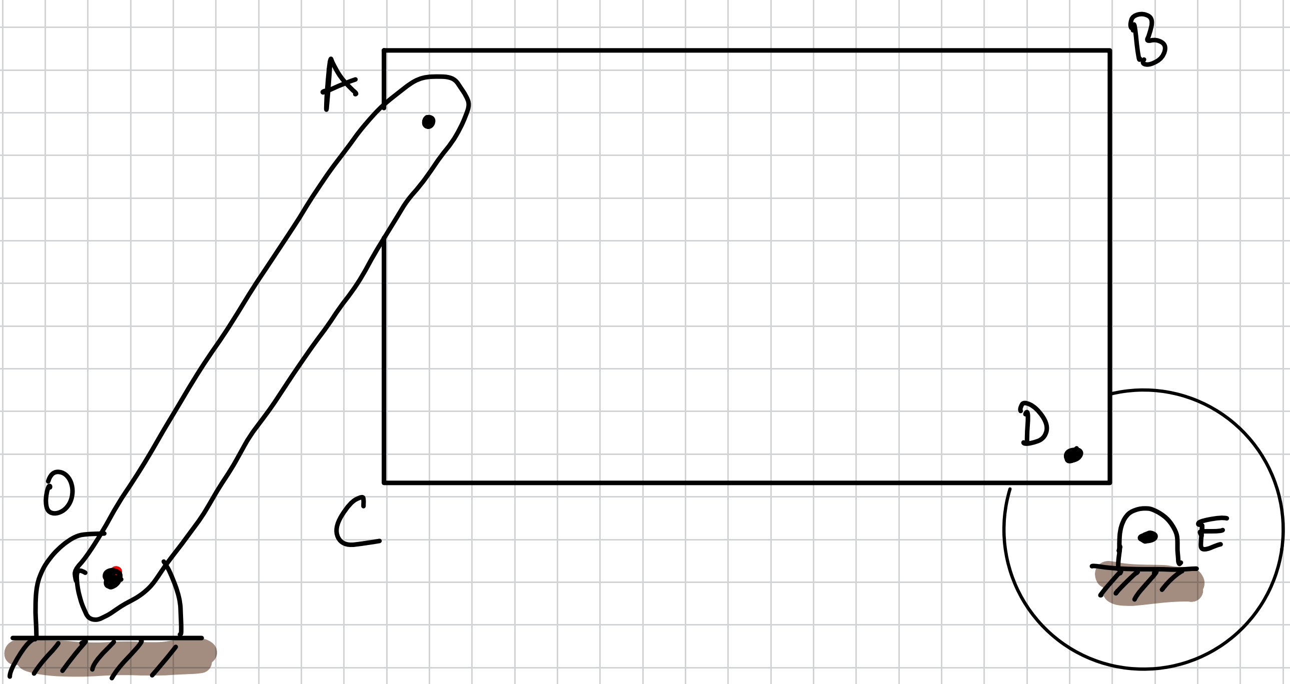
1/28/2026

KINEMATICS OF RIGID BODIES

$$\vec{v}_B = \boxed{\vec{v}_A} + \boxed{\vec{\omega} \times \vec{r}_{B/A}}$$

$$\vec{a}_B = \boxed{\vec{a}_A} + \boxed{\vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}] + \vec{\alpha} \times \vec{r}_{B/A}}$$

what is a RB ?



