KINEMATICS OF RELATIVE & CONSTRAINED MOTION

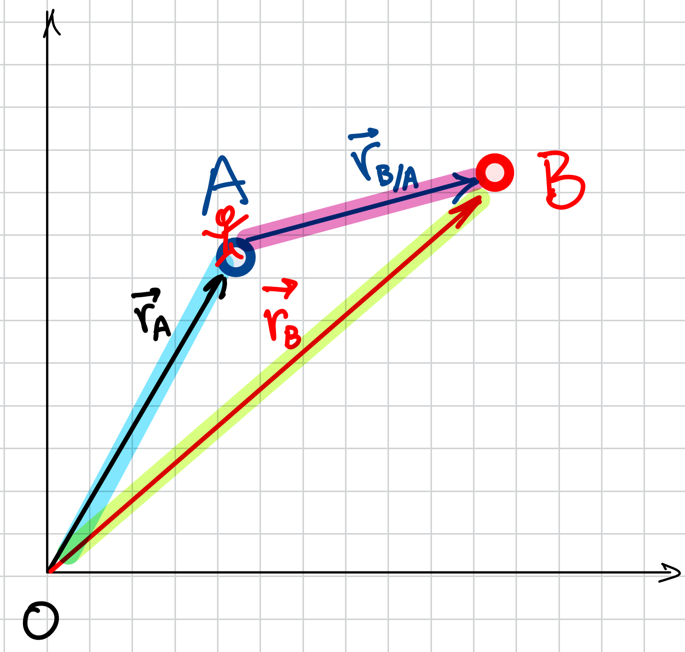
1/23/2026

In dynamics, we will need to identify the position of a particle (often, a point of a complex mechanism) with respect to another point (reference).

From there, through differentiation, we will be able to understand the full set of kinematic interactions $(\vec{r}, \vec{v}, \vec{a})$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

"The position of particle B is the sum of the position of particle A plus the relative position from A to B"



Now let's derive $\vec{v}$ and $\vec{a}$ from $\vec{r}$:

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A \quad \text{(Position of B wrt A)}$$

$$\frac{d}{dt}\left(\vec{r}_{B/A}\right) = \frac{d}{dt}\vec{r}_B - \frac{d}{dt}\vec{r}_A$$

$$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\frac{d^2}{dt^2}\left(\vec{r}_{B/A}\right) = \frac{d^2}{dt^2}\vec{r}_B - \frac{d^2}{dt^2}\vec{r}_A$$

$$\Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

In these expressions, $\vec{v}_{B/A}$ and $\vec{a}_{B/A}$ are the velocity of B wrt A, and the acceleration of B wrt A, respectively.

Also said the velocity/accel. of B seen by an observer in A

Example:

Example 1.D.1

Given: At the instant shown, car B is traveling with a speed of 50 km/hr and is slowing down at a rate of 10 km/hr². Car A is moving with a speed of 80 km/hr, a speed that is increasing at a rate of 10 km/hr². At this instant, A and B are traveling in the same direction.

Find: What acceleration does a passenger in car A observe for car B?

Solution:

Given $\rho = 500$ m

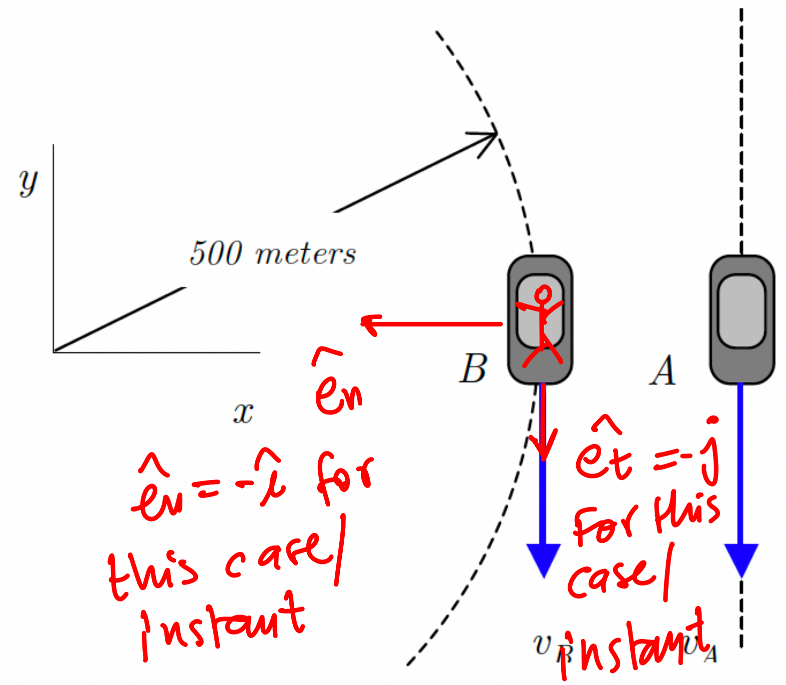
$v_B = 50$ km/hr

$\dot{v}_B = -10$ km/hr²

$v_A = 80$ km/hr

$\dot{v}_A = +10$ km/hr²

Find $\vec{a}_{B/A}$



The problem asks for the acceleration of B seen by an observer in A, $\vec{a}_{B/A}$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

We know the path geometry ($\rho = 500m$). So, let's use path description.

$$\vec{a}_B = \dot{v}_B \hat{e}_t + \frac{v^2}{\rho} \hat{e}_n$$

Now, the problem provides hints of the motion of A, i.e.,

$$\vec{a}_A = -\dot{v}_A \hat{j} \quad , \quad \dot{v}_A = 7.7E-4 \ m/s^2$$

Since we have $\vec{a}_A$ in Cartesian coordinates, let's unify systems: (for this specific case)

$\hat{e}_n = -\hat{\imath}$

$\hat{e}_t = -\hat{j}$

$$\vec{a}_B = -\frac{v_B^2}{\rho} \hat{\imath} - \dot{v}_B \hat{j}$$

Finally:

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

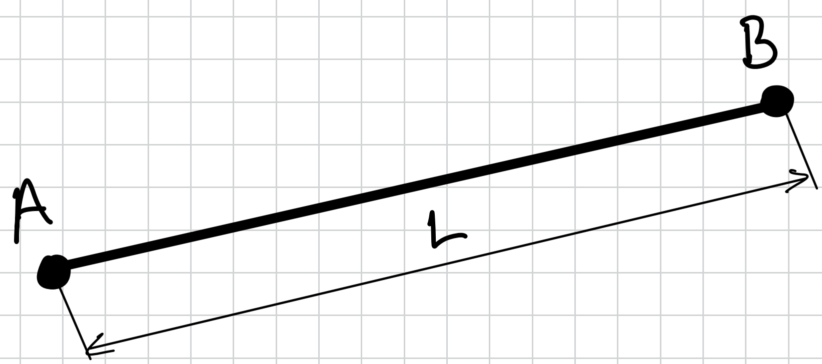
$$\boxed{\vec{a}_{B/A} = -\frac{v_B^2}{\rho}\hat{i} + (\dot{v}_A - \dot{v}_B)\hat{j}}$$

$\vec{a}_{B/A}$ is also interpreted as acceleration of B relative to A

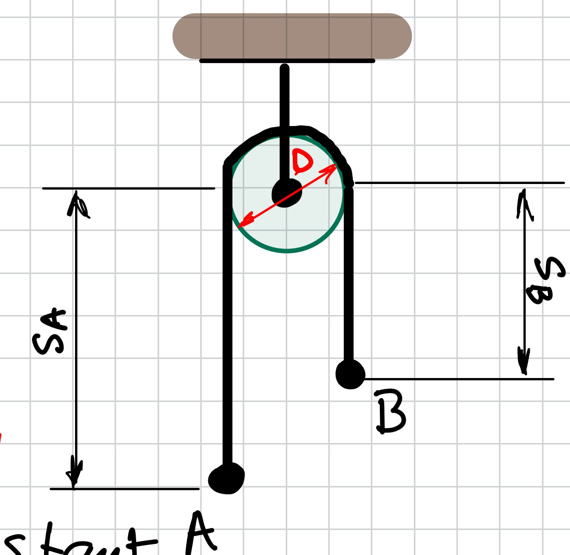
CONSTRAINED MOTION

In general, relative motion kinematics are used to describe the motion of a point relative to another point. Often, these points have a constraint. That is, the motion of one point will affect the motion of the other.

So, in addition to the relative motion equations, we will need to find additional equations to describe such constraints (usually, cable lengths).



$\Rightarrow$



assume very small

$$L = S_A + S_B + \frac{1}{2}\pi D = \text{constant}$$

$$L = S_A + S_B = C$$

$$\frac{dL}{dt} = v_A + v_B = 0$$

$$\frac{d^2L}{dt^2} = a_A + a_B = 0$$

PULLEY DIAMETERS (D)

D does not play a role in the motion of points A and B (We can assume D = 0)

ROPE | CABLE LENGHTS (L)

L remains constant (assumed inextensible) for all motion of A & B. However, distances between points A & B doe not remain constant.

