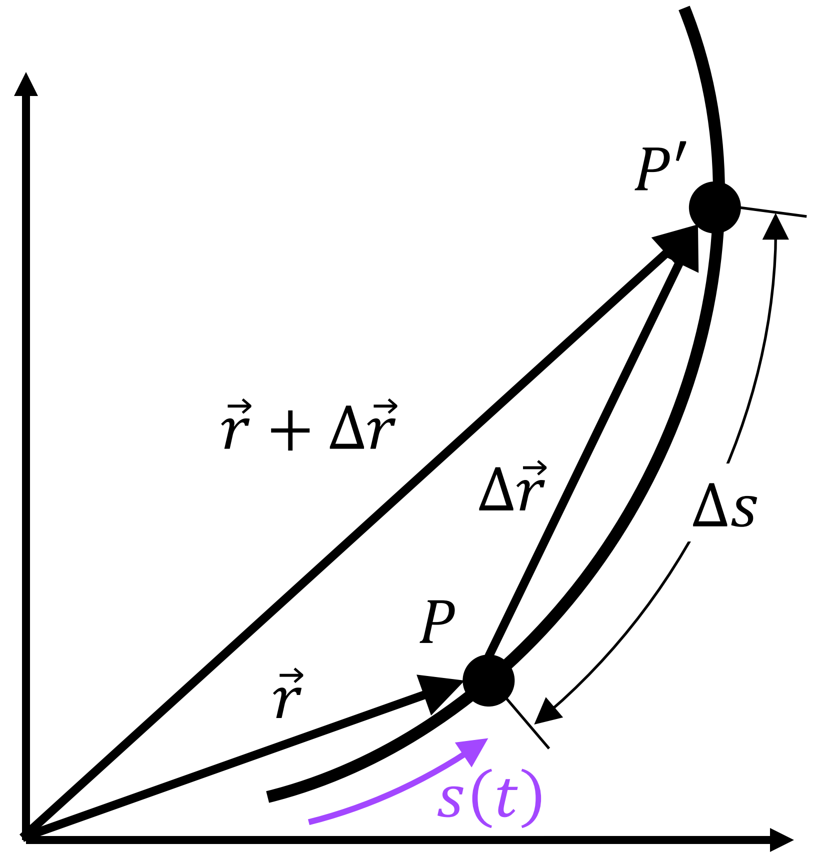
2. Path Kinematics

What can we tell about the direction?

By definition:

$$\frac{d\vec{r}}{ds} = \lim_{\Delta s \to 0} \frac{\Delta \vec{r}}{\Delta s}$$

What happens as $\Delta s \to 0$?



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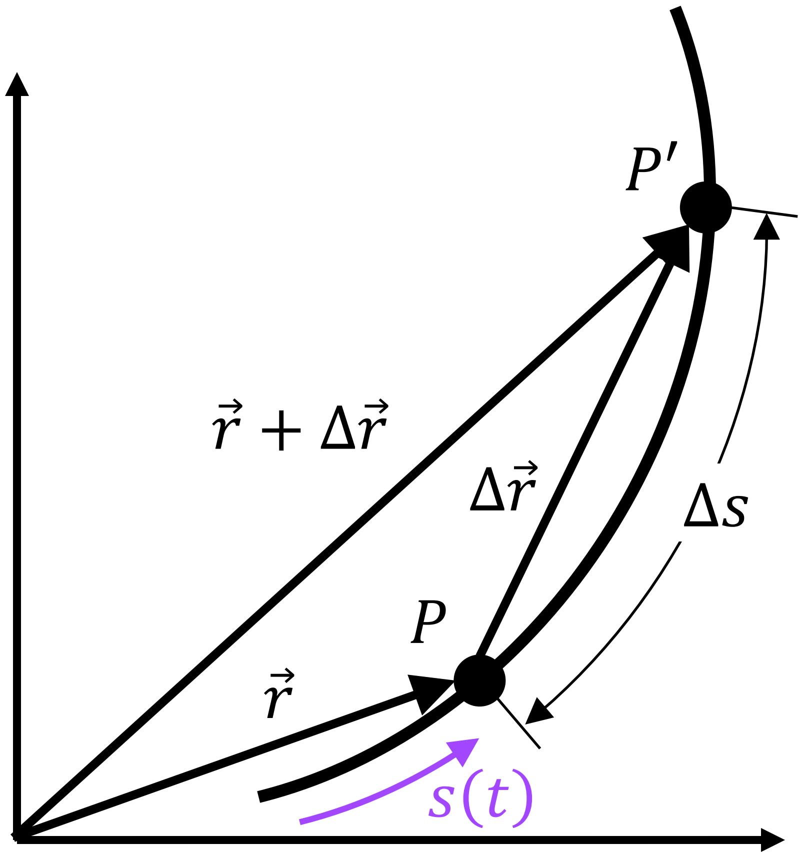

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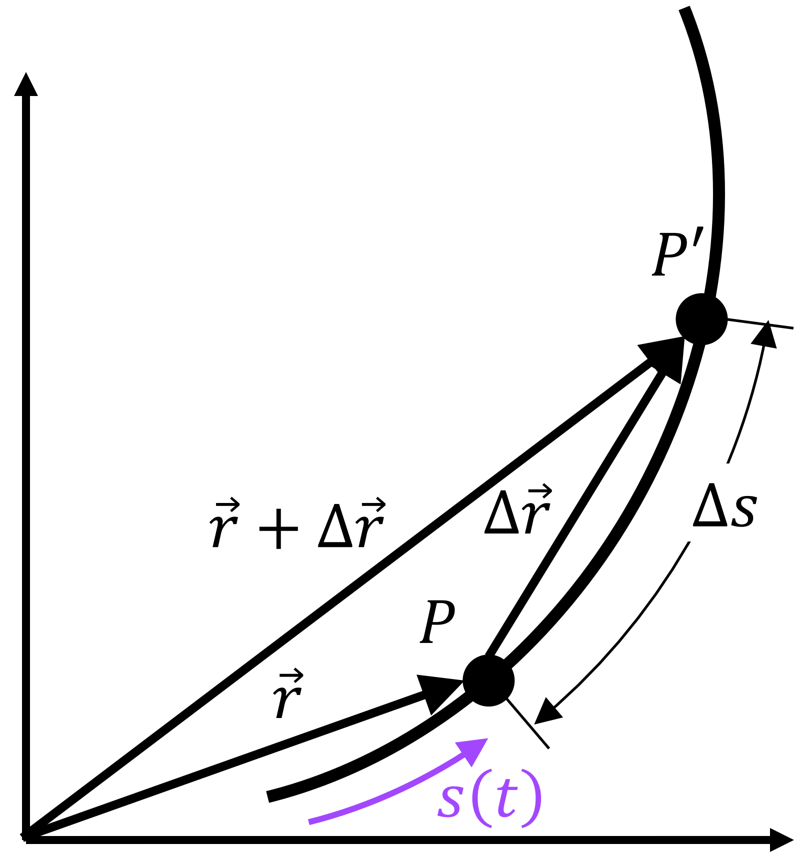


2. Path Kinematics

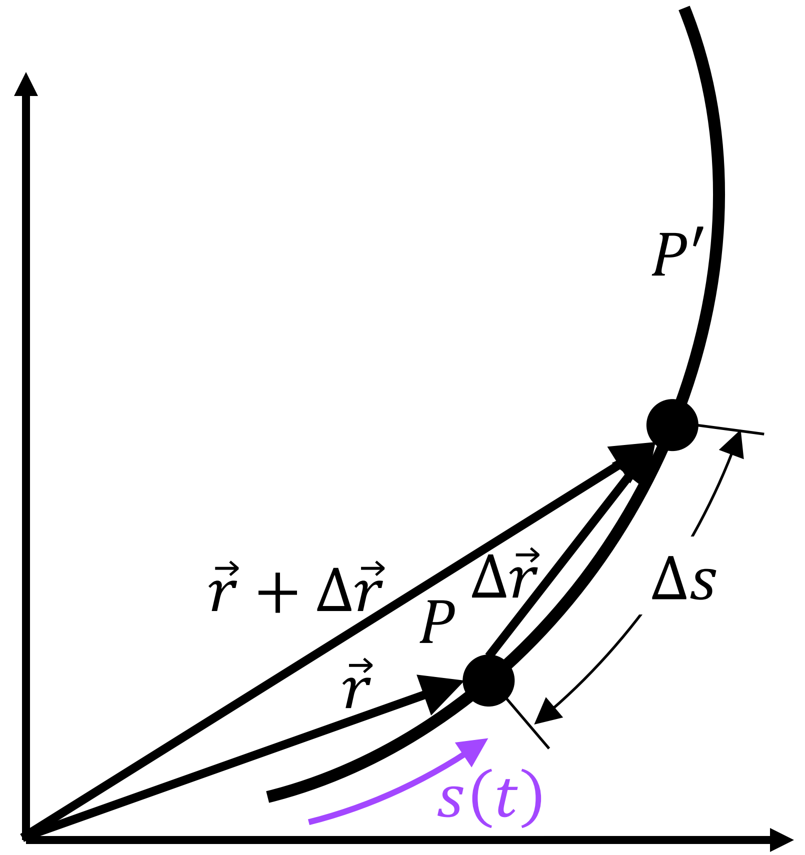
What can we tell about the direction?

