ME 274: Basic Mechanics II

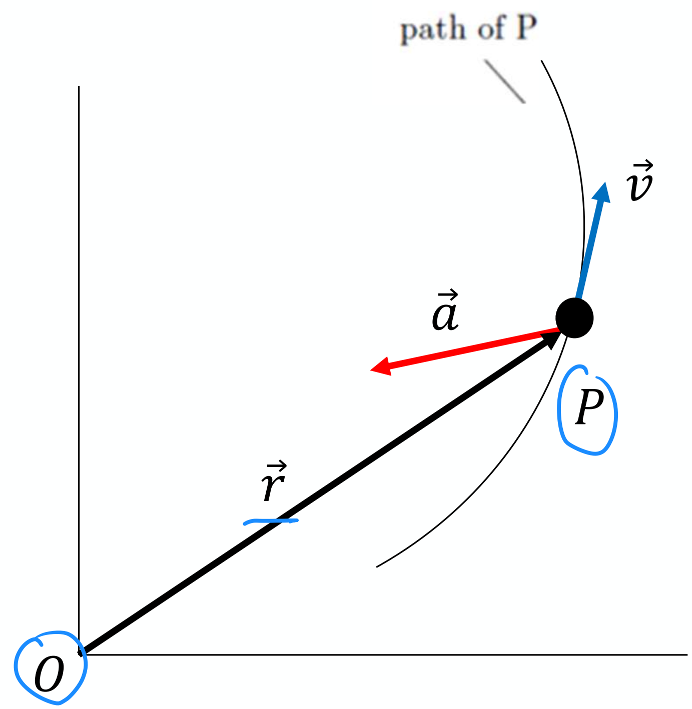
Lecture 5: Relative and Constrained Motion

PURDUE UNIVERSITY | School of Mechanical Engineering

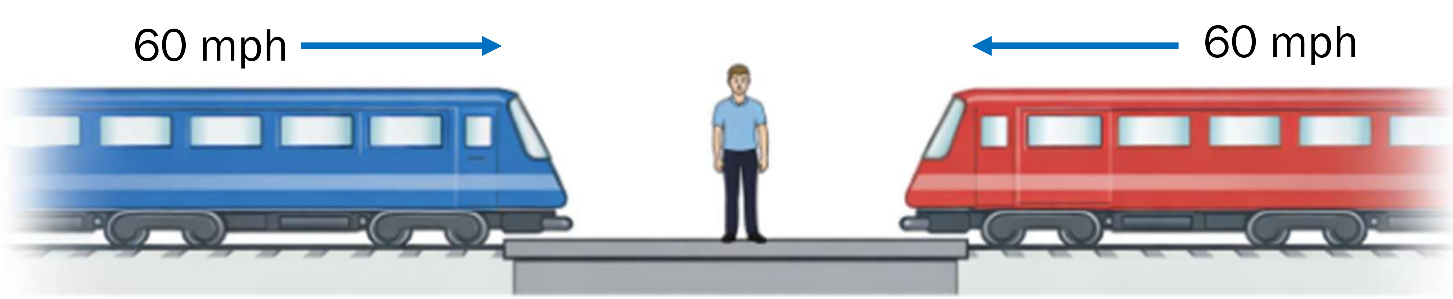
Motion is measured relative to a point

So far, we have described:
- Motion of a single point
- Measured relative to a fixed reference point, O

Motion is not absolute ⟶ depends on the observer
- What do we do if we have multiple moving points?
- How do we describe motion of one body as seen from another?



1D Example: trains crossing at a station



What is the speed of the red train observed from the platform? − 60mph

What is the speed of the red train observed from the blue train? − 120mph

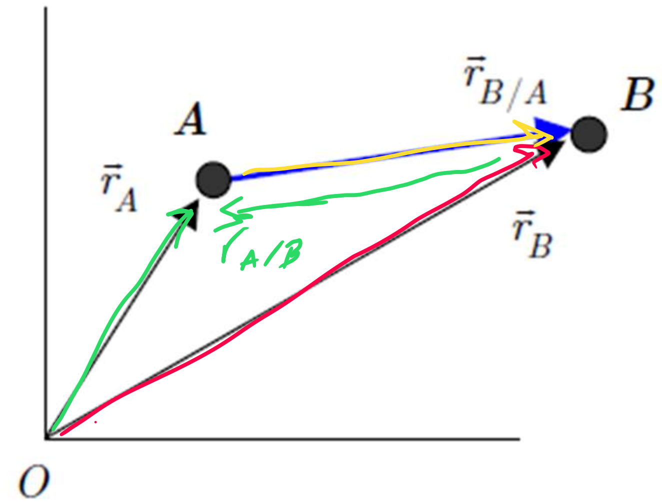
Relative position: where one point is as seen by another

