Example 1.A.3 pg. 43

Given: A jet is flying on the path shown below with a speed of v. At position A on the loop, the speed of the jet is $v = 600$ km/hr, the magnitude of the acceleration is 2.5g and the tangential component of acceleration is $a_t = 5$ m/s^2.

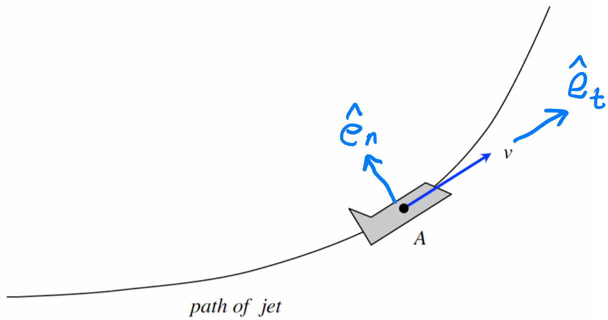
Find: The radius of curvature of the path of the jet at A.

$\hat{e}_{tangent} = \hat{e}_t$

$\hat{e}_{normal} = \hat{e}_n$

$\wedge$ = unit vector

$\rightharpoonup$ = vector



$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$= a_t\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$|\vec{a}| = (2.5)(9.806)\frac{m}{s^2}$$

$$|\vec{a}|^2 = a_t^2 + \frac{v^4}{\rho^2}$$

$$\rho = \sqrt{\frac{v^4}{|\vec{a}|^2 - a_t^2}}$$

$\dot{v} \neq |\vec{a}|$ bc

Additional lecture Example 1.1 In course website

Given: An automobile P is entering a freeway along a "clothoid-shaped" entrance ramp whose radius of curvature ρ is given by $\rho = (a + bs)^{-1}$, where a and b are constants, and s is the distance traveled along the entrance ramp. The speed of P is known as a function of position s on the entrance ramp to be: $v(s) = c + ds$, where c and d are constants.

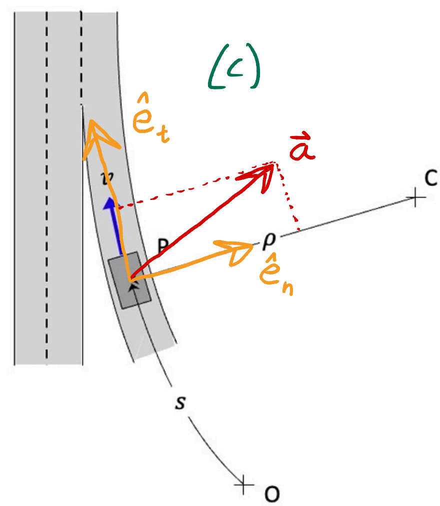
Find:

(a) Determine the velocity and acceleration vectors for P. Express these vectors in terms of their path coordinates, and in terms of, at most: s, a, b, c and d.

(b) Determine the numerical values of the velocity and acceleration vectors for P at the position at $s = 200$ ft.

(c) Make a sketch of these velocity and acceleration vectors, including the path unit vectors $\hat{e}_t$ and $\hat{e}_n$.

Use the following parameters in your work: $a = 0.005/\text{ft}$, $b = 1 \times 10^{-5}/\text{ft}^2$, $c = 25$ ft/s and $d = 0.25/\text{s}$.



$$\vec{v} = v\hat{e}_t = (c+ds)\hat{e}_t$$

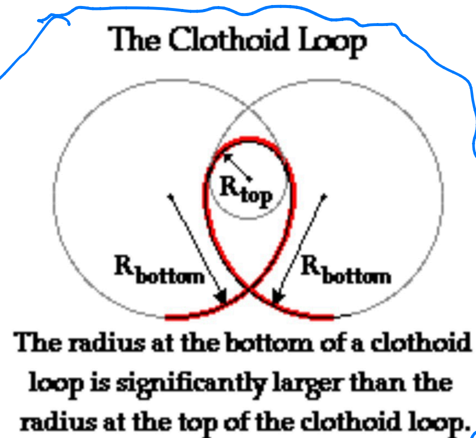
(a) $\vec{v}$

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

w/ $$\dot{v} = \frac{dv}{dt} = \underbrace{\frac{dv}{ds}\overset{=v}{\frac{ds}{dt}}}_{\text{chain rule}} = v\frac{dv}{ds} = (c+ds)d$$