By definition:

$$\frac{d\vec{r}}{ds} = \lim_{\Delta s \to 0} \frac{\Delta\vec{r}}{\Delta s}$$

What happens as $\Delta s \to 0$?

- $|\Delta\vec{r}| \approx \Delta s$
- $\Delta\vec{r}$ becomes tangent to the path of P



2. Path Kinematics

Thus,

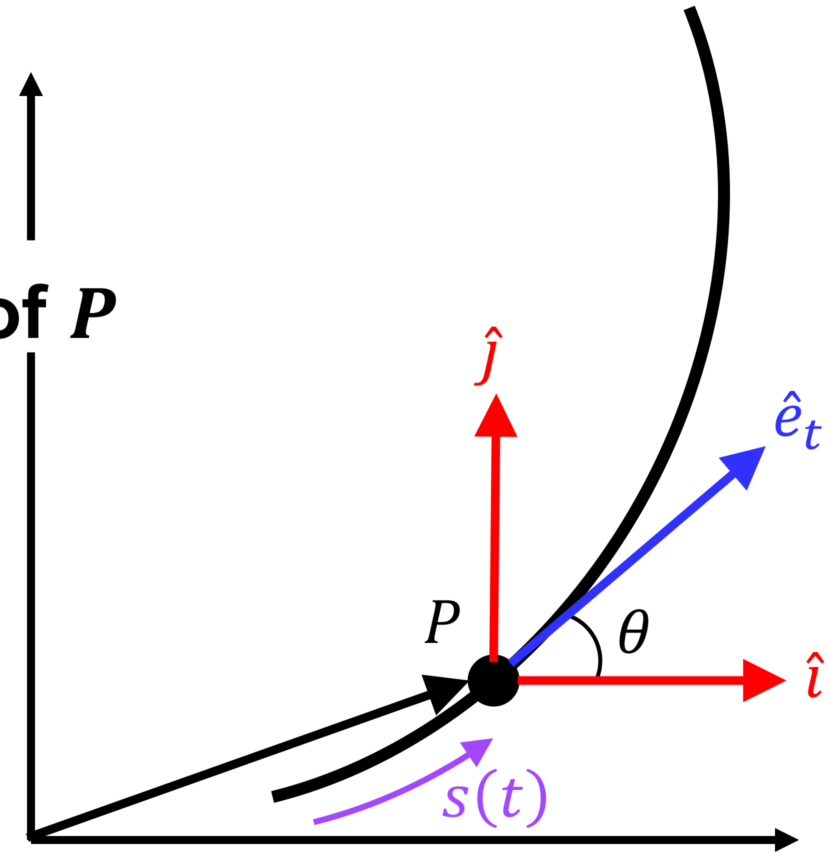
$\frac{d\vec{r}}{ds} = \hat{e}_t$: **unit vector tangent to the path of $\boldsymbol{P}$**

It can be expressed as:

$$\hat{e}_t = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

We can write the velocity as:

$$\vec{v} = v\hat{e}_t$$



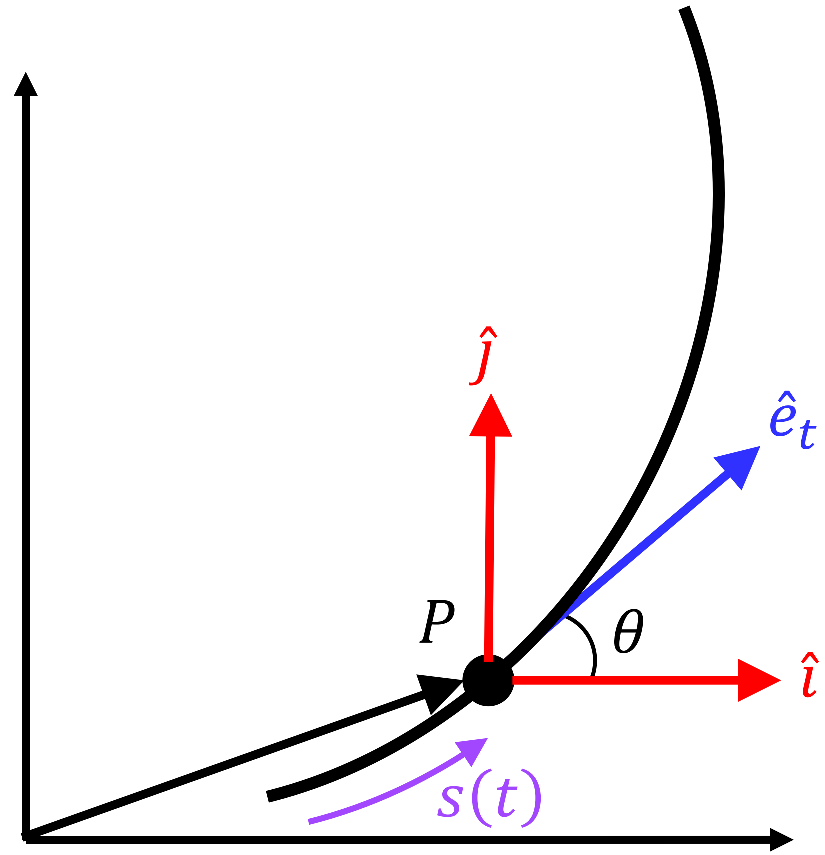
2. Path Kinematics

Now, for the acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt}(v\hat{e}_t)$$

$$= \frac{dv}{dt}\hat{e}_t + v\frac{d\hat{e}}{dt} \quad \text{(product rule)}$$



2. Path Kinematics

Remember that:

$$\hat{e}_t = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

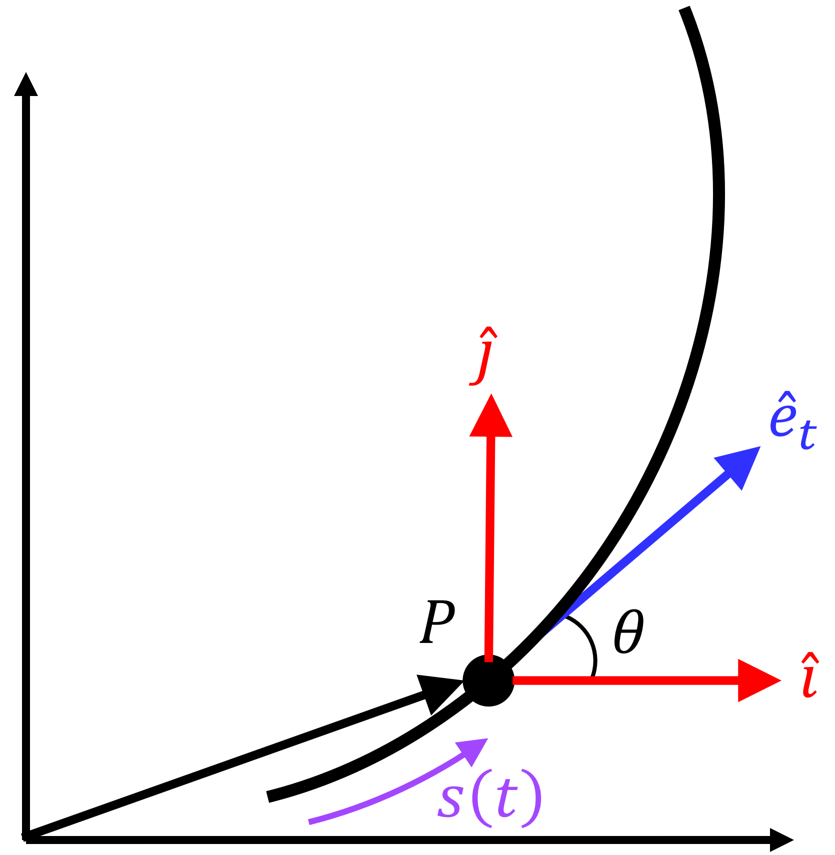
Thus, $\hat{e}_t = f(\theta)$:

$$\vec{a} = \frac{dv}{dt}\hat{e}_t + v\frac{d\hat{e}}{dt} \qquad \text{(product rule)}$$

$$= \dot{v}\hat{e}_t + v\frac{d\hat{e}_t}{d\theta}\frac{d\theta}{dt} \qquad \text{(chain rule)}$$

Also note that: $\theta = f\big(s(t)\big)$:

$$\vec{a} = \dot{v}\hat{e}_t + v\frac{d\hat{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt} = \dot{v}\hat{e}_t + v^2\frac{d\hat{e}_t}{d\theta}\frac{d\theta}{ds}$$



2. Path Kinematics

We define:

$$\hat{e}_n = \frac{d\hat{e}_t}{d\theta} = \frac{d}{d\theta}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

