Problem H1.Ha – solution using projection onto polar unit vectors

Given: Particle P is constrained to move within a circular slot with a radius of R and center at point C. P is also constrained to move within the straight slot cut in an arm, with the arm rotating about end O with a constant rate of $\dot{\theta} = \omega$. O is located within the circular slot immediately to the left of C.

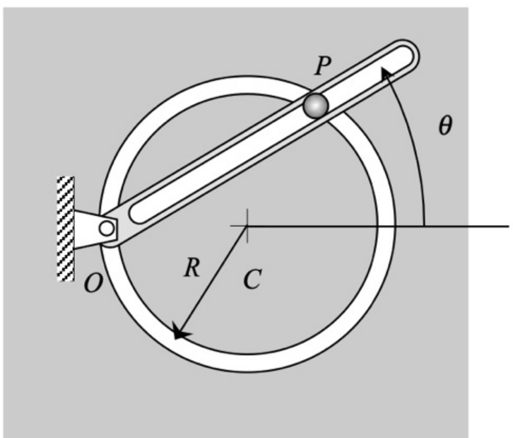
Find: For the position of $\theta = 45°$:

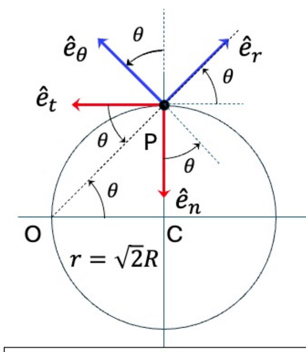
(a) show the path unit vectors $\hat{e}_t$ and $\hat{e}_n$, along with polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$, in a sketch. Note that the polar variable r is measured from point O to P, thus defining the direction for $\hat{e}_r$.

(b) determine numerical values for the rate of change of speed $\dot{v}_P$ of P and for $\dot{r}$, $\ddot{r}$ and $\ddot{\theta}$.

(c) is the speed of P increasing or decreasing? Explain.

Use the following parameters in your analysis: $R = 8$ in and $\omega = 4$ rad/s.





$$\hat{e}_r = -cos\theta\hat{e}_t - sin\theta\hat{e}_n$$
$$\hat{e}_\theta = sin\theta\hat{e}_t - cos\theta\hat{e}_n$$

Velocity

$$r\dot{\theta} = \vec{v}\cdot\hat{e}_\theta = (v\hat{e}_t)\cdot(sin\theta\hat{e}_t - cos\theta\hat{e}_n) = vsin\theta \quad \Rightarrow \quad v = \frac{r\dot{\theta}}{sin\theta} = \frac{\sqrt{2}(8)(4)}{\sqrt{2}/2} = 64\ in/s \longleftarrow$$

$$\dot{r} = \vec{v}\cdot\hat{e}_r = (v\hat{e}_t)\cdot(-cos\theta\hat{e}_t - sin\theta\hat{e}_n) = -vcos\theta = -64(\sqrt{2}/2) = -32\sqrt{2}\ in/s \longleftarrow$$

Acceleration

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \vec{a}\cdot\hat{e}_\theta = \left(\dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n\right)\cdot(sin\theta\hat{e}_t - cos\theta\hat{e}_n) = \dot{v}sin\theta - \frac{v^2}{\rho}cos\theta \quad \Rightarrow$$

$$\dot{v} = \frac{1}{sin\theta}\left[2\dot{r}\dot{\theta} + \frac{v^2}{\rho}cos\theta\right] = \frac{1}{\sqrt{2}/2}\left[2(-32\sqrt{2})(4) + \frac{64^2}{8}(\sqrt{2}/2)\right] = 0 \longleftarrow$$

$$\ddot{r} - r\dot{\theta}^2 = \vec{a}\cdot\hat{e}_r = \left(\dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n\right)\cdot(-cos\theta\hat{e}_t - sin\theta\hat{e}_n) = -\dot{v}cos\theta - \frac{v^2}{\rho}sin\theta \quad \Rightarrow$$

$$\ddot{r} = r\dot{\theta}^2 - \dot{v}cos\theta - \frac{v^2}{\rho}sin\theta = \sqrt{2}(8)(4)^2 - 0 - \frac{(64)^2}{8}(\sqrt{2}/2) = -128\sqrt{2}\ in/s^2 \longleftarrow$$

Problem H1.Hb – solution using projection onto path unit vectors

Given: Particle P is constrained to move within a circular slot with a radius of R and center at point C. P is also constrained to move within the straight slot cut in an arm, with the arm rotating about end O with a constant rate of $\dot{\theta} = \omega$. O is located within the circular slot immediately to the left of C.

