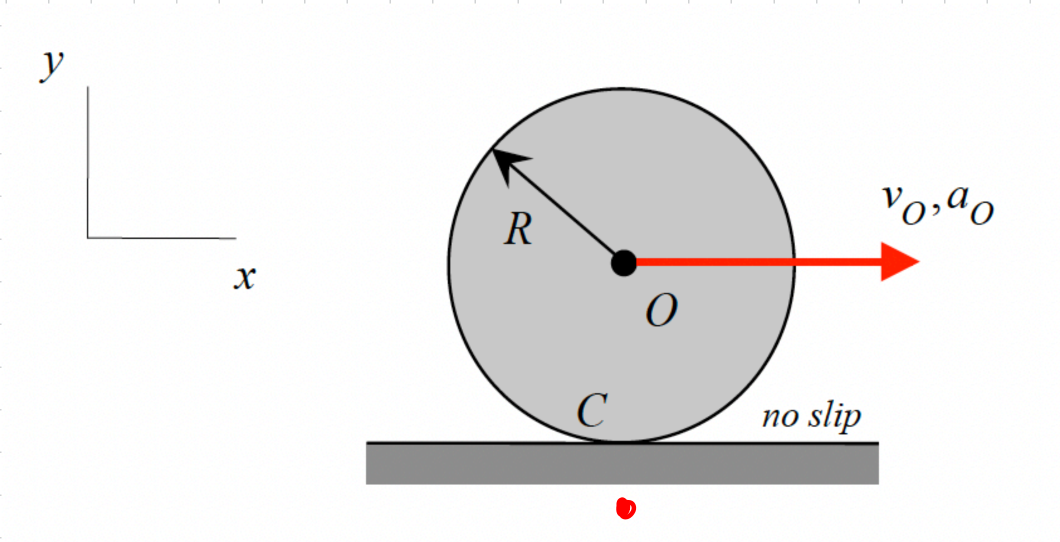
1|30|2026

Rolling without slipping

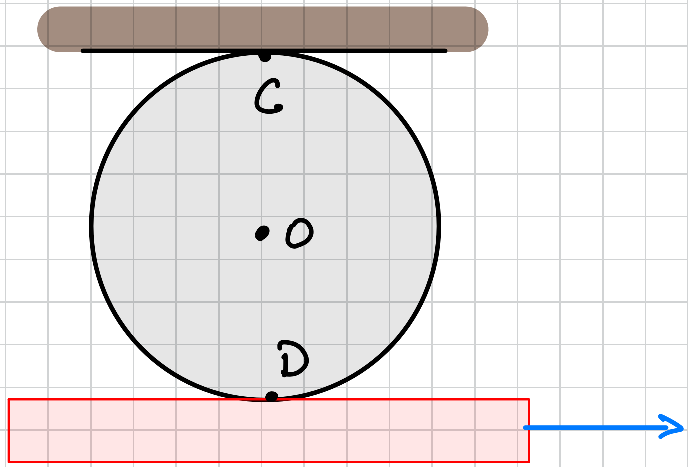
Consider the wheel shown. As it moves, sufficient friction acts between the wheel and the surface such that contact point C doesn't slip