Any body with points of interest within its boundaries

e.g. O-A bar

A-D plate

D-E disk

Example 2.A.7

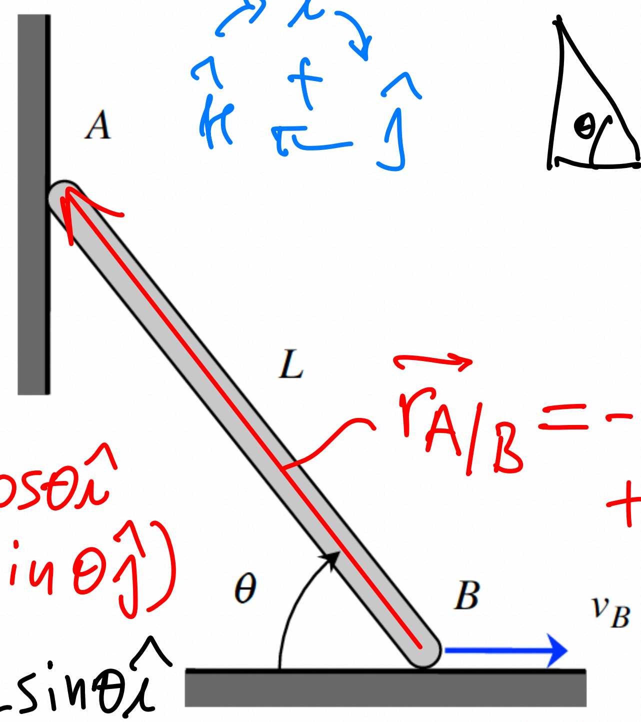
Given: End B of the link moves to the right with a constant speed v_B. $\quad a_B = 0$

Find: Determine:
(a) The angular velocity of link AB; and
(b) The angular acceleration of link AB.

Use the following parameters in your analysis: $v_B = 3$ m/s, $L = 0.5$ m and $\theta = 36.87°$. Also, be sure to express your answers as vectors.

Solution: set up $\vec{v}$ and $\vec{a}$ equ from least known qties to most known

$\hat{i} \to \hat{j} \to \hat{k}$ (+)



$$\vec{v}_A = \vec{v}_B + \omega_{AB} \times \vec{r}_{A/B}$$

$$\vec{r}_{A/B} = -L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}$$

$$v_A\hat{j} = v_B\hat{i} + \omega_{AB}\hat{k} \times (-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j})$$

$$v_A\hat{j} = v_B\hat{i} - \omega_{AB}L\cos\theta\,\hat{j} - \omega_{AB}L\sin\theta\,\hat{i}$$

$\hat{i}: \quad 0 = v_B - \omega_{AB}L\sin\theta \quad \Rightarrow \quad \boxed{\omega_{AB} = \frac{v_B}{L\sin\theta}}$

$\hat{j}: \quad v_A = -\omega_{AB}L\cos\theta$

$$\omega_{AB} = \frac{3}{(0.5)(0.6)} = 10\,\hat{k}\ \frac{rad}{s}$$

Similarly for $\vec{a}_A$ $\qquad\qquad\qquad\qquad v_A = -v_B \cot\theta$

$$\vec{a}_A = \overset{0}{\cancel{\vec{a}_B}} + \vec{\alpha}_{AB} \times \vec{r}_{A/B} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{A/B})$$

$$= \alpha_{AB}\hat{k} \times (-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}) + \omega_{AB} k \times$$

$$= \qquad \omega_{AB} k \times (-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j})$$