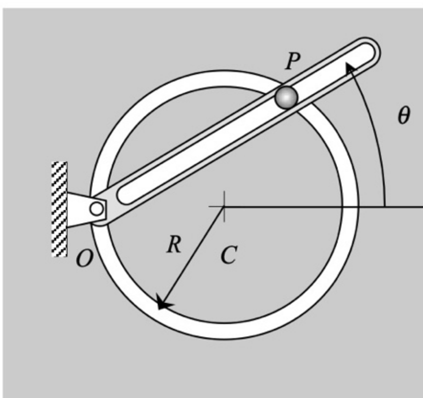
Find: For the position of $\theta = 45°$:

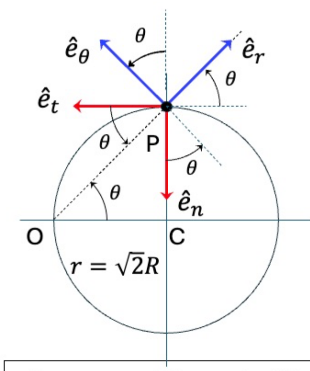
(a) show the path unit vectors $\hat{e}_t$ and $\hat{e}_n$, along with polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$, in a sketch. Note that the polar variable r is measured from point O to P, thus defining the direction for $\hat{e}_r$.

(b) determine numerical values for the rate of change of speed $\dot{v}_P$ of P and for $\dot{r}$, $\ddot{r}$ and $\ddot{\theta}$.

(c) is the speed of P increasing or decreasing? Explain.

Use the following parameters in your analysis: $R = 8$ in and $\omega = 4$ rad/s.





$$\hat{e}_t = -cos\theta\,\hat{e}_r + sin\theta\,\hat{e}_\theta$$
$$\hat{e}_n = -sin\theta\,\hat{e}_r - cos\theta\,\hat{e}_\theta$$

Velocity

$$0 = \vec{v}\cdot\hat{e}_n = \left(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\right)\cdot\left(-sin\theta\,\hat{e}_r - cos\theta\,\hat{e}_\theta\right) = -\dot{r}sin\theta - r\dot{\theta}cos\theta \quad \Rightarrow$$
$$\dot{r} = -r\dot{\theta}cot\theta = -\left(8\sqrt{2}\right)(4)(1) = -32\sqrt{2}\ in/s$$

$$v = \vec{v}\cdot\hat{e}_t = \left(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\right)\cdot\left(-cos\theta\,\hat{e}_r + sin\theta\,\hat{e}_\theta\right) = -\dot{r}cos\theta + r\dot{\theta}sin\theta$$
$$= -\left(-32\sqrt{2}\right)\left(\sqrt{2}/2\right) + \left(8\sqrt{2}\right)(4)\left(\sqrt{2}/2\right) = 64\ in/s$$

Acceleration

$$\frac{v^2}{\rho} = \vec{a}\cdot\hat{e}_n = \left[\left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta\right]\cdot\left(-sin\theta\,\hat{e}_r - cos\theta\,\hat{e}_\theta\right)$$
$$= -\left(\ddot{r} - r\dot{\theta}^2\right)sin\theta - \left(2\dot{r}\dot{\theta}\right)cos\theta \quad \Rightarrow$$
$$\ddot{r} = -\frac{1}{sin\theta}\left[\frac{v^2}{\rho} + \left(2\dot{r}\dot{\theta}\right)cos\theta\right] + r\dot{\theta}^2 = -128\sqrt{2}\ in/s^2$$

$$\dot{v} = \vec{a}\cdot\hat{e}_t = \left[\left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta\right]\cdot\left(-cos\theta\,\hat{e}_r + sin\theta\,\hat{e}_\theta\right)$$
$$= -\left(\ddot{r} - r\dot{\theta}^2\right)cos\theta + \left(2\dot{r}\dot{\theta}\right)sin\theta = 0$$

Problem H1.Hc – solution using <u>balancing coefficients</u> using polar unit vectors

Given: Particle P is constrained to move within a circular slot with a radius of R and center at point C. P is also constrained to move within the straight slot cut in an arm, with the arm rotating about end O with a constant rate of $\dot{\theta} = \omega$. O is located within the circular slot immediately to the left of C.