(a) $\vec{a}$

$$\vec{a} = \underbrace{(c+ds)d}_{\substack{>0 \\ \text{(positive value)}}}\hat{e}_t + \underbrace{(c+ds)^2(a+bs)}_{\substack{>0 \\ \text{(positive value)}}}\hat{e}_n$$



Additional lecture Example 1.2 In course website

Given: Particle P moves along a path with its position on the path given by the arc length of s. The speed of P is given as a function of s as: $v_P = bs^2$, where s is given in meters and v_P in terms of meters/second. The radius of curvature of the path is given by ρ and the path tangent is at an angle of θ with respect to the direction of the x-axis.

Find: At the position of P where $s = 3$ m:

(a) Make a sketch of the path unit vectors $\hat{e}_t$ and $\hat{e}_n$.

(b) Determine the velocity and acceleration of P in terms of path unit vectors $\hat{e}_t$ and $\hat{e}_n$.

(c) Determine the velocity and acceleration of P in terms of Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

(d) Determine the xy-components of location of the center of curvature, C, for the path.

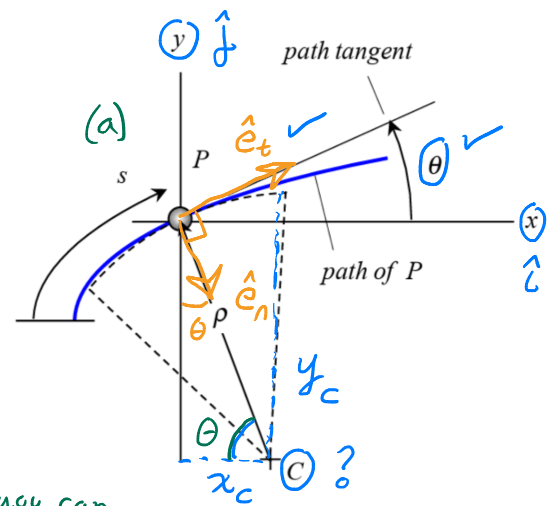
Use the following parameters in your work: $b = 0.5$/m-s, $\rho = 5$ m and $\theta = 30°$.

(b)

✓ $\vec{v}_p = v_p \hat{e}_t = (bs^2)\hat{e}_t$

✓ $\vec{a}_p = \dot{v}_p \hat{e}_t + \frac{v_p^2}{\rho}\hat{e}_n$

$= \dot{v}_p \hat{e}_t + \frac{(bs^2)^2}{\rho}\hat{e}_n$



w/ $\dot{v}_p = \frac{dv_p}{dt} = \underbrace{\frac{dv_p}{ds}\frac{ds}{dt}}_{\text{chain rule}} = v_p \frac{dv_p}{ds}$ ($\frac{ds}{dt} = v_p$)

$= (bs^2)(2bs)$

$= 2b^2s^3$

$=$ ____

w/ this you can plugin & solve for (b)

(c)

From figure: $\hat{e}_t = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{e}_n = \sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}$

$\therefore$

$\vec{v}_p = bs^2(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$ ⟵ (c)'s $\vec{v}_p$

$\vec{a}_p = 2b^2s^3(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) + \frac{(bs^2)^2}{\rho}(\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath})$

$= \left[2b^2s^3\cos\theta + \frac{(bs^2)^2}{\rho}\sin\theta\right]\hat{\imath} + \left[2b^2s^3\sin\theta - \frac{(bs^2)^2}{\rho}\cos\theta\right]\hat{\jmath}$ ⟵ (c)'s $\vec{a}_p$

(d) $x_c = \rho\sin\theta$, $y_c = -\rho\cos\theta$