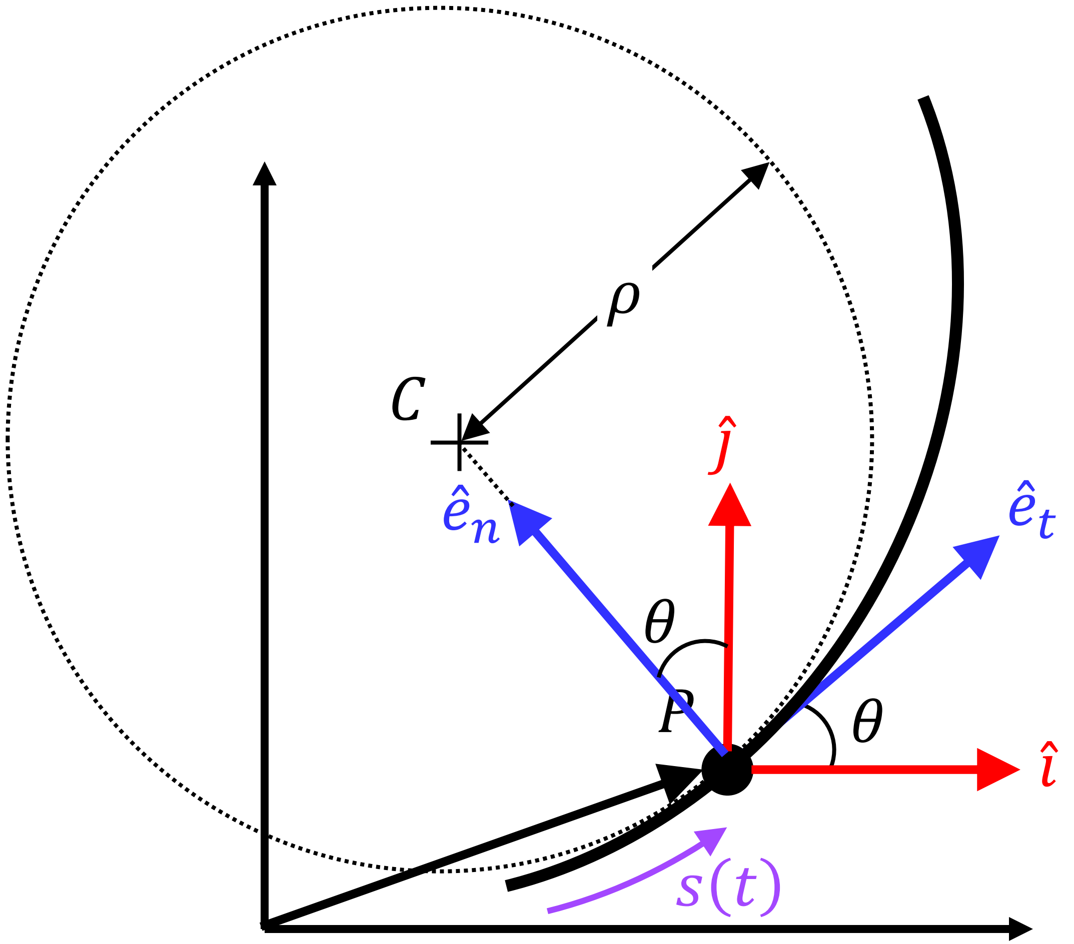
$$\hat{e}_n = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

From the arc length formula:

$$ds = \rho d\theta$$

So:

$$\frac{d\theta}{ds} = \frac{1}{\rho}$$



NOTE: see the lecture book for the derivation in the case where
$$\hat{e}_n = -d\hat{e}_t/d\theta$$

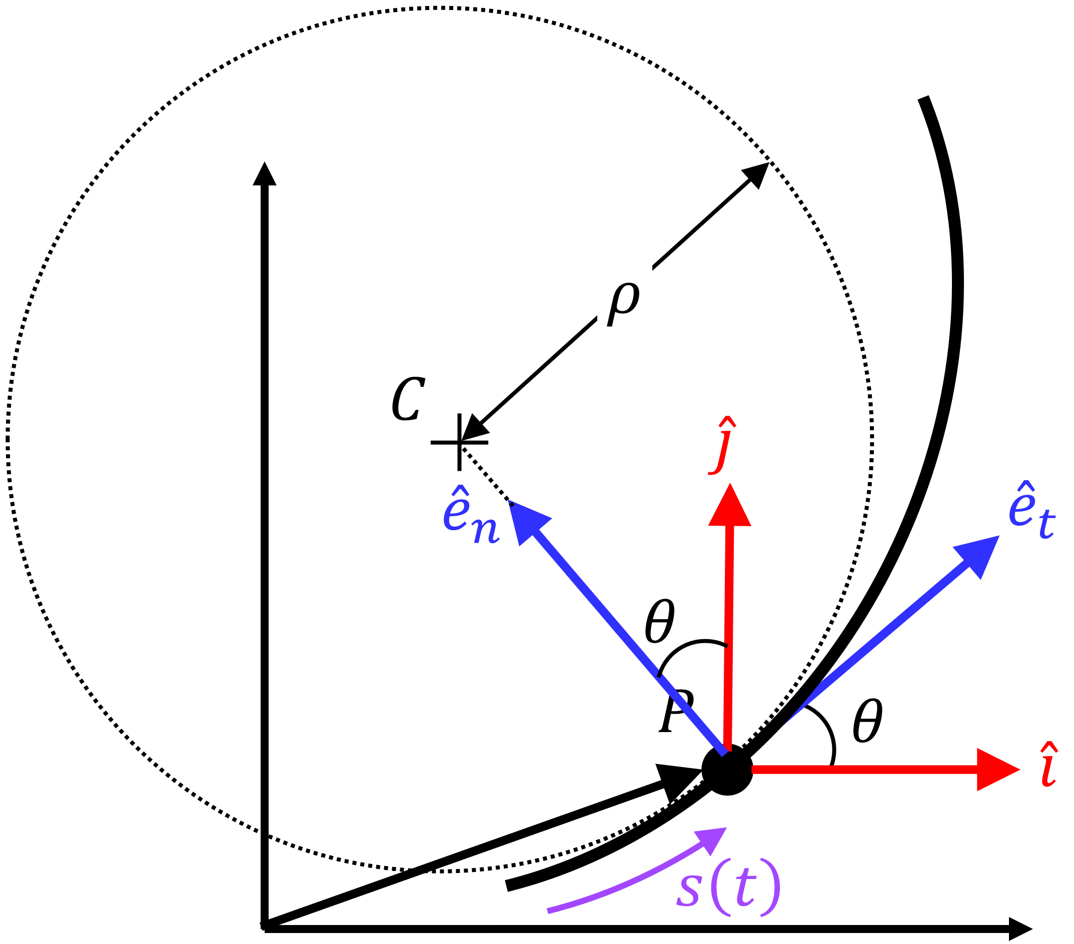
2. Path Kinematics

Finally:

$$\vec{a} = \dot{v}\hat{e}_t + v^2 \frac{d\hat{e}_t}{d\theta}\frac{d\theta}{ds}$$

$$\vec{a} = \dot{v}\hat{e}_t + v^2 \hat{e}_n \cdot \frac{1}{\rho}$$

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$



2. Path Kinematics

For the path description we have:

$$\vec{v} = v\hat{e}_t$$

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

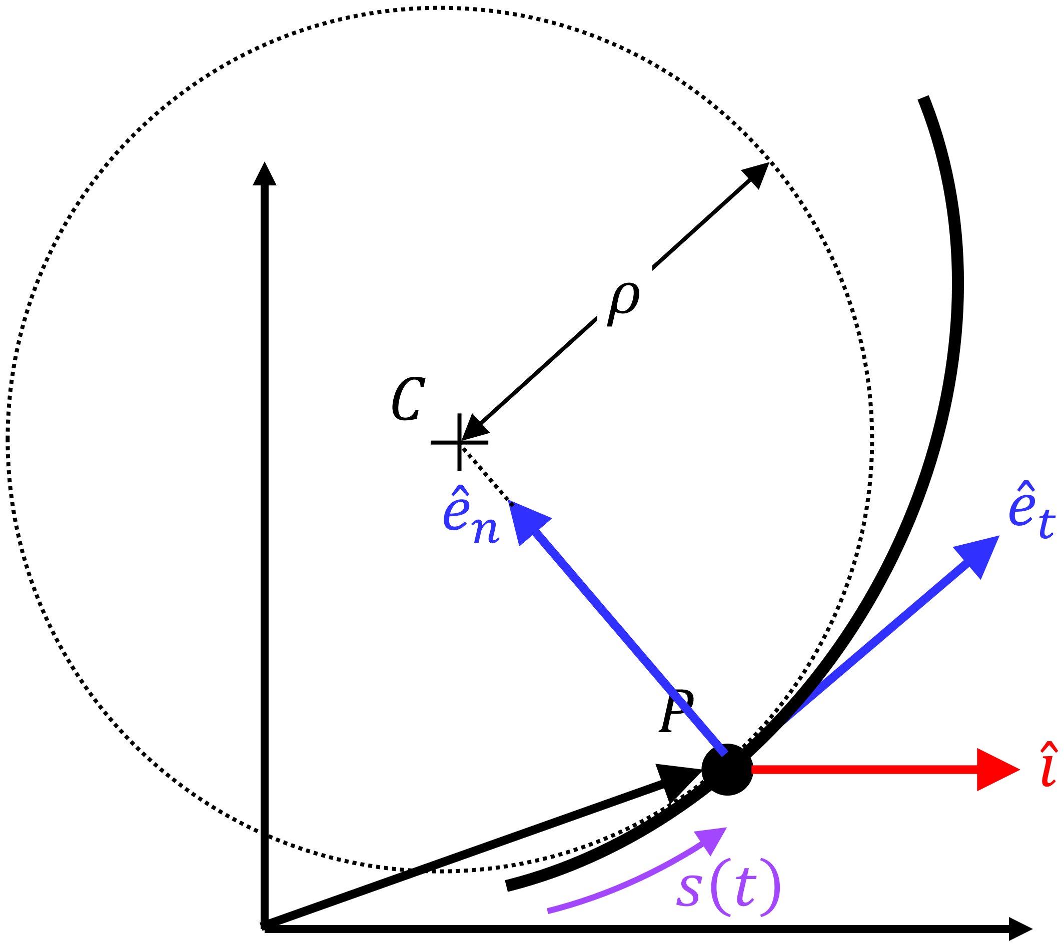
v : speed (how fast?)