The position of point B relative to point A is given by the vector $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

Note:

B/A → B with respect to A

- $\vec{r}_{B/A}$ points from A TO B
- $\vec{r}_{B/A} \neq \vec{r}_{A/B}$ ← same magnitude, opposite direction



Relative velocity and acceleration can be found by taking time derivatives

$$\vec{V}_{B/A} = \frac{d}{dt}\vec{r}_{B/A} = \frac{d\vec{r}_B}{dt} - \frac{d\vec{r}_A}{dt} = \vec{V}_B - \vec{V}_A$$

$$\vec{a}_{B/A} = \frac{d}{dt}\vec{V}_{B/A} = \frac{d\vec{V}_B}{dt} - \frac{d\vec{V}_A}{dt} = \vec{a}_B - \vec{a}_A$$

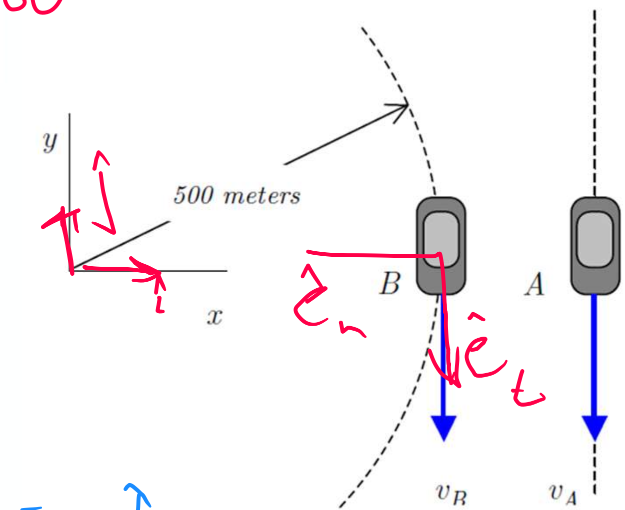
Example 1.D.1

$\dot{v}_B = -10$ $\qquad$ $v_B = 50$

Given: At the instant shown, car B is traveling with a speed of 50 km/hr and is slowing down at a rate of 10 km/hr^2. Car A is moving with a speed of 80 km/hr, a speed that is increasing at a rate of 10 km/hr^2. At this instant, A and B are traveling in the same direction.

$\dot{v}_A = 10$ $\qquad$ $v_A = 80$

Find: What acceleration does a passenger in car A observe for car B?



$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

$$\vec{a}_B = \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\,\hat{e}_n$$

$$\vec{a}_B = -10\,\hat{e}_t + \frac{50^2}{(0.5)}\,\hat{e}_n$$

$$= -10\,\hat{e}_t + 5000\,\hat{e}_n$$

$$\hat{e}_t = -\hat{j}$$
$$\hat{e}_n = -\hat{i}$$

$$\vec{a}_B = 10\hat{j} - 5000\hat{i}$$

$$\vec{a}_A = -\dot{v}_A\,\hat{j} = -10\hat{j}$$

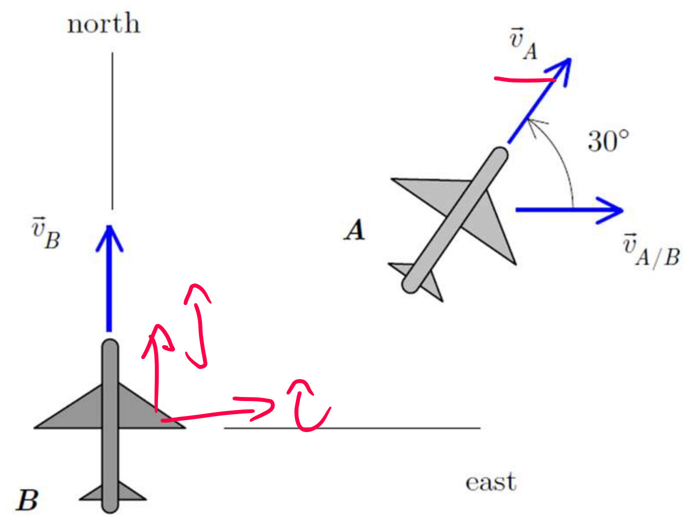
$$\vec{a}_{B/A} = 10\hat{j} - 5000\hat{i} - (-10\hat{j})$$

$$= -5000\hat{i} + 20\hat{j} \text{ km/hr}^2$$

Example 1.D.2

Given: Jet B is traveling due north with a speed of $v_B = 600$ km/hr. Passengers on jet B observe A to be flying sideways and moving due east.