PROCEDURE

1) Carefully define set of coordinates from a fixed point to each moving particle, with direction : $s_A, s_B, s_C, \ldots$

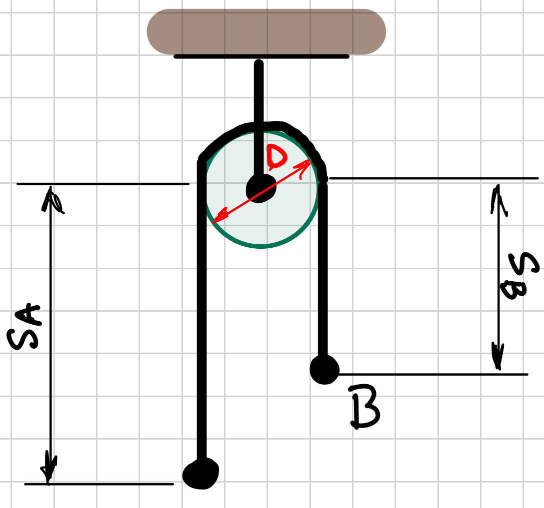
2) Write an equation for the length L of cable

$$s_A + s_B + \text{Constant} = L$$

3) Take time derivatives of the L expression (here, L does NOT change)

$$\dot{s}_A + \dot{s}_B = 0$$

$$\ddot{s}_A + \ddot{s}_B = 0$$



4) Relate v and a to $\dot{s}$ and $\ddot{s}$

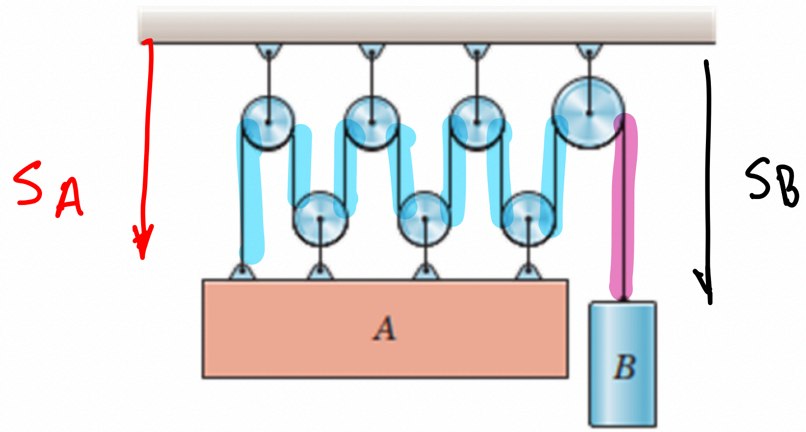
$\vec{v}_A, \vec{a}_A$ in $\hat{j}$ => $v_A = -\dot{s}_A$

$a_A = -\ddot{s}_A$

$\vec{v}_B, \vec{a}_B$ in $\hat{j}$ => $v_B = \dot{s}_B$ $\quad a_B = \ddot{s}_B$

Quiz Question

- A system has a taut cable connecting block A and B with a series of ideal pulleys as shown in the figure. If B is moving down with a speed of 8 m/s, what is the speed and moving direction of block A? (Modified from Meriam Textbook 7th edition)