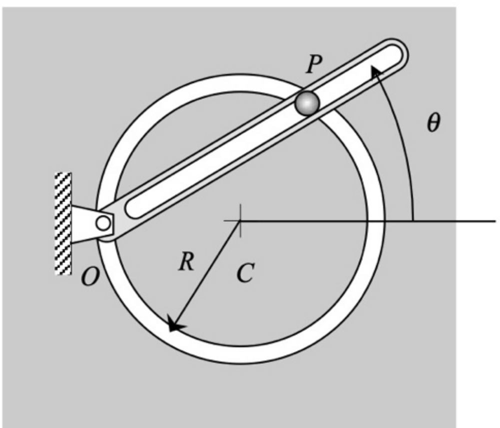
Find: For the position of $\theta = 45°$:

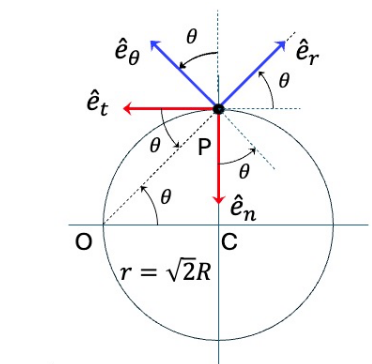
(a) show the path unit vectors $\hat{e}_t$ and $\hat{e}_n$, along with polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$, in a sketch. Note that the polar variable r is measured from point O to P, thus defining the direction for $\hat{e}_r$.

(b) determine numerical values for the rate of change of speed $\dot{v}_P$ of P and for $\dot{r}$, $\ddot{r}$ and $\ddot{\theta}$.

(c) is the speed of P increasing or decreasing? Explain.

Use the following parameters in your analysis: $R = 8$ in and $\omega = 4$ rad/s.





$$\hat{e}_t = -cos\theta\,\hat{e}_r + sin\theta\,\hat{e}_\theta$$

$$\hat{e}_n = -sin\theta\,\hat{e}_r - cos\theta\,\hat{e}_\theta$$

Velocity

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = v\hat{e}_t = v(-cos\theta\,\hat{e}_r + sin\theta\,\hat{e}_\theta) \quad \Rightarrow$$

$$\hat{e}_\theta:\ r\dot{\theta} = vsin\theta \quad \Rightarrow \quad v = r\dot{\theta}/sin\theta = 8\sqrt{2}(4)/(\sqrt{2}/2) = 64\,in/s \longleftarrow$$

$$\hat{e}_r:\ \dot{r} = -vcos\theta = -(64)(\sqrt{2}/2) - 32\sqrt{2}\,in/s \longleftarrow$$

Acceleration

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$= \dot{v}(-cos\theta\,\hat{e}_r + sin\theta\,\hat{e}_\theta) + \frac{v^2}{\rho}(-sin\theta\,\hat{e}_r - cos\theta\,\hat{e}_\theta) \quad \Rightarrow$$

$$\hat{e}_\theta:\ 2\dot{r}\dot{\theta} = \dot{v}sin\theta - \frac{v^2}{\rho}cos\theta \quad \Rightarrow \quad \dot{v} = \frac{1}{sin\theta}\left[2\dot{r}\dot{\theta} + \frac{v^2}{\rho}cos\theta\right] = \frac{1}{\sqrt{2}/2}\left[2(-32)(4) + \frac{64^2}{8}\frac{\sqrt{2}}{2}\right] = 0 \longleftarrow$$

$$\hat{e}_r:\ \ddot{r} - r\dot{\theta}^2 = -\dot{v}cos\theta - \frac{v^2}{\rho}sin\theta \quad \Rightarrow$$

$$\ddot{r} = r\dot{\theta}^2 - \frac{v^2}{\rho}sin\theta = (8\sqrt{2})(4)^2 - \frac{64^2}{8}(\sqrt{2}/2) = -128\sqrt{2}\,in/s^2 \longleftarrow$$

Problem H1.Hd – solution using <u>balancing coefficients</u> using path unit vectors

Given: Particle P is constrained to move within a circular slot with a radius of R and center at point C. P is also constrained to move within the straight slot cut in an arm, with the arm rotating about end O with a constant rate of $\dot{\theta} = \omega$. O is located within the circular slot immediately to the left of C.