ρ : radius of curvature of the path

$\hat{e}_t$: **unit** vector **tangent** to the path

$\hat{e}_n$: **unit** vector **normal** to the path

(ALWAYS pointing **inward** to the path!)



2. Path Kinematics

$$\vec{v} = v\hat{e}_t$$

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

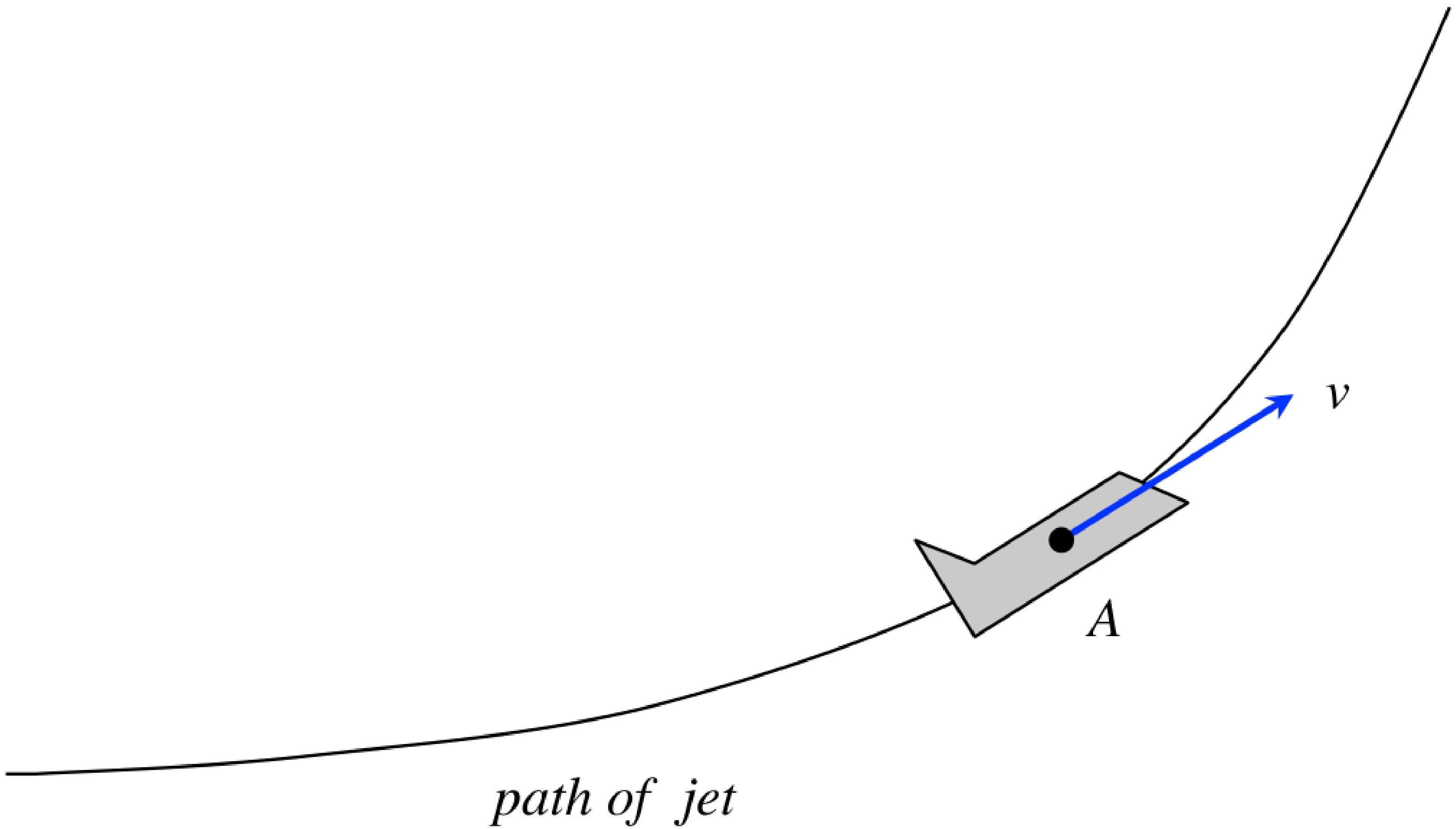
NOTE: Do not confuse $\dot{v}$ (rate of change of speed) with $|\vec{a}|$ (magnitude of acceleration)!

$$|\vec{a}| = \sqrt{\dot{v}^2 + \left(\frac{v^2}{\rho}\right)^2}$$

Example 1.A.3

Given: A jet is flying on the path shown below with a speed of v. At position A on the loop, the speed of the jet is $v = 600$ km/hr, the magnitude of the acceleration is 2.5g and the tangential component of acceleration is $a_t = 5$ m/s^2.

Find: The radius of curvature of the path of the jet at A.



Additional lecture Example 1.2

Given: Particle P moves along a path with its position on the path given by the arc length of s. The speed of P is given as a function of s as: $v_P = bs^2$, where s is given in meters and v_P in terms of meters/second. The radius of curvature of the path is given by ρ and the path tangent is at an angle of θ with respect to the direction of the x-axis.

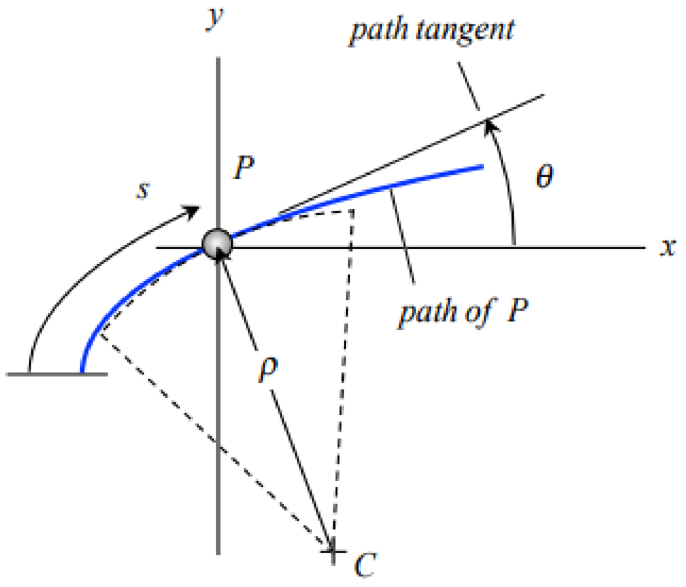
Find: At the position of P where $s = 3$ m:

(a) Make a sketch of the path unit vectors $\hat{e}_t$ and $\hat{e}_n$.

(b) Determine the velocity and acceleration of P in terms of path unit vectors $\hat{e}_t$ and $\hat{e}_n$.

(c) Determine the velocity and acceleration of P in terms of Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

(d) Determine the xy-components of location of the center of curvature, C, for the path.

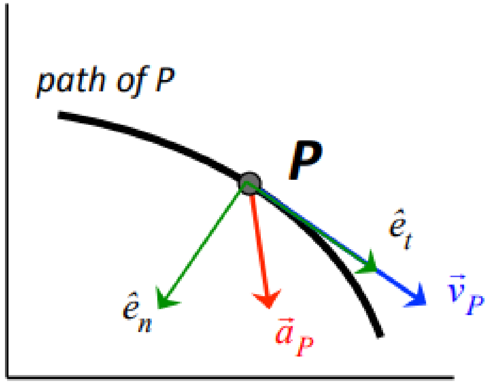


Use the following parameters in your work: $b = 0.5$/m-s, $\rho = 5$ m and $\theta = 30°$.

Summary: Particle Kinematics – Path Description

1. *PROBLEM*: Motion of a point described in path variables.