(A) Not Moving

(B) Up, $1m/s$

(C) Up, $8/7m/s$

(D) Down, $1m/s$

(E) Down, $8/7m/s$

$L = 7S_A + S_B = \text{constant.}$

Example 1.D.6

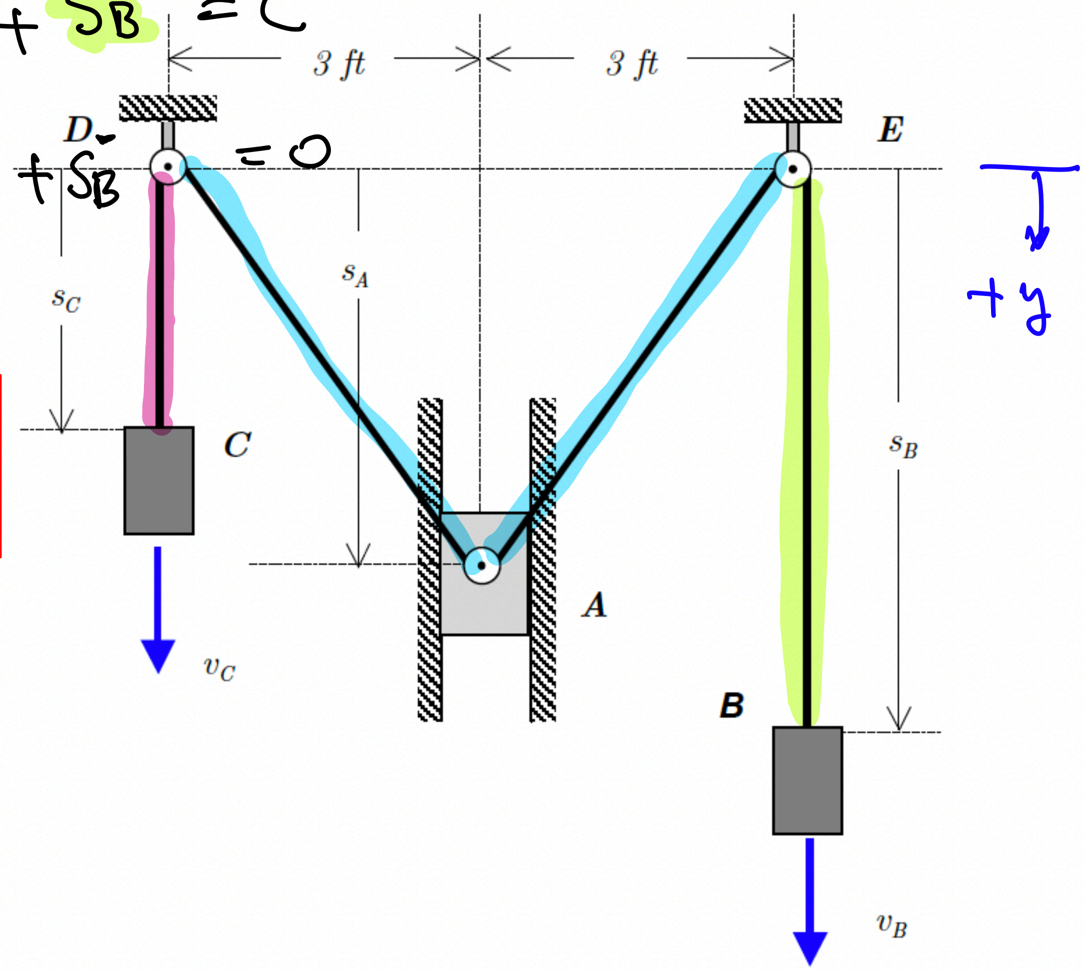
Given: Blocks B and C are connected by a single inextensible cable, with this cable being wrapped around pulleys at D and E. In addition, the cable is wrapped around a pulley attached to block A as shown. Assume the radii of the pulleys to be small. Blocks B and C move downward with speeds of $v_B = 6$ ft/s and $v_C = 18$ ft/s, respectively.

Find: Determine the velocity of block A when $s_A = 4$ ft.

$$L = s_C + 2\sqrt{s_A^2 + 3^2} + s_B = C$$

$$\dot{s}_C + 2\left(\tfrac{1}{2}\right)\left(s_A^2 + 3^2\right)^{-1/2} \cdot (2 s_A)\underbrace{(\dot{s}_A)}_{v_A} + \dot{s}_B = 0$$