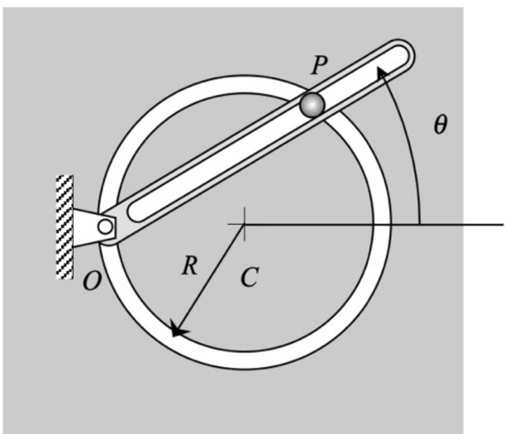
Find: For the position of $\theta = 45°$:

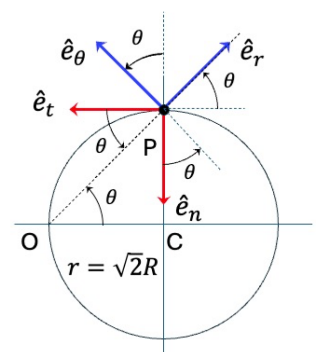
(a) show the path unit vectors $\hat{e}_t$ and $\hat{e}_n$, along with polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$, in a sketch. Note that the polar variable r is measured from point O to P, thus defining the direction for $\hat{e}_r$.

(b) determine numerical values for the rate of change of speed $\dot{v}_P$ of P and for $\dot{r}$, $\ddot{r}$ and $\ddot{\theta}$.

(c) is the speed of P increasing or decreasing? Explain.

Use the following parameters in your analysis: $R = 8$ in and $\omega = 4$ rad/s.





$$\hat{e}_r = -cos\theta\hat{e}_t - sin\theta\hat{e}_n$$
$$\hat{e}_\theta = sin\theta\hat{e}_t - cos\theta\hat{e}_n$$

Velocity

$$\vec{v} = v\hat{e}_t = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$
$$= \dot{r}(-cos\theta\hat{e}_t - sin\theta\hat{e}_n) + r\dot{\theta}(sin\theta\hat{e}_t - cos\theta\hat{e}_n) \Rightarrow$$
$$\hat{e}_n:\ 0 = -\dot{r}sin\theta - r\dot{\theta}cos\theta \quad \Rightarrow \quad \dot{r} = -r\dot{\theta}cot\theta = -32\sqrt{2}\, in/s \longleftarrow$$
$$\hat{e}_t:\ v = -\dot{r}cos\theta + r\dot{\theta}sin\theta = 64\, in/s \longleftarrow$$

Acceleration

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$
$$= \left(\ddot{r} - r\dot{\theta}^2\right)(-cos\theta\hat{e}_t - sin\theta\hat{e}_n) + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)(sin\theta\hat{e}_t - cos\theta\hat{e}_n) \Rightarrow$$
$$\hat{e}_n:\ \frac{v^2}{\rho} = -\left(\ddot{r} - r\dot{\theta}^2\right)sin\theta - \left(2\dot{r}\dot{\theta}\right)cos\theta \quad \Rightarrow$$
$$\ddot{r} = -\frac{1}{sin\theta}\left[\frac{v^2}{\rho} + \left(2\dot{r}\dot{\theta}\right)cos\theta\right] + r\dot{\theta}^2 = -128\sqrt{2}\,\frac{in}{s^2} \longleftarrow$$
$$\hat{e}_t:\ \dot{v} = -\left(\ddot{r} - r\dot{\theta}^2\right)cos\theta + \left(2\dot{r}\dot{\theta}\right)sin\theta = 0 \longleftarrow$$