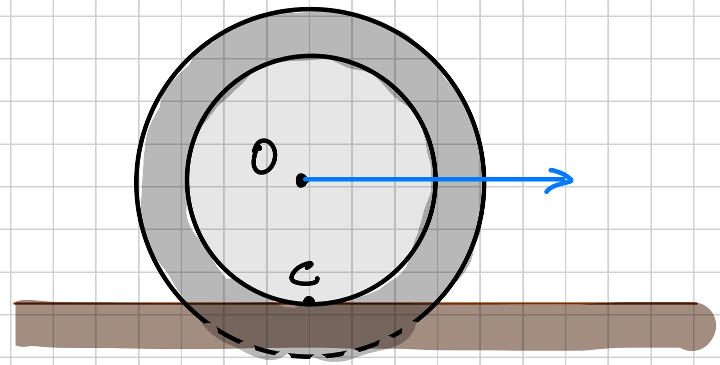


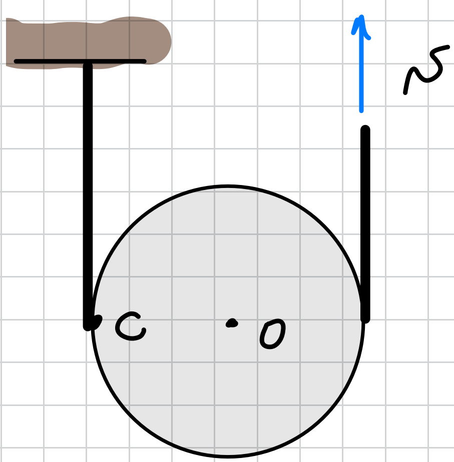
NO SLIP CONDITION AT C
$$\begin{cases} v_{Cx} = 0 \\ a_{Cx} = 0 \end{cases}$$

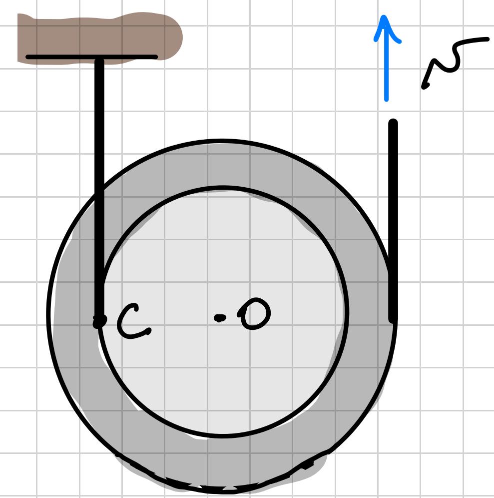
Both v_{Cx} and a_{Cx} need to be zero for no slip. otherwise, point C will slip

Many problems througout ME 274 will have rolling w/o slipping conditions:

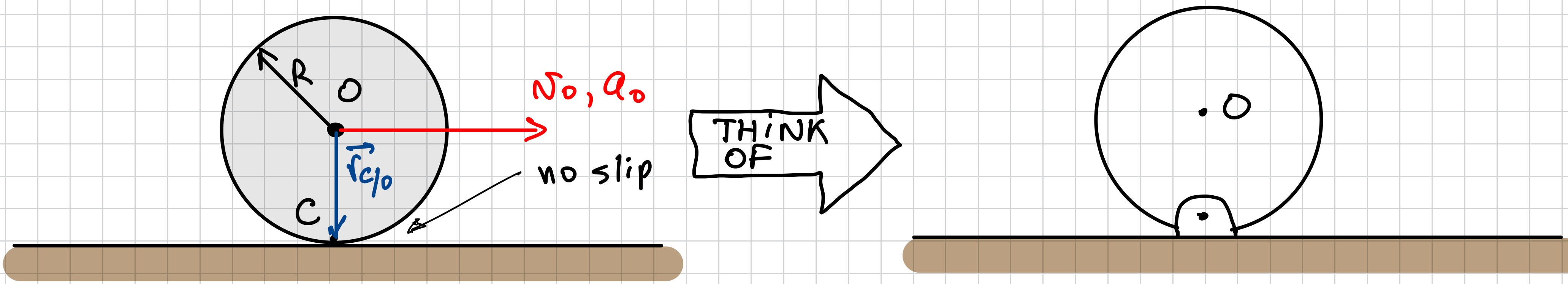








If C is a no-slipping point, what are its y velocity and acceleration vectors?



Treat the no-slip point as fixed.

$$\vec{v}_C = \vec{v}_O + \vec{\omega} \times \vec{r}_{c/o}$$

$$v_{cy}\hat{\jmath} = v_o\hat{\imath} + \omega\hat{k} \times (-R\hat{\jmath})$$

$$v_{cy}\hat{\jmath} = (v_o + \omega R)\hat{\imath}$$

$\hat{\imath}$

$\hat{k} + \hat{\jmath}$

↳ $\hat{\imath}$: $0 = v_o + \omega R \Rightarrow \vec{v}_o = -\omega R\,\hat{\imath}$

$\hat{\jmath}$: $v_{cy} = 0$

Now, we work the acceleration:

$$\vec{a}_C = \vec{a}_O + \vec{\alpha}_O \times \vec{r}_{C/O} - \omega_O^2 \, \vec{r}_{C/O}$$