Find: Determine:
 (a) The speed of A; and
 (b) The speed of A as observed by the passengers on jet B.



$$\vec{V}_B = V_B\,\hat{j}$$

$$\vec{V}_{A/B} = V_{A/B}\,\hat{i}$$

$$\vec{V}_A = V_A(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

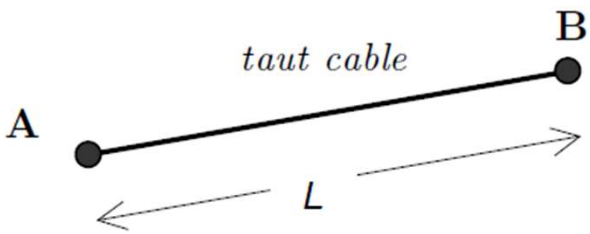
$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B} = \underline{V_{A/B}\,\hat{i}} + \underline{V_B\,\hat{j}} = \underline{V_A(\cos\theta\,\hat{i}} + \sin\theta\,\hat{j})$$

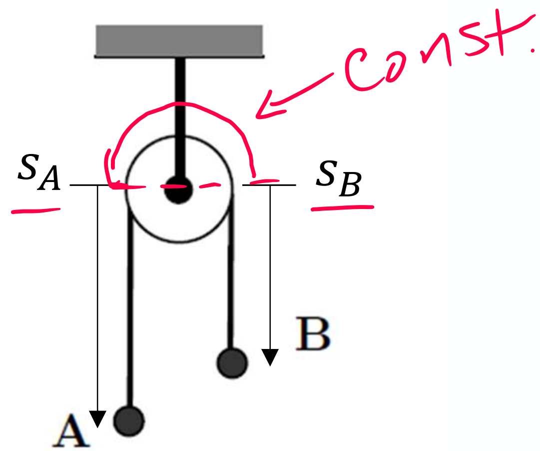
$$V_{A/B} = V_A\cos\theta$$

$$V_B = V_A\sin\theta \Rightarrow V_A = \frac{V_B}{\sin\theta} = \frac{60}{1/2} = 1200\,\frac{km}{hr}$$

$$V_{A/B} = V_A\cos\theta = 1200\cdot\frac{\sqrt{3}}{2}\ \frac{km}{hr}$$

Constrained Motion – Inextensible Cable





Constrained motion →the motion of one point depends on the motion of the other

→ constraint eqns

Important considerations:

- Length, L, does not change → $\frac{dL}{dt} = 0$
- The cable is flexible & can go around pulleys
    - L remains constant, but the distance between A and B does not
    - The diameter of the pulley does not affect the constraint equation

constraint eqn: $L = s_A + s_B + \text{const.}$

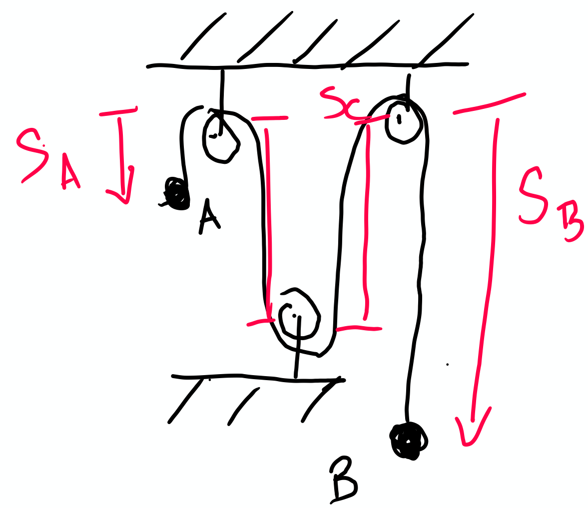
$$\frac{dL}{dt} = \frac{ds_A}{dt} + \frac{ds_B}{dt} + \underbrace{\cancel{\frac{d(\text{const})}{dt}}}_{0}$$