2. *FUNDAMENTAL EQUATIONS*:



$$\vec{v}_P = v_P \hat{e}_t = velocity\ of\ P$$

$$\vec{a}_P = \dot{v}_P \hat{e}_t + \frac{v_P^2}{\rho} \hat{e}_n = acceleration\ of\ P$$

where $\hat{e}_t$ and $\hat{e}_n$ are unit vectors tangent and (inwardly) normal to the path.

3. *OBSERVATIONS: In regard to the path description kinematics, we see*

- Velocity is ALWAYS tangent to the path.
- Acceleration, in general, has BOTH normal and tangential components.
- Note that acceleration depends on three factors: speed v_P, rate of change of speed $\dot{v}_P$ and radius of curvature of the path ρ.
- Rate of change of speed is the projection of acceleration onto the unit tangent vector: $\dot{v}_P = \vec{a}_P \bullet \hat{e}_t$
- Rate of change of speed is NOT equal to the magnitude of acceleration:

$$\left|\vec{a}_P\right| = \sqrt{\dot{v}_P^2 + \left(v_P^2 / \rho\right)^2} \neq \left|\dot{v}_P\right|$$

ME 274: Basic Mechanics II

Week 1 *– Friday, January 16*

Particle kinematics: Polar description

Instructor: Manuel Salmerón

Attendance!

- Log in with your Purdue email (NOT Gmail)
- You have 30 seconds to answer each question
- The questions will only appear in the slides

Access:

Question 1

Speed is a…

a) …vector, $\hat{v}$, denoting the direction of the velocity

b) …scalar, v, denoting the magnitude of the velocity

c) …vector, $\vec{v}$, denoting the velocity of the particle

d) …scalar, $\dot{v}$, denoting the rate of change of speed

Question 2

The velocity, $\vec{v}$, is always _______________ to the path

a) normal

b) pointing

c) tangent

d) parallel

Question 3

$\hat{e}_n$ is the _______________ component of acceleration

a) tangent

b) only

c) x

d) normal

Question 4

$\hat{e}_n$ is always…

a) …directed outward to the path

b) …directed inward to the path

c) …tangent to the path

d) …parallel to the path

Correct answers

Q1: b) Speed is a scalar, v, denoting the magnitude of the velocity

Q2: c) The velocity, $\vec{v}$, is always tangent to the path

Q3: d) $\hat{e}_n$ is the normal component of acceleration

Q4: b) $\hat{e}_n$ is always directed inward to the path

Today’s Agenda

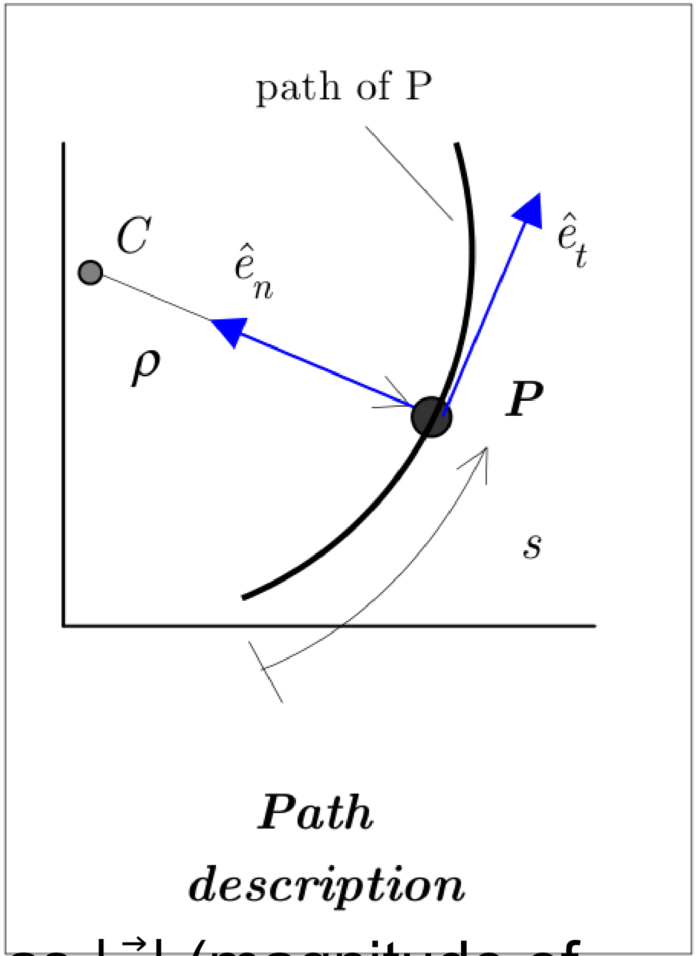
1. Recap: path coordinates
2. Path coordinates example
3. Polar coordinates
4. Example
5. Summary

1. Recap: Path Coordinates

Kinematic Equations for Path Coordinates:

$$\vec{v}(t) = v\hat{e}_t$$

$$\vec{a}(t) = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$



To keep in mind:

- v is a scalar (speed), $\vec{v}$ is a vector (velocity)
- $\vec{v}$ is always tangent to the path
- $\dot{v} = ds/dt$ (rate of change of speed) is NOT the same as $|\vec{a}|$ (magnitude of acceleration)
- $\dot{v}$ is the (scalar) projection of $\vec{a}$ onto $\hat{e}_t$: $\dot{v} = \vec{a} \cdot \hat{e}_t$
- $\hat{e}_n$ is always directed inward to the path

Additional lecture Example 1.2

Given: Particle P moves along a path with its position on the path given by the arc length of s. The speed of P is given as a function of s as: $v_P = bs^2$, where s is given in meters and v_P in terms of meters/second. The radius of curvature of the path is given by ρ and the path tangent is at an angle of θ with respect to the direction of the x-axis.

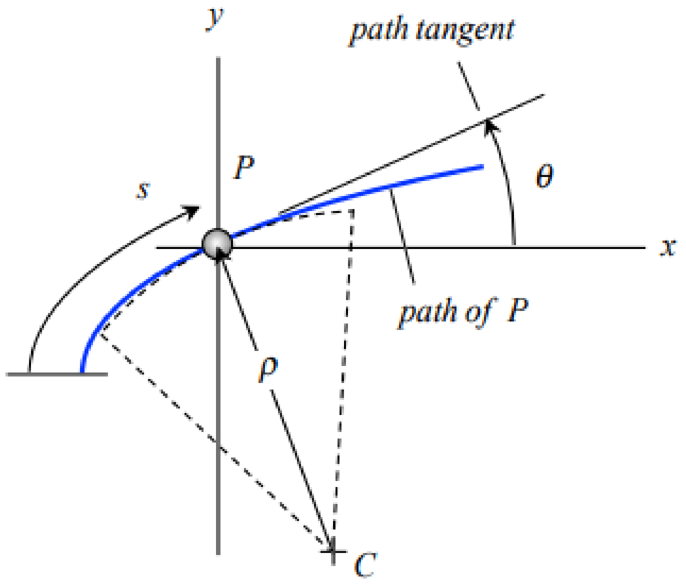
Find: At the position of P where $s = 3$ m:

(a) Make a sketch of the path unit vectors $\hat{e}_t$ and $\hat{e}_n$.

(b) Determine the velocity and acceleration of P in terms of path unit vectors $\hat{e}_t$ and $\hat{e}_n$.

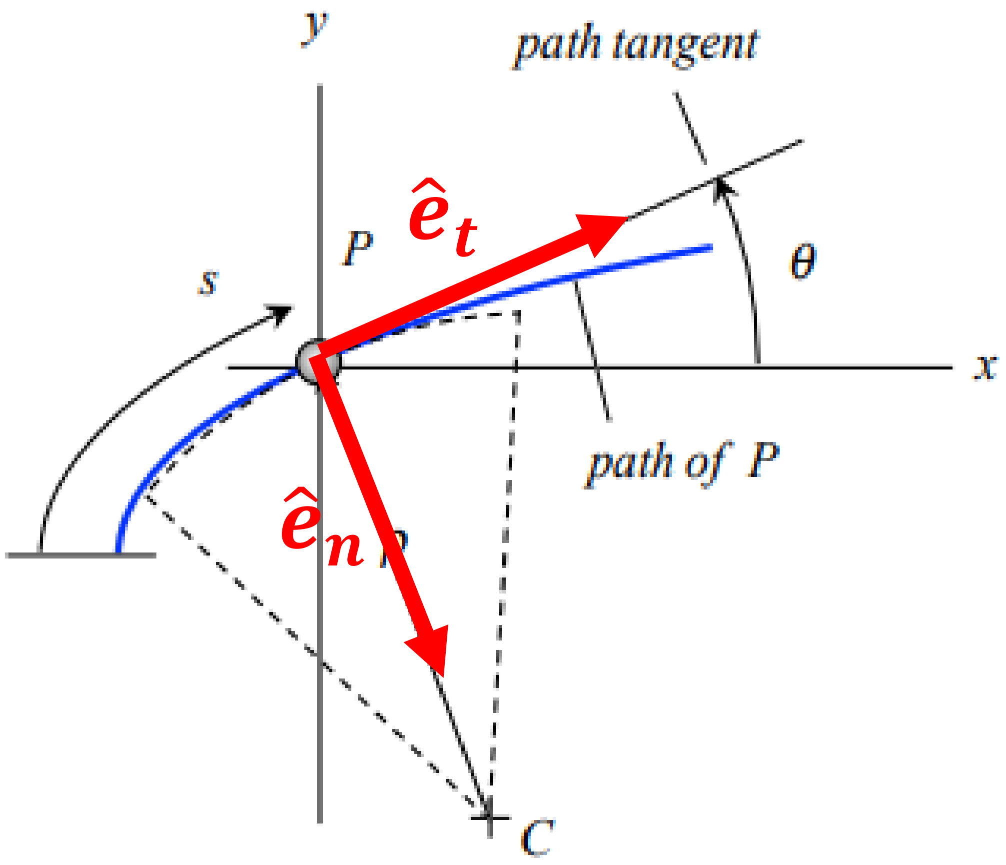
(c) Determine the velocity and acceleration of P in terms of Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

(d) Determine the xy-components of location of the center of curvature, C, for the path.