$$\boxed{v_A = -\frac{(v_B + v_C)\sqrt{s_A^2 + 9}}{2 s_A}}$$



Example 1.D.2

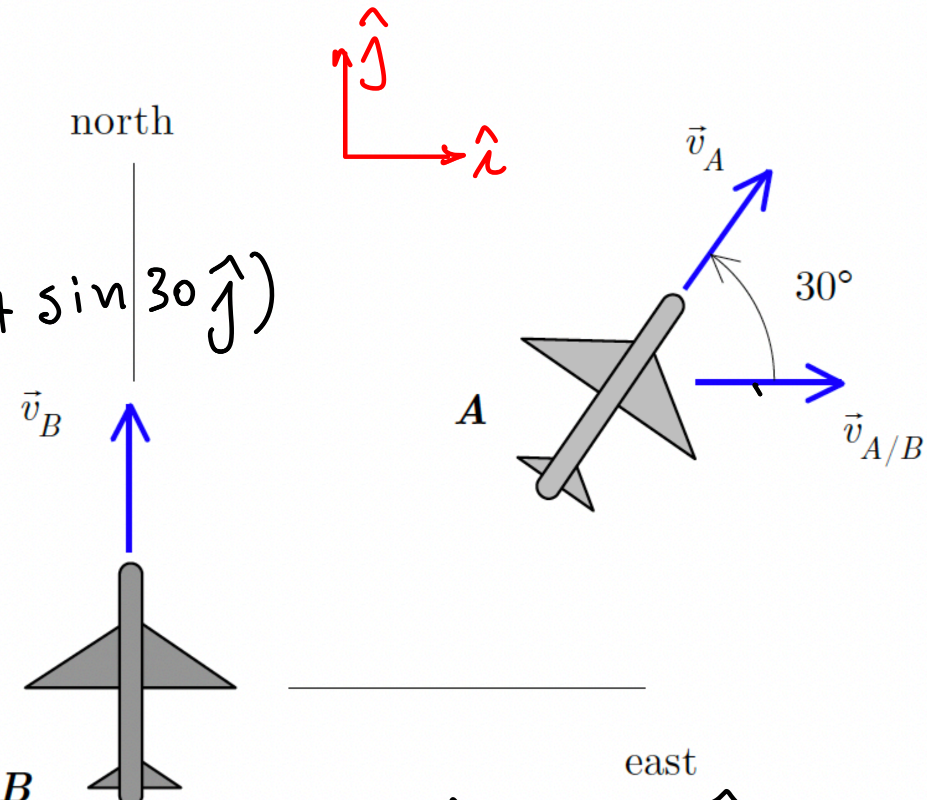
Given: Jet B is traveling due north with a speed of $v_B = 600$ km/hr. Passengers on jet B observe A to be flying sideways and moving due east.

Find: Determine:

(a) The speed of A; and

(b) The speed of A as observed by the passengers on jet B.

Solution:



$$\vec{v}_B = 600\hat{j}$$

$$\vec{v}_A = v_A(\cos 30\hat{i} + \sin 30\hat{j})$$

$$\vec{v}_{A/B} = v_{A/B}\,\hat{i}$$

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

$$v_{A/B}\,\hat{i} = v_A \cos 30\hat{i} + v_A \sin 30\hat{j} - 600\hat{j}$$

Separate in $\hat{i}$ and $\hat{j}$ components

$\hat{i}$: $\quad N_{A/B} = N_A \cos 30 \Rightarrow N_{A/B} = 1200 \frac{\sqrt{3}}{2} = 1040 \ \mathrm{km/hr}$

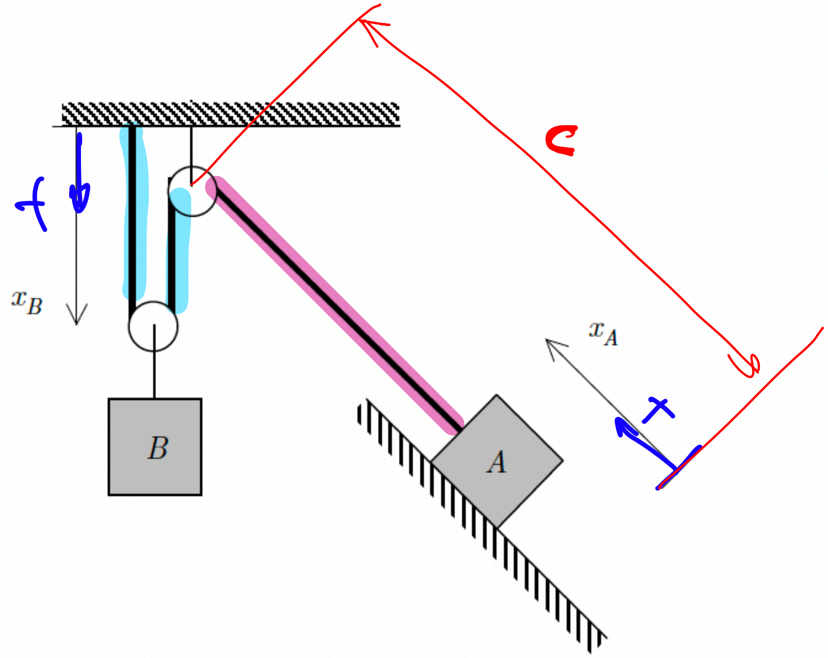
$\hat{j}$: $\quad 0 = N_A \sin 30 - N_B \Rightarrow 0 = \frac{1}{2} N_A - 600$

$$\boxed{N_A = 1200 \ \mathrm{km/hr}}$$

Example 1.D.4

Given: Block A moves with an acceleration of $\ddot{x}_A = a_A = 0.44\ \text{m/s}^2$.