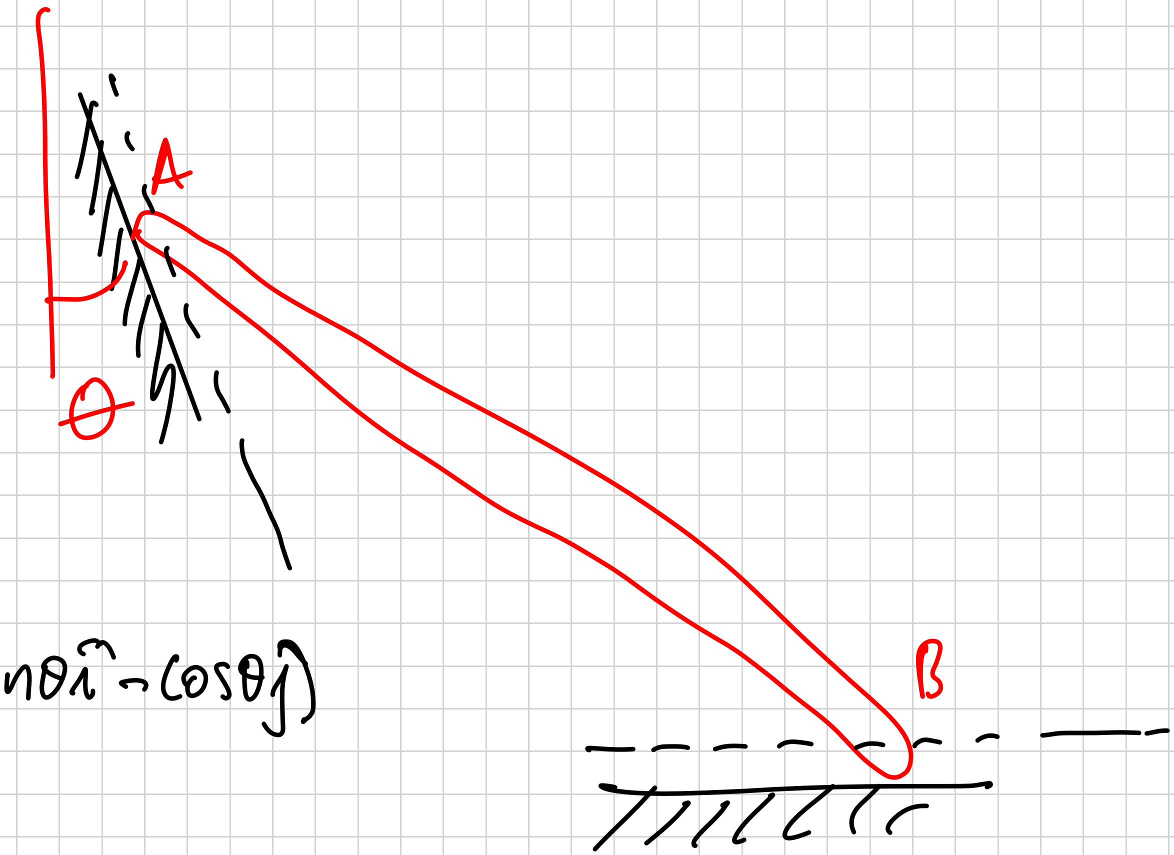
$$a_A\hat{j} = -\alpha_{AB} L\cos\theta\,\hat{j} - \alpha_{AB} L\sin\theta\,\hat{i} + \omega_{AB}^2 L\cos\theta\,\hat{i} - \omega_{AB}^2 L\sin\theta\,\hat{j}$$

$$\hat{i}: \quad 0 = -\alpha_{AB}\cancel{L}\sin\theta + \omega_{AB}^2\cancel{L}\cos\theta$$

$$\alpha_{AB} = \frac{\omega_{AB}^2\cos\theta}{\sin\theta} = \left(\frac{v_B}{L\sin\theta}\right)^2 \frac{\cos\theta}{\sin\theta}$$

$$\boxed{\vec{\alpha}_{AB} = \frac{v_B^2\cos\theta}{L^2\sin^3\theta}\,\hat{k} \quad \left[\frac{rad}{s}\right]}$$

$$\therefore \quad \alpha_{AB} = \frac{(3)^2(0.8)}{(0.5)^2(0.6)^3} = 133.3 \ rad/s^2$$



$$\vec{N}_A = N_A\left(+\sin\theta\,\hat{i} - \cos\theta\,\hat{j}\right)$$

Example 2.A.8

$\ddot{\theta} = 0$

Given: A flywheel rotates in the clockwise sense with a constant angular speed $\dot{\theta}$ about a shaft passing through its center O. The flywheel is connected to a piston A through connecting rod BA. The piston is constrained to slide along a horizontal surface.