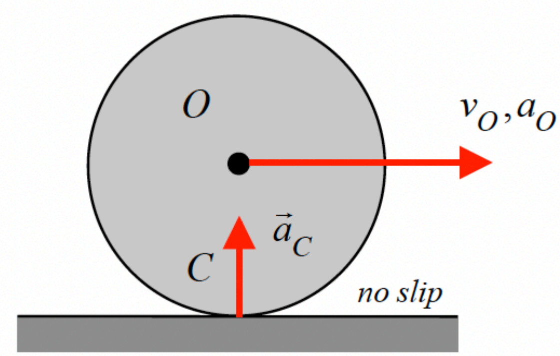
$$a_C \hat{j} = a_O \hat{i} + \alpha_O \hat{k} \times (-R\hat{j}) - \omega_O^2 (-R\hat{j})$$

$$a_C \hat{j} = a_O \hat{i} + \alpha_O R \hat{i} + \omega_O^2 R \hat{j}$$

$$\hat{i}: \quad 0 = a_O + \alpha_O R \implies a_O = -\alpha_O R \hat{k}$$

$$\hat{j}: \quad a_{Cy} = \omega_O^2 R \neq 0$$

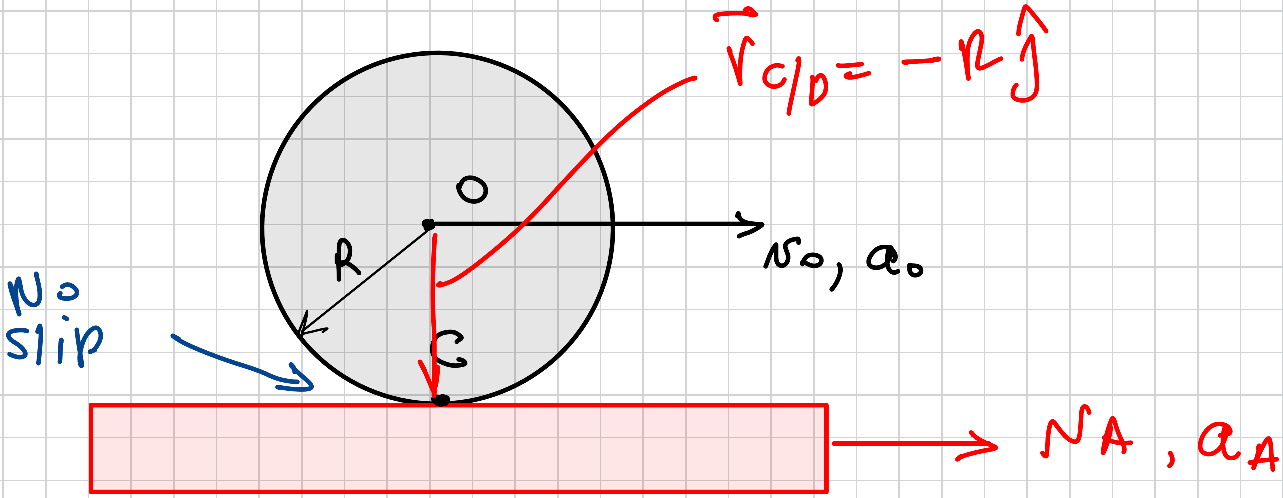
So, the velocity of a no-slip point is zero, but its acceleration is NOT ZERO



$\vec{v}_C = \vec{0}$

$\vec{a}_C = R\omega^2 \hat{j}$

What about when the slip point is moving?



and

$$\vec{v}_c = \vec{v}_o + \vec{\omega} \times \vec{r}_{c/o}$$

$$v_A\hat{i} = v_o\hat{i} + \omega\hat{k} \times (-R\hat{j})$$

$$v_A\hat{i} = v_o\hat{i} + \omega R\hat{i} = (v_o + \omega R)\hat{i}$$

So

$$v_{c_x} = v_A$$

$$\vec{a}_c = \vec{a}_o + \alpha \times \vec{r}_{c/o} - \omega^2 \vec{r}_{c/o}$$

$$a_{cx}\hat{i} + a_{cy}\hat{j} = a_o\hat{i} + \alpha\hat{k} \times (-R\hat{j}) - \omega^2(-R\hat{j})$$

$$a_{cx}\hat{i} + a_{cy}\hat{j} = a_o\hat{i} + \alpha R\hat{i} + \omega^2 R\hat{j}$$