Find: Determine the acceleration of block B.



Solution

$$2x_B + (c - x_A) = L$$

$$2\dot{x}_B - \dot{x}_A = 0$$

$$2\ddot{x}_B - \ddot{x}_A = 0$$

$$\ddot{x}_B = \frac{\ddot{x}_A}{2}$$

$$\Rightarrow \boxed{\vec{a}_B = -\frac{a_A}{2}\hat{\jmath}}$$

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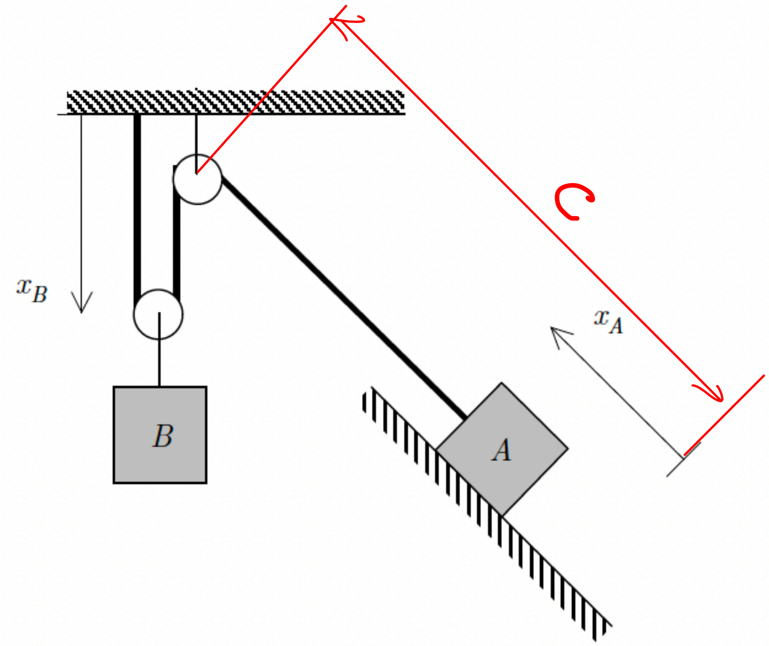


Equation of rope

$$2x_B + (C - x_A) = L$$

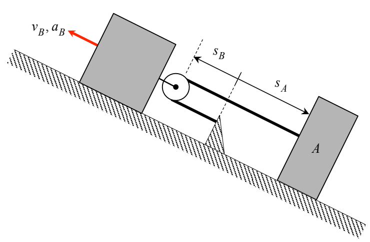
$$2\ddot{x}_B - \ddot{x}_A = 0$$

$$\ddot{x}_B = \frac{\ddot{x}_A}{2}$$

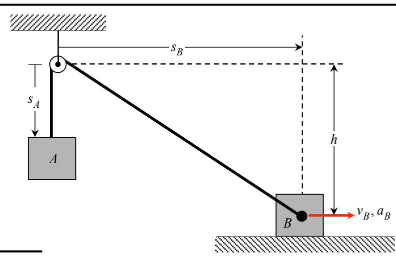
$$\vec{a}_B = -\frac{a_A}{2}\,\hat{j}$$

Summary: Particle Kinematics – Constrained and Relative Motion

PROBLEM: Two bodies connected by inextensible cable.
Write down the length of the cable in terms of motion variables and differentiate.



$$L = s_A + 2s_B + const = const \;\Rightarrow\; \dot{s}_A = -2\dot{s}_B$$



$$L = s_A + \sqrt{h^2 + s_B^2} + const = const \;\Rightarrow\; \dot{s}_A = -\frac{s_B \dot{s}_B}{\sqrt{h^2 + s_B^2}}$$

PROBLEM: The motion of one point relative to another point.

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = position\ of\ B\ with\ respect\ to\ A$$

$$\vec{v}_{B/A} = \frac{d\vec{r}_{B/A}}{dt} = \vec{v}_B - \vec{v}_A = velocity\ of\ B\ with\ respect\ to\ A$$

$$\vec{a}_{B/A} = \frac{d\vec{v}_{B/A}}{dt} = \vec{a}_B - \vec{a}_A = acceleration\ of\ B\ with\ respect\ to\ A$$