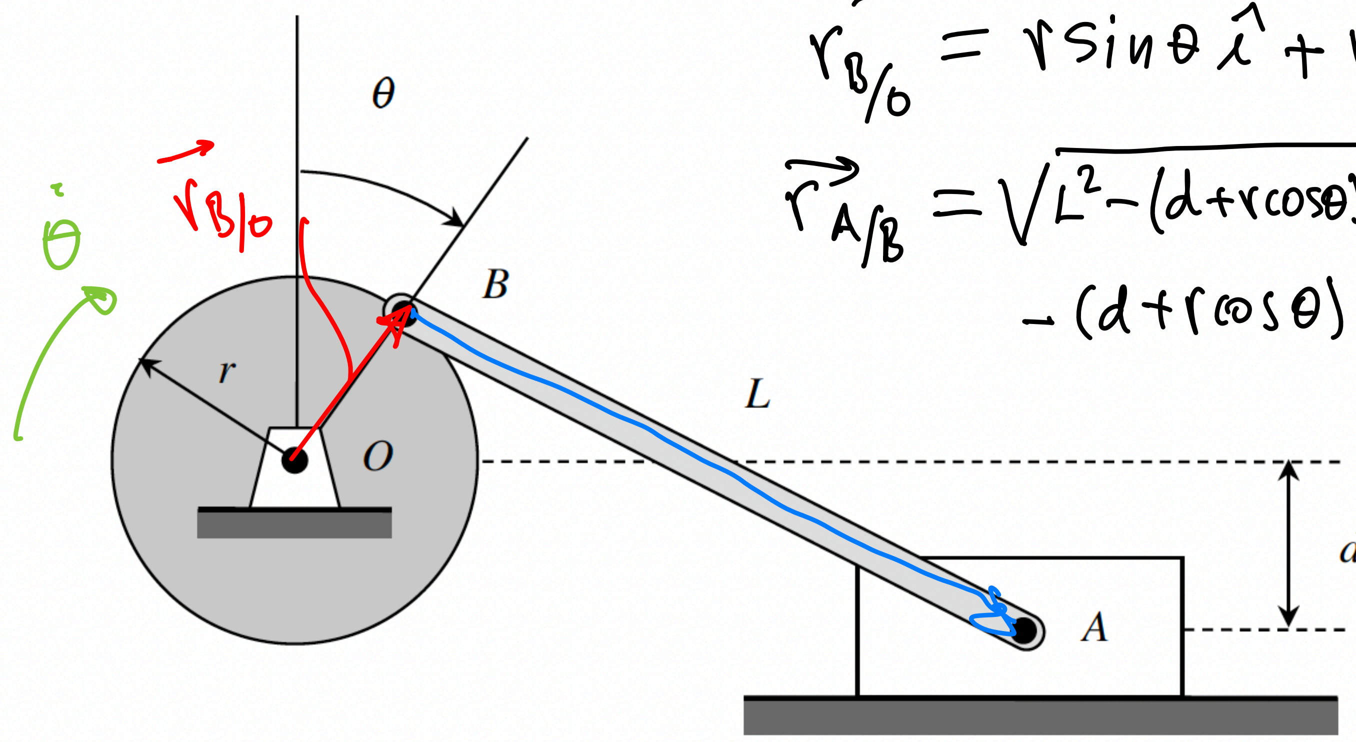
Find: Determine:

(a) The acceleration of piston A; and

(b) The angular acceleration of connecting rod arm BA.

Use the following parameters in your analysis: $\dot{\theta} = 10$ rad/s, $\theta = 90°$, $r = 0.1$ m, $d = 0.2$ m and $L = 0.45$ m.

Solution

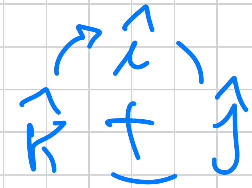


$$\vec{r}_{B/O} = r\sin\theta\,\hat{\imath} + r\cos\theta\,\hat{\jmath}$$

$$\vec{r}_{A/B} = \sqrt{L^2-(d+r\cos\theta)^2}\,\hat{\imath} - (d+r\cos\theta)\,\hat{\jmath}$$

@ $90° \Rightarrow \quad r_{A/B}\,\hat{\imath} = \sqrt{L^2-d^2}\,, \quad r_{A/B}\,\hat{\jmath} = d$

$$\vec{v}_B = \cancel{\vec{v}_0}^{\,0} + \vec{\omega}_{OB} \times \vec{r}_{B/O}$$



$$= -\dot{\theta}\,\hat{k} \times \left(r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j} \right)$$

