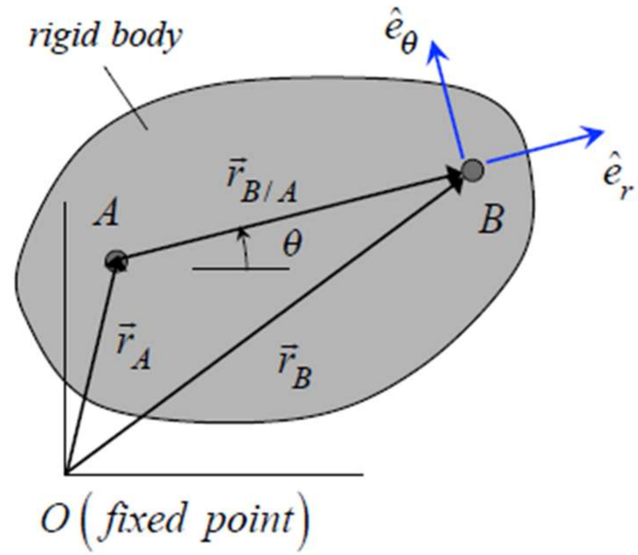
ME 274: Basic Mechanics II

Lecture 7: Rigid Body Motion

PURDUE UNIVERSITY | School of Mechanical Engineering

Review: Rigid Body Motion

If we know the motion of one point and the rotation of the body, we can determine the motion of any point on the body



Rigid body:

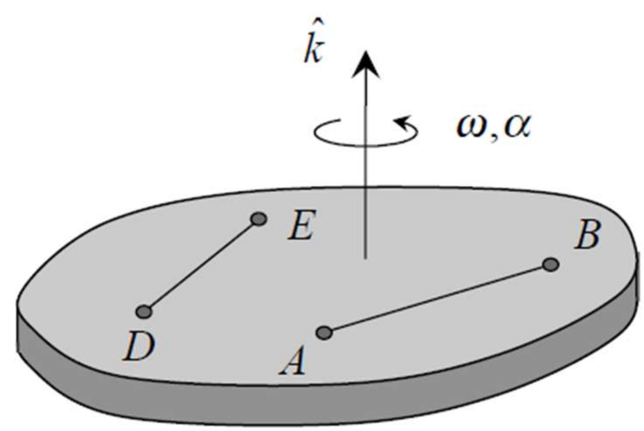
$$r = \left| \vec{r}_{B/A} \right| = \text{CONSTANT and } \dot{r} = 0$$

Velocity and acceleration of point B with respect to point A:

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A} \right] + \vec{\alpha} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}.$$



Angular velocity ω, and angular acceleration α are properties of the body

- $\vec{\omega} = \omega \hat{k} = \dot{\theta} \hat{k}$, $\vec{\alpha} = \alpha \hat{k} = \ddot{\theta} \hat{k}$
- Sign of ω, α determines direction of rotation (using right hand rule)

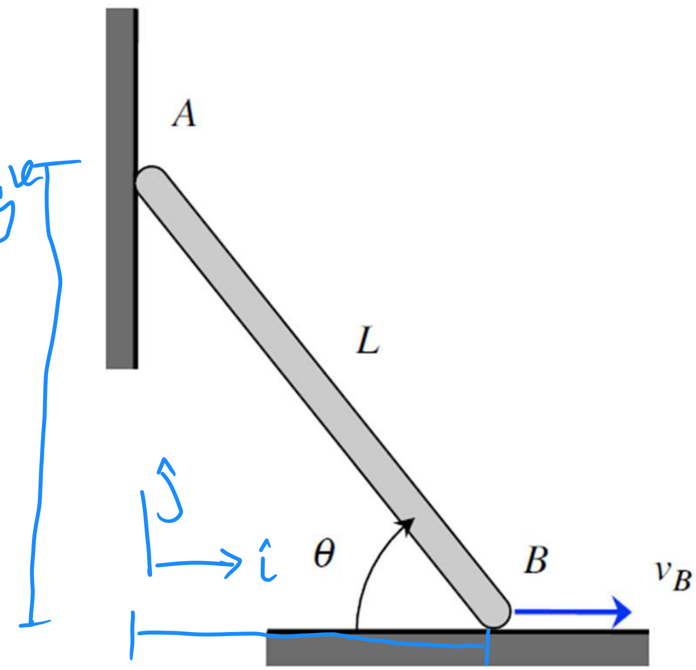
Example 2.A.7

Given: End B of the link moves to the right with a constant speed v_B.