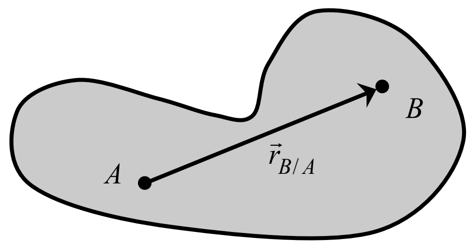
$$\hat{i}: \quad a_{cx} = a_o + \alpha R = a_B$$

$$\hat{j}: \quad a_{cy} = \omega^2 R \neq 0$$

Summary: Rigid Body Kinematics 2

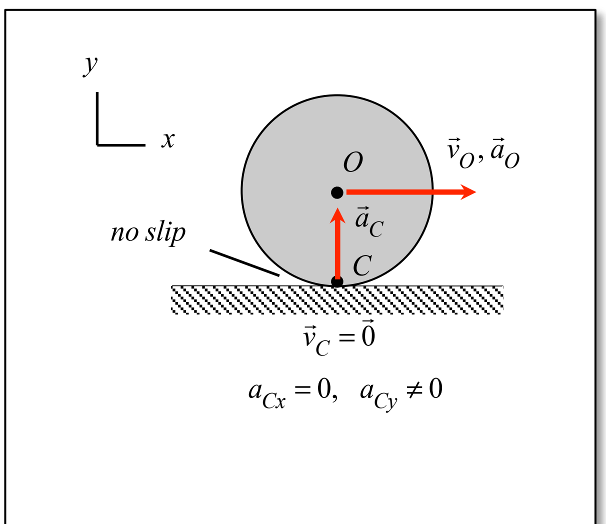
PROBLEM: Two points A and B on the same rigid body undergoing planar motion.