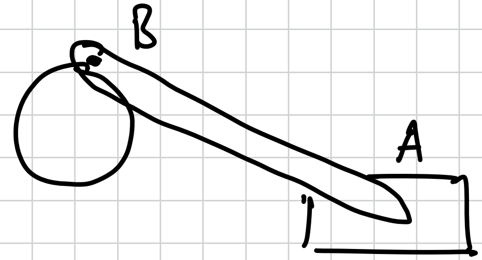
$$\vec{v}_B = \dot{\theta}\, r\cos\theta\,\hat{i} - \dot{\theta}\, r\sin\theta\,\hat{j}$$

$$\vec{a}_B = \cancel{\vec{a}_O}^{\,0} + \cancel{\vec{\alpha}_{OB}}^{\,0} \times \vec{r}_{B/O} + \bar{\omega}_{OB} \times \left(\bar{\omega}_{OB} \times \vec{r}_{B/O} \right)$$

$$= -\dot{\theta}\,\hat{k} \times \left[\left(-\dot{\theta}\,\hat{k}\right) \times \left(r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j} \right) \right]$$

$$-\dot{\theta}\,\hat{k} \quad \left(-\dot{\theta}\, r\sin\theta\,\hat{i} + \dot{\theta}\, r\cos\theta\,\hat{j} \right)$$

$$\vec{a}_B = -\dot{\theta}^2 r\sin\theta\,\hat{i} - \dot{\theta}^2 r\cos\theta\,\hat{j}$$



Bar AB

$$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$$

$$v_A \hat{i} = \dot{\theta} r\cos\theta\, \hat{i} - \dot{\theta} r\sin\theta\, \hat{j} + \omega_{AB}\hat{k} \times (r_{A/Bx}\hat{i} + r_{A/By}\hat{j})$$

$$v_A \hat{i} = \dot{\theta} r\cos\theta\, \hat{i} - \dot{\theta} r\sin\theta\, \hat{j} + \omega_{AB}\cdot r_{A/Bx}\hat{j} - \omega_{AB}\cdot r_{A/By}\hat{i}$$

$$v_A \hat{i} = (\dot{\theta} r\cos\theta - \omega_{AB}\cdot r_{A/By})\hat{i} + (-\dot{\theta} r\sin\theta + \omega_{AB} r_{A/Bx})\hat{j}$$

$\hat{i}$: $v_A = \dot{\theta} r\cos\theta - \omega_{AB} R_{A/By}$

$\hat{j}$: $0 = -\dot{\theta} r\sin\theta + \omega_{AB} R_{A/Bx}$

$\Rightarrow \quad \omega_{AB} = \dfrac{\dot{\theta} r\sin\theta}{\boxed{R_{A/Bx}}}$ $\leftarrow$ x component of $\vec{r}_{A/B}$

@ 90° : $\boxed{\omega_{AB} = \dfrac{\dot{\theta} r}{\sqrt{L^2-d^2}}}$ $\hat{k}$ rad/s

Similarly, For acceleration:

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times \vec{r}_{A/B} + \vec{\omega}_{AB} \times \left(\vec{\omega}_{AB} \times \vec{r}_{A/B}\right)$$

$$a_A \hat{i} = -\dot{\theta}^2 r \sin\theta \hat{i} - \dot{\theta}^2 r \cos\theta \hat{j} + \alpha_{AB} \hat{k} \times \left(R_{A/Bx} \hat{i} + R_{A/By} \hat{j}\right)$$
$$+ \omega_{AB} \hat{k} \times \left[\omega_{AB} \hat{k} \times \left(R_{A/Bx} \hat{i} + R_{A/By} \hat{j}\right)\right]$$