Find: Determine:
- (a) The angular velocity of link AB; and
- (b) The angular acceleration of link AB.

Use the following parameters in your analysis: $v_B = 3$ m/s, $L = 0.5$ m and $\theta = 36.87°$. Also, be sure to express your answers as vectors.

3,4,5 triangle



$L\sin\theta$ (0.6)

$L\cos\theta = (0.8)$

1) draw unit vectors

2) what do we know?

$\vec{V}_B = V_B\hat{i}$, $\vec{V}_A = V_A\hat{j}$

$\vec{r}_{A/B} = (-L\cos\theta\hat{i} + L\sin\theta\hat{j})$

3) solve for V_A in terms of V_B

$\vec{V}_A = \vec{V}_B + \vec{\omega} \times \vec{r}_{A/B}$

$V_A\hat{j} = V_B\hat{i} + \omega\hat{k} \times (-L\cos\theta\hat{i} + L\sin\theta\hat{j})$

$= V_B\hat{i} - \omega L\cos\theta\hat{j} - \omega L\sin\theta\hat{i}$

4) split components

$\hat{i}$: $0 = V_B - \omega L \sin\theta$

$\hat{j}$: $V_A = -\omega L \cos\theta$

solve for ω & V_A

$$\vec{\omega} = \frac{V_B}{L\sin\theta} = \frac{3}{0.5(0.6)} = 10\hat{k}$$

$$V_A = -10(0.5)(0.8) = -4 \text{ m/s} \qquad \vec{V}_A = -4\hat{j} \text{ m/s}$$

5) solve for $\vec{a}_A$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$$

$$a_A\hat{j} = a_B\hat{i} + \alpha\hat{k} \times (-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}) - \omega^2(-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j})$$

$$= a_B\hat{i} - \alpha L\cos\theta\,\hat{j} - \alpha L\sin\theta\,\hat{i} + \omega^2 L\cos\theta\,\hat{i} - \omega^2 L\sin\theta\,\hat{j}$$

6) split components

$\hat{i}$: $0 = a_B - \alpha L \sin\theta + \omega^2 L \cos\theta$

$\hat{j}$: $a_A = -\alpha L \cos\theta - \omega^2 L \sin\theta$

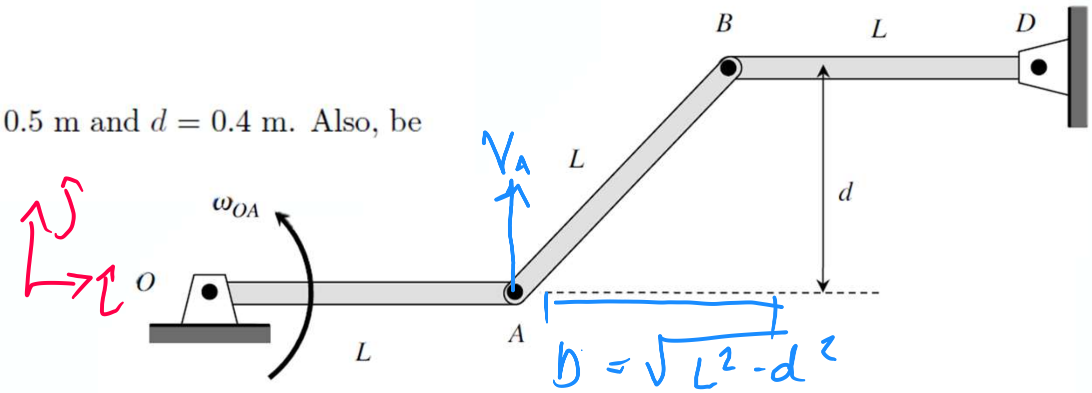
sub in known/found values & solve

Example 2.A.10

Given: At the instant shown, link OA rotates counterclockwise about pin O with a constant angular speed of ω_{OA}. At the instant shown, links OA and BD are horizontal.