Use the following parameters in your work: $b = 0.5$/m-s, $\rho = 5$ m and $\theta = 30°$.

2. Additional Lecture Example 1.2



(b) Get $\vec{v}$ and $\vec{a}$:

Write the fundamental equations:

$$\vec{v} = v\hat{e}_t$$

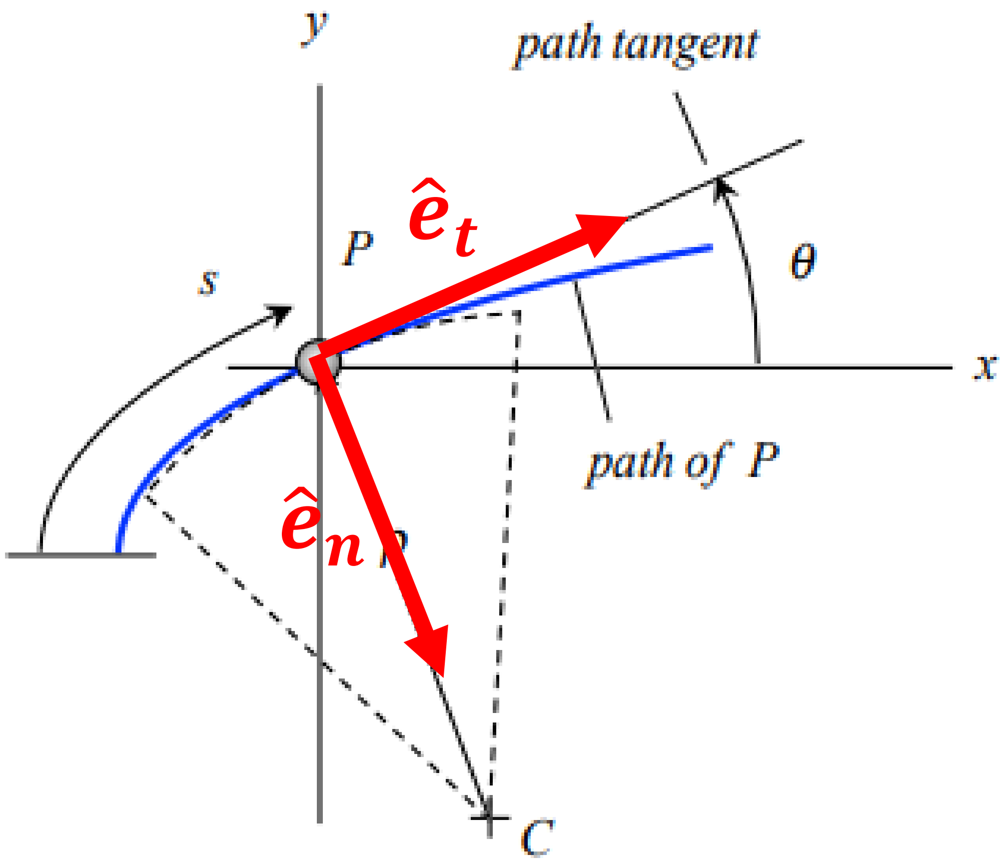
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Do we have everything we need? If not, what are we missing?

$v = v_P = bs^2$ (given)

$$\dot{v} = \dot{v}_P = \frac{dv}{ds}\frac{ds}{dt} = 2bsv = 2b^2s^3$$

2. Additional Lecture Example 1.2



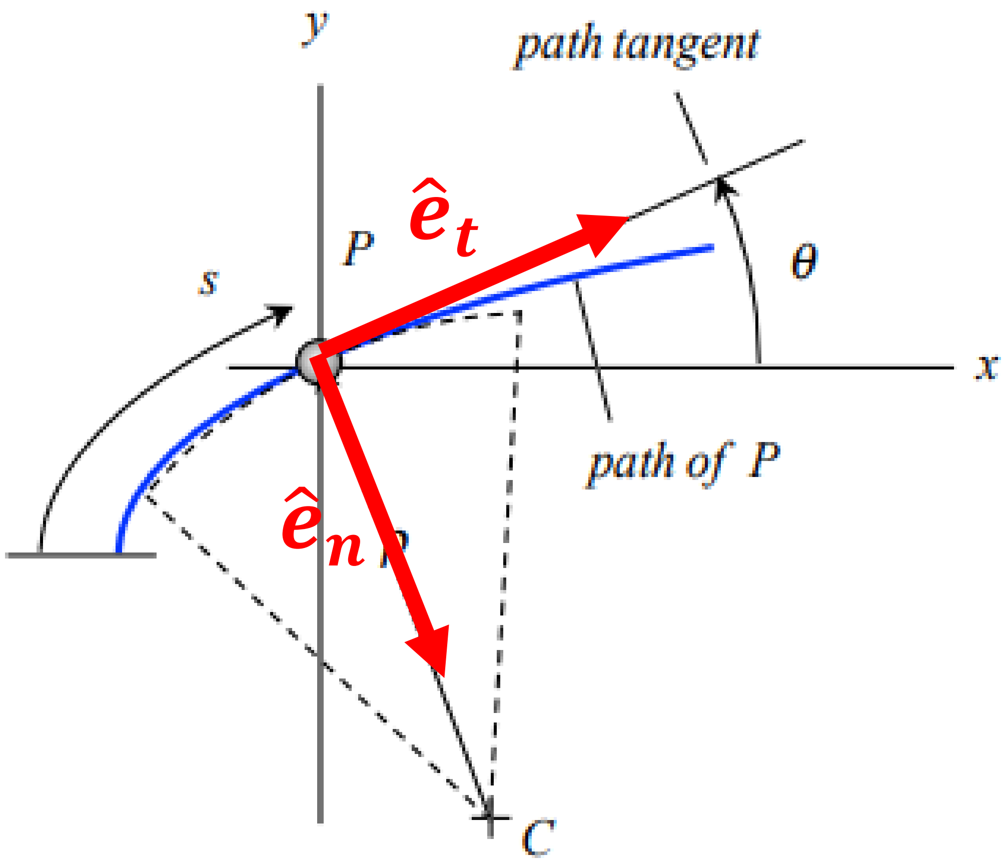
(b) Get $\vec{v}$ and $\vec{a}$:

Plug in values:

$\vec{v} = 4.5\hat{e}_t$ m/s

$\vec{a} = 13.5\hat{e}_t + 4.05\hat{e}_n$ m/s^2

2. Additional Lecture Example 1.2



(c) Get $\vec{v}$ and $\vec{a}$ in Cartesian coordinates

From figure:

$$\hat{e}_t = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

$$\hat{e}_n = \sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}$$

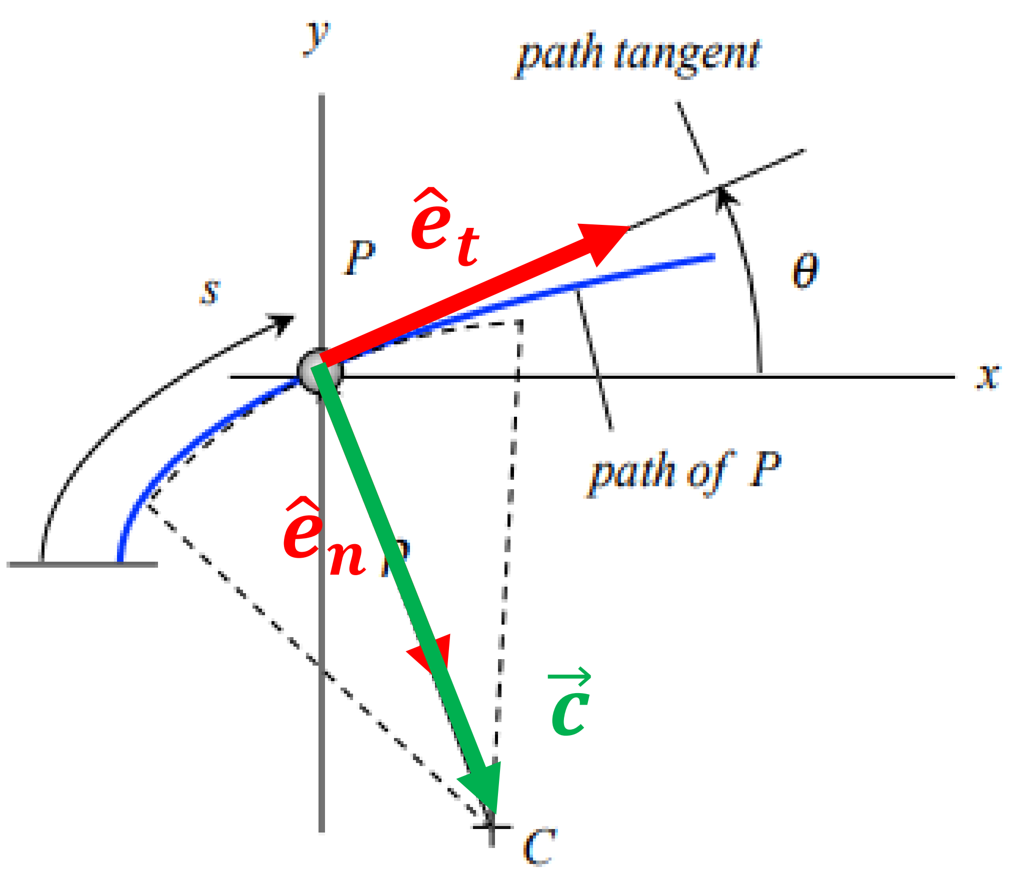
Substitute $\hat{e}_t$ and $\hat{e}_n$ into $\vec{v}$ and $\vec{a}$:

$$\vec{v} = 4.5(\cos\theta\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$\vec{a} = 13.5(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) + 4.05(\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath})$$

Then, plug in θ and you're done!

2. Additional Lecture Example 1.2



(c) xy-components of C

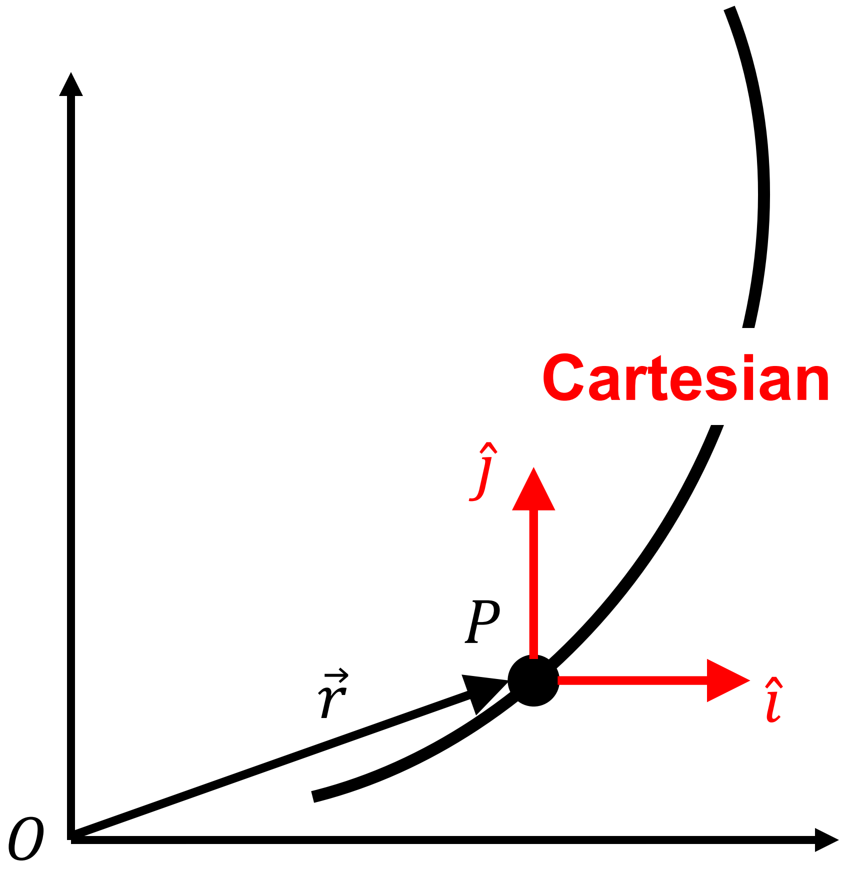
Remember: $\hat{e}_n$ is ALWAYS directed inward to the path (see figure)

Thus, the position vector of C is:

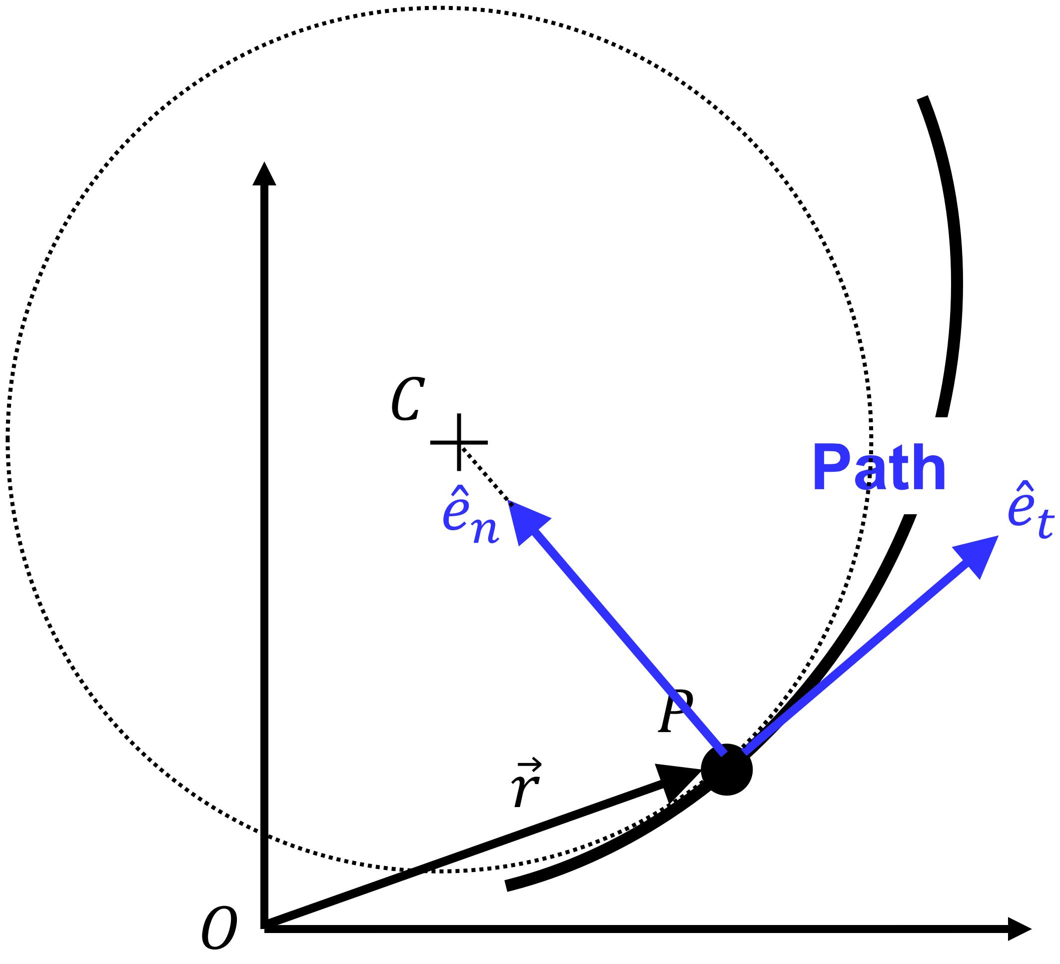
$$\vec{c} = \rho \hat{e}_n$$

$$\vec{c} = 5(\sin 30° \,\hat{\imath} - \cos 30° \,\hat{\jmath})$$

3. Polar Coordinates

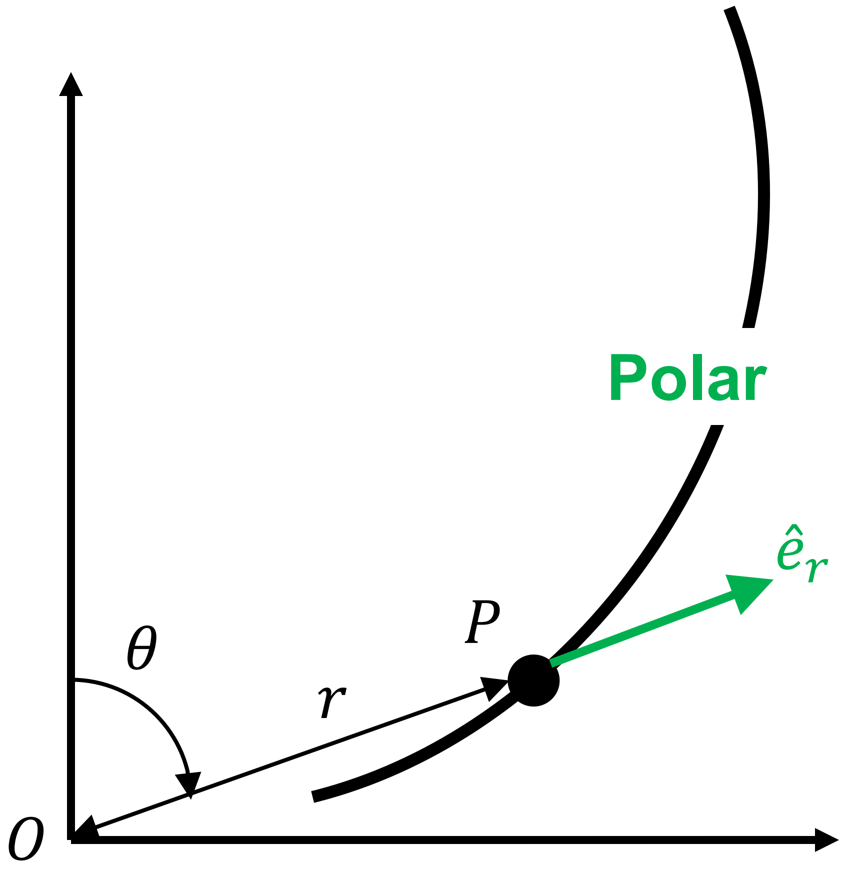


3. Polar Coordinates



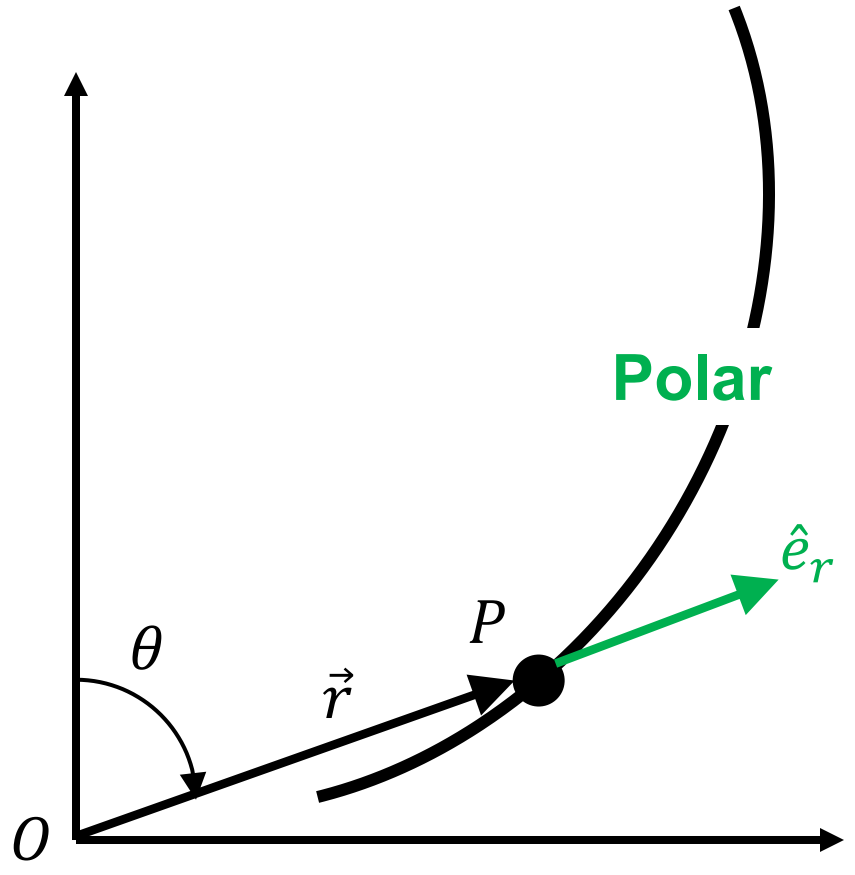
3. Polar Coordinates

Position:



3. Polar Coordinates

Position: $\vec{r}(t) = r\hat{e}_r$

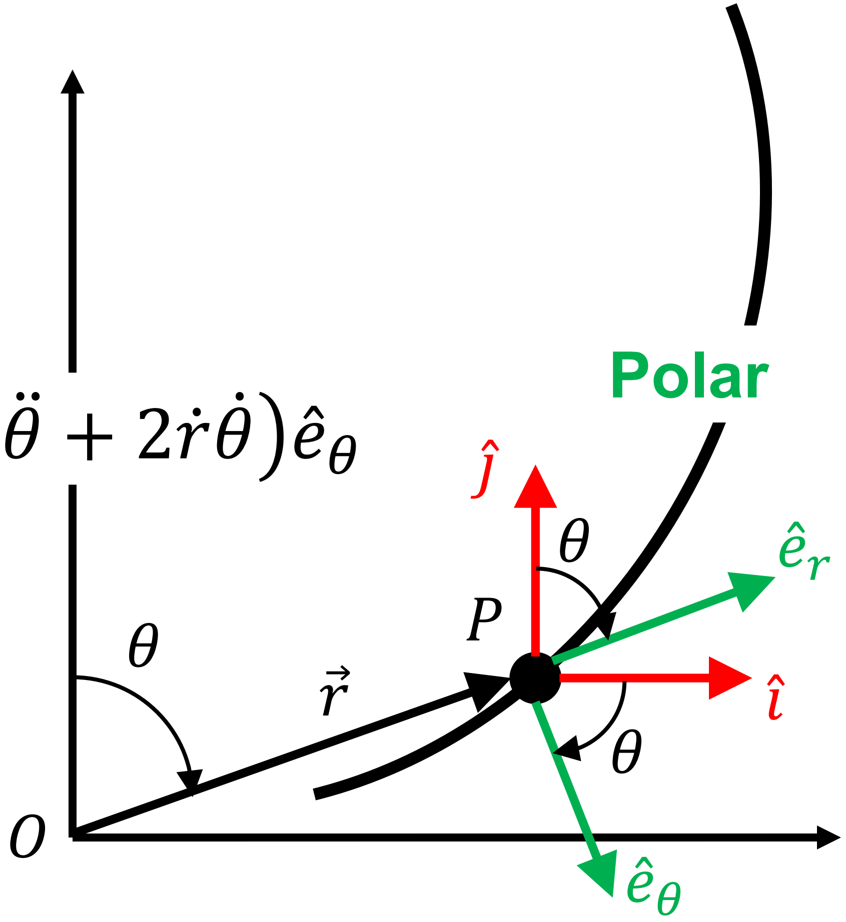


3. Polar Coordinates

Position: $\vec{r}(t) = r\hat{e}_r$

Velocity: $\vec{v}(t) = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

Acceleration: $\vec{a}(t) = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$



NOTES:

- θ depends on O: choose wisely, be consistent
- If $r = r(\theta)$, use chain rule to get $\dot{r}$ and $\ddot{r}$

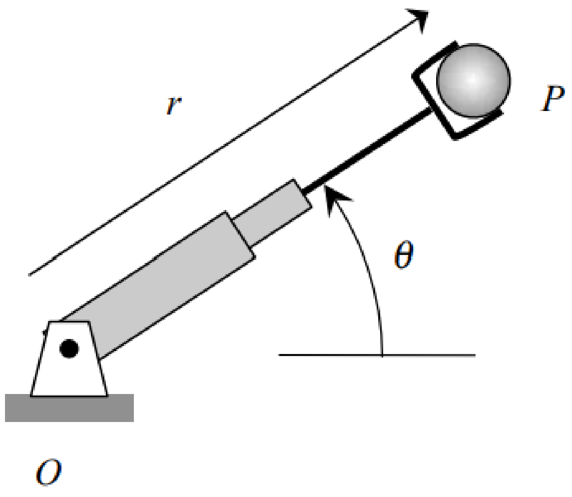
4. Additional Lecture Example 1.4

Given: A rotating and telescoping robotic arm is gripping a small sphere P in its end effector. The arm is rotating counterclockwise with a constant angular speed of $\dot{\theta}$. The arm is extending such that the radial distance from O to P is related to the rotation angle θ by the following equation:

$$r(\theta) = R_0 + R_1 \cos 2\theta$$

where r and θ are given in terms of meters and radians, respectively.

Find: Determine the velocity and acceleration of the sphere P. Write your answers as vectors in terms of the polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$.



Use the following parameters in your analysis: $R_0 = 2$ m, $R_1 = 0.5$ m, $\theta = \pi/2$ rad and $\dot{\theta} = 2$ rad/s.

Solution:

We need $\vec{v}$ and $\vec{a}$:

$$\vec{v}(t) = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a}(t) = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

What do we have? What are we missing?

$\dot{r}$ = need!

r = given!

$\dot{\theta}$ = given!

$\ddot{r}$ = need!

$\ddot{\theta} = 0$