$$a_A \hat{i} = -\dot{\theta}^2 r \sin\theta \hat{i} - \dot{\theta} r \cos\theta \hat{j} + \alpha_{AB} R_{A/Bx} \hat{j} - \alpha_{AB} R_{A/By} \hat{i}$$
$$- \omega_{AB}^2 R_{A/Bx} \hat{i} - \omega_{AB}^2 R_{A/By} \hat{j}$$

$$a_A \hat{i} = \left(-\dot{\theta}^2 r \sin\theta - \alpha_{AB} R_{A/By} - \omega_{AB}^2 R_{A/Bx}\right) \hat{i}$$
$$+ \left(-\dot{\theta} r \cos\theta + \alpha_{AB} R_{A/Bx} - \omega_{AB}^2 R_{A/By}\right) \hat{j}$$

$\hat{i}$: $\quad a_A = -\dot{\theta}^2 r \sin\theta - \alpha_{AB} R_{A/By} - \omega_{AB}^2 R_{A/Bx} \quad$ (1)

$\hat{j}$: $\quad 0 = -\dot{\theta} r \cos\theta + \alpha_{AB} R_{A/Bx} - \omega_{AB}^2 R_{A/By} \quad$ (2)

From eq 2

$$\alpha_{AB} = \frac{\dot{\theta} r \cos\theta + \omega_{AB}^2 R_{A/B\,y}}{R_{A/B\,x}}$$

@ 90°:

$$\alpha_{AB} = \frac{0 + \left(\frac{\dot{\theta} r}{\sqrt{L^2 - d^2}}\right)^2 \cdot d}{\sqrt{L^2 - d^2}}$$

$$\boxed{\alpha_{AB} = \frac{\dot{\theta}^2 r^2 d}{\sqrt{(L^2 - d^2)^3}}} \quad \hat{K} \qquad \text{rad}/s^2$$

Practice Problem

Recall:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

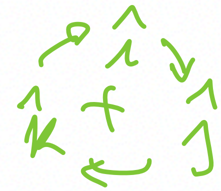
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}]$$

$$= \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

Given: $\vec{\omega}_{OA} = 4$ rad/s

$\vec{\alpha}_{OA} = 0$

$\overline{OA} = 0.3L$, $\overline{AB} = L$



Find: $\vec{\alpha}_{AB}$

5/152 For a short interval of motion, link OA has a constant angular velocity $\omega =$ ~~4 rad/s~~ ω_{OA}. Determine the angular acceleration α_{AB} of link AB for the instant when OA is parallel to the horizontal axis through B.

Solution

1) relate points A and O

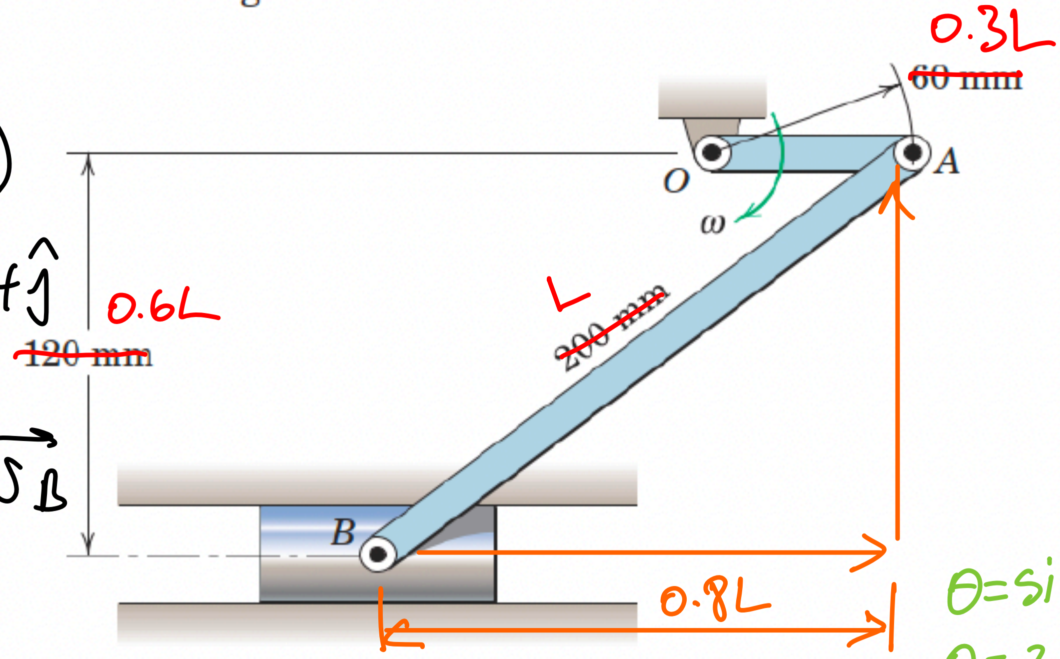
$$\vec{v}_A = \cancel{\vec{v}_O}^{\,0} + \vec{\omega}_{OA} \times \vec{r}_{A/O}$$