Find: Determine at this instant:
(a) The angular acceleration of link AB; and
(b) The angular acceleration of link BD.

Use the following parameters in your analysis: $\omega_{OA} = 3$ rad/s, $L = 0.5$ m and $d = 0.4$ m. Also, be sure to express your answers as vectors.



1) draw unit vectors

2) define geometry

$$b = \sqrt{L^2 - d^2}$$

3) find V_A from V_o

$$\vec{V}_A = \underline{\vec{V}_o} + \vec{\omega} \times r_{A/O}$$

$$0 + \omega_{OA}\hat{k} \times L\hat{i}$$

$$\vec{V}_A = \omega_{OA} L \hat{j}$$

4) find V_B using V_A

$$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times r_{B/A}$$

$$= V_A\hat{j} + \omega_{AB}\hat{k} \times (b\hat{i} - d\hat{j})$$

$$= V_A\hat{j} + \omega_{AB} b\hat{j} - \omega_{AB} d\hat{i}$$

5) find V_B using V_D

$$\vec{V}_B = \vec{V}_D + \vec{\omega}_{BD} \times r_{B/D}$$

$$= 0 + \omega_{BD}\hat{k} \times (-L\hat{i})$$

$$\vec{V}_B = -\omega_{BD} L \hat{j}$$

6) set eqns for V_B equal

$$-\omega_{AB} d\,\hat{i} + (V_A + \omega_{AB} b)\hat{j} = -\omega_{BD} L\,\hat{j}$$

7) split components

$\hat{i}$: $-\omega_{AB} d = 0 \Rightarrow \omega_{AB} = 0 \qquad \vec{\omega}_{AB} = \vec{0}$

$\hat{j}$: $V_A + \omega_{AB} b = -\omega_{BD} L$

$$V_A = -\omega_{BD} L \Rightarrow \omega_{BD} = \frac{-V_A}{L} = \frac{\omega_{OA} L}{L}$$

$$\vec{\omega}_{BD} = -\omega_{OA}\,\hat{k}$$

now for acceleration:

find a_A from a_O

$$\vec{a}_A = \cancel{\vec{a}_O}^{\,0} + \cancel{\vec{\alpha}_{OA}}^{\,0} \times \vec{r}_{A/O} - \omega_{OA}^2 \vec{r}_{A/O}$$

$\leftarrow$ given

fixed pt.

$$\vec{a}_A = -\omega_{OA}^2 L \hat{i}$$

a_B from a_A

$$\vec{a}_B = \vec{a}_A + \alpha_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$= \vec{a}_A \hat{i} + \alpha_{AB} \hat{k} \times (b\hat{i} - d\hat{j}) - \cancel{\omega_{AB}^2 (b\hat{i} - d\hat{j})}^{\,0}$$

$$= \vec{a}_A \hat{i} + \alpha_{AB} b \hat{j} - \alpha_{AB} d \hat{i}$$

a_B from a_D $\leftarrow$ fixed pt.

$$\vec{a}_B = \cancel{\vec{a}_D}^{\,0} + \alpha_{BD} \times r_{B/D} - \omega_{BD}^2 \vec{r}_{B/D}$$

$$+ \alpha_{BD} \hat{k} \times -L\hat{i} - \omega_{BD}^2 (-L)\hat{i}$$

$$\vec{a}_B = -\alpha_{BD} L \hat{j} + \omega_{BD}^2 L \hat{i}$$

Set eqns for $\vec{a}_B$ =

$$\vec{a}_A \hat{i} + \alpha_{AB} b \hat{j} - \alpha_{AB} d \hat{i} = -\alpha_{BD} L \hat{j} + \omega_{BD}^2 L \hat{i}$$

Separate terms:

$$\hat{i} : \vec{a}_A - \alpha_{AB} d = \omega_{BD}^2 L$$

$$\hat{j} : \alpha_{AB} b = -\alpha_{BD} L$$

$>$ 2 eqn, 2 unknown

Sub in values and solve