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

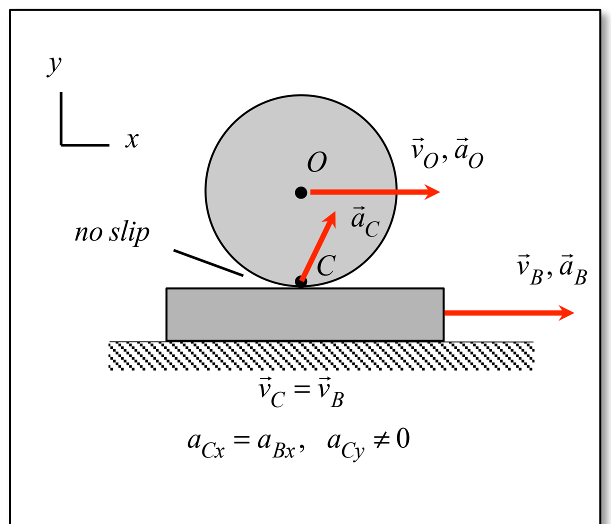


SPECIAL TOPIC: Rolling without slipping

rolling on fixed surface



rolling on moving surface



$$\vec{v}_B = \boxed{\vec{v}_A} + \boxed{\vec{\omega} \times \vec{r}_{B/A}}$$

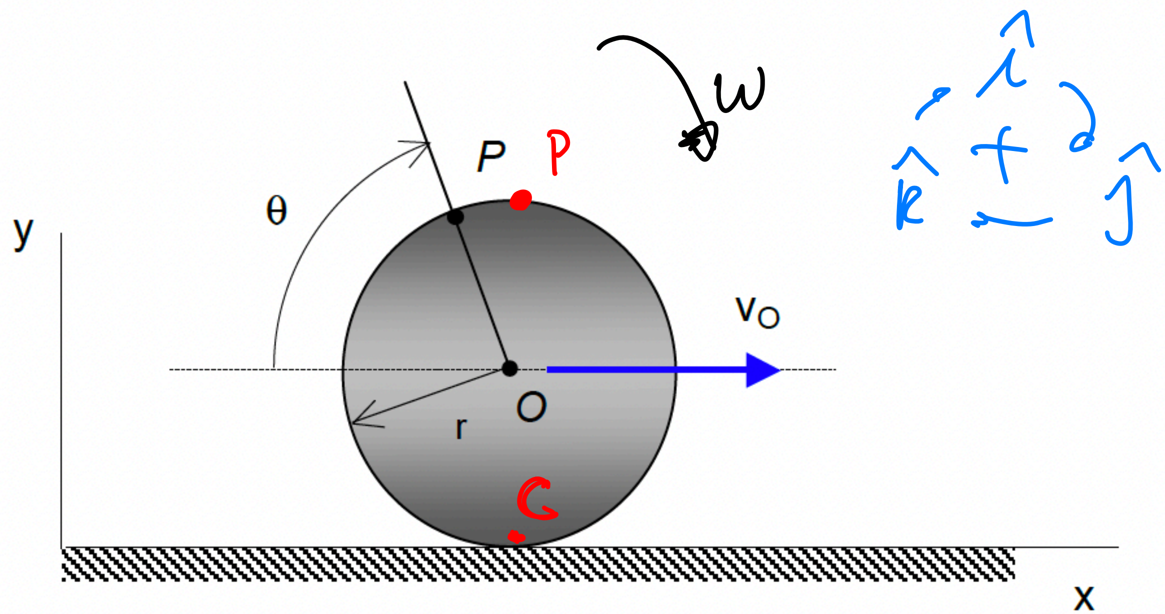
$$\vec{a} \qquad \vec{a}_A \; + \; \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}] + \vec{\alpha} \times \vec{r}_{B/}$$

Example 2.A.5

Given: A wheel rolls without slipping on a rough horizontal surface. At one instant, when $\theta = 90°$, the center of the wheel O is moving to the right with a speed of $v_O = 5$ ft/s with this speed decreasing at a rate of 3 ft/s^2.

Find: Determine the acceleration of point P on the circumference of the wheel at this instant, if $r = 2$ ft. Make a sketch of this acceleration vector at P.

Solution



$$\vec{v}_O = \cancel{\vec{v}_C}^{\,0} + \vec{w} \times \vec{r}_{O/C}$$

$$= w\hat{k} \times (r\hat{j})$$

$$v_O\hat{i} = -wr\hat{i}$$

$$\vec{w} = -\frac{v_O}{r}\hat{k}$$

$$\vec{a}_O = \vec{a}_C + \vec{\alpha} \times \vec{r}_{O/C} \underbrace{- w^2 \vec{r}_{O/C}}_{\text{triple cross prod. for 2D}}$$

$$a_O\hat{i} = a_C\hat{j} + \alpha\hat{k} \times (r\hat{j}) - w^2(r\hat{j})$$

$$a_o \hat{i} = a_c \hat{j} - \alpha r \hat{i} - \left(-\frac{v_o}{r}\right)^2 r \hat{j}$$

$$a_o \hat{i} = a_c \hat{j} - \alpha r \hat{i} - \frac{v_o^2}{r} \hat{j} = (-\alpha r)\hat{i} + \left(a_c - \frac{v_o^2}{r}\right)\hat{j}$$

$$\hat{i}: \quad a_o = -\alpha r \quad \Rightarrow \quad \vec{\alpha} = -\frac{a_o}{r} \hat{k}$$

$$\hat{j}: \quad 0 = a_c - \frac{v_o^2}{r} \quad \Rightarrow \quad \vec{a_c} = +\frac{v_o}{r} \hat{j}$$

Now, let's focus on P

$$\vec{v_p} = \vec{v_o} + \omega \times \vec{r}_{P/O}$$

$$= v_o \hat{i} + \left(-\frac{v_o}{r} \hat{k}\right) \times (r\hat{j}) = v_o \hat{i} + v_o \hat{i}$$

$$\vec{v_p} = 2 v_o \hat{i}$$

$$\vec{a_p} = \vec{a_o} + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O}$$

$$= a_o \hat{i} + \left(-\frac{a_o}{r} k\right) \times (r\hat{j}) - \left(-\frac{v_o}{r}\right)^2 (r\hat{j})$$

$$\vec{a_p} = a_o \hat{i} + a_o \hat{i} - \frac{v_o^2}{r} \hat{j}$$

$$\vec{a}_p = 2a_0\,\hat{\imath} - \frac{v_0^2}{r}\,\hat{\jmath}$$

$$= -6\,\hat{\imath} - \frac{25}{2}\,\hat{\jmath}$$