$$= (-\omega_{OA}\hat{k}) \times (0.3L\,\hat{i})$$

$$= -0.3L\,\omega_{OA}\,\hat{j} = -0.24\,\hat{j}$$

2) Once $\vec{v}_A$ is known, use this to find $\vec{v}_B$

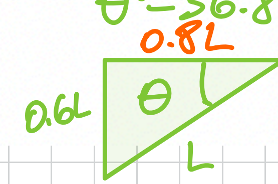
$$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$$



Problem 5/152

$\theta = \sin^{-1}\left(\frac{0.6L}{L}\right)$

$\theta = 36.86°$



$$\vec{v}_A \hat{j} = v_B \hat{i} + \omega_{AB} \hat{k} \times [0.8L\hat{i} + 0.6L\hat{j}]$$

$$v_A \hat{j} = v_B \hat{i} + (0.8L\omega_{AB}\hat{j} - 0.6L\omega_{AB}\hat{i})$$

$$\hat{i}: \quad 0 = v_B - 0.6L\omega_{AB}$$

$$\hat{j}: \quad v_A = 0.8L\omega_{AB}$$

$$\Rightarrow \omega_{AB} = \frac{0.24}{(0.8)(0.2)}$$

$$\therefore \omega_{AB} = 1.5\,\hat{k} \text{ rad/s}$$

$$\vec{a}_A = \cancel{\vec{a}_O}^{\,0} + \cancel{\vec{\alpha}_{OA}}^{\,0} \times \vec{r}_{A/O} - \omega_{OA}^2 \vec{r}_{A/O}$$

$$= -\omega_{OA}^2 (0.3L\hat{i}) = -0.3L\omega_{OA}^2 \hat{i}$$

$$= -0.3(0.2)(4)^2 \hat{i}$$

$$\therefore \vec{a}_A = -0.96\,\hat{i} \text{ m/s}^2$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times \vec{r}_{A/B} - \omega_{AB}^2 \vec{r}_{A/B}$$

$$a_A \hat{\imath} = a_B \hat{\imath} + \alpha_{AB} \hat{k} \times (0.8L\hat{\imath} + 0.6L\hat{\jmath}) - \omega_{AB}^2 (0.8L\hat{\imath} + 0.6L\hat{\jmath})$$

$$a_A \hat{\imath} = a_B \hat{\imath} + 0.8L\alpha_{AB}\hat{\jmath} - 0.6L\alpha_{AB}\hat{\imath} - 0.8L\omega_{AB}^2\hat{\imath} - 0.6L\omega_{AB}^2\hat{\jmath}$$

$\hat{\imath}$: $$a_A = a_B - 0.6L\alpha_{AB} - 0.8L\omega_{AB}^2$$

$\hat{\jmath}$: $$0 = 0.8\cancel{L}\alpha_{AB} - 0.6\cancel{L}\omega_{AB}^2$$

$$\hookrightarrow \quad \alpha_{AB} = \frac{0.6}{0.8}\omega_{AB}^2 = \frac{0.6(1.5)^2}{0.8}$$

$$\boxed{\alpha_{AB} = 1.69 \; \hat{k} \quad \frac{rad}{s^2}}$$