$$0 = v_A + v_B$$

$$0 = a_A + a_B$$

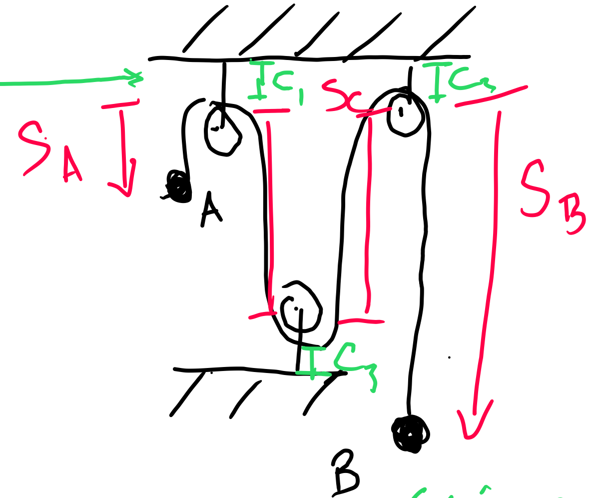
Solving Inextensible Cable Problems

1. Carefully define a set of coordinates that describe the motion of the various particles in the system. → unique lengths of cable
2. For each cable, write an expression for its length, L, in terms of an appropriate set of coordinates defined above in step 1. $L = S_A + 2S_C + S_B$
3. Differentiate (with respect to time) the above expression for the cable length L and set $dL/dt = 0$ to determine the velocity constraint. $0 = V_A + 2V_C + V_B$
4. Differentiate again with respect to time to determine the acceleration constraint.
5. Repeat steps 2 through 4 for each cable in the system. $0 = a_A + 2a_C + a_B$



a note on connectors in pulley problems:

in these problems, you may see small "connector" lengths



- note in many problems (like the one pictured) you can define your length such that you do not need to include the connectors in your constraint eqns.

- unless otherwise specified, you can disregard these connectors.
- if including them helps your book keeping, you can label c_1, c_2, c_3 & group as a constant

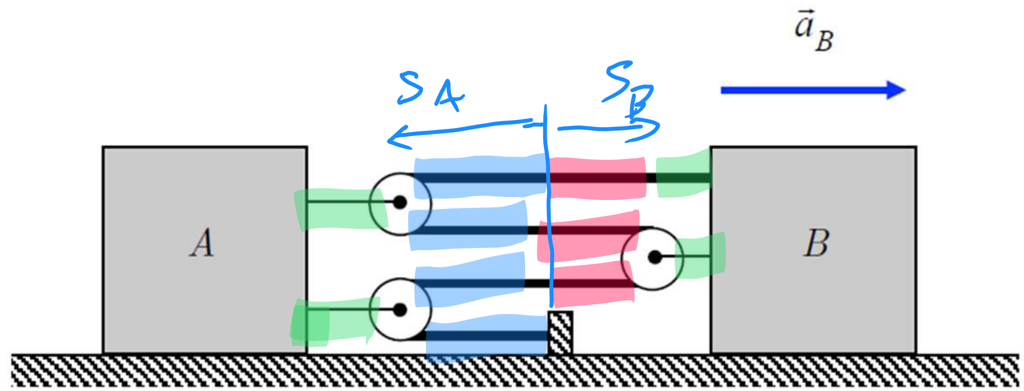
i.e. $L = S_A + 2S_C + S_B + \underbrace{(c_1 + c_2 + c_3)}_{=\text{const}}$

- when you take your derivatives for $\vec{v}$ or $\vec{a}$, this term $= 0$

Example 1.D.3

Given: Block B has a constant acceleration of $a_B = 3\ \text{m/s}^2$ to the right. At the instant shown, B has a velocity of 2 m/s to the right.

Find: Determine:
(a) The velocity of block A; and
(b) The acceleration of block A.



Solution:
1) Define a set of coordinates

2) Write an expression for cable length, L, in terms of coordinates

$$L = 4s_A + 3s_B$$

3) Differentiate the expression for L and set $\frac{dL}{dt} = 0$ to determine velocity constraint

$$0 = 4v_A + 3v_B \qquad v_A = -\frac{3v_B}{4} \Rightarrow - \quad \vec{V}_A = -\frac{3V_B}{4}(-\hat{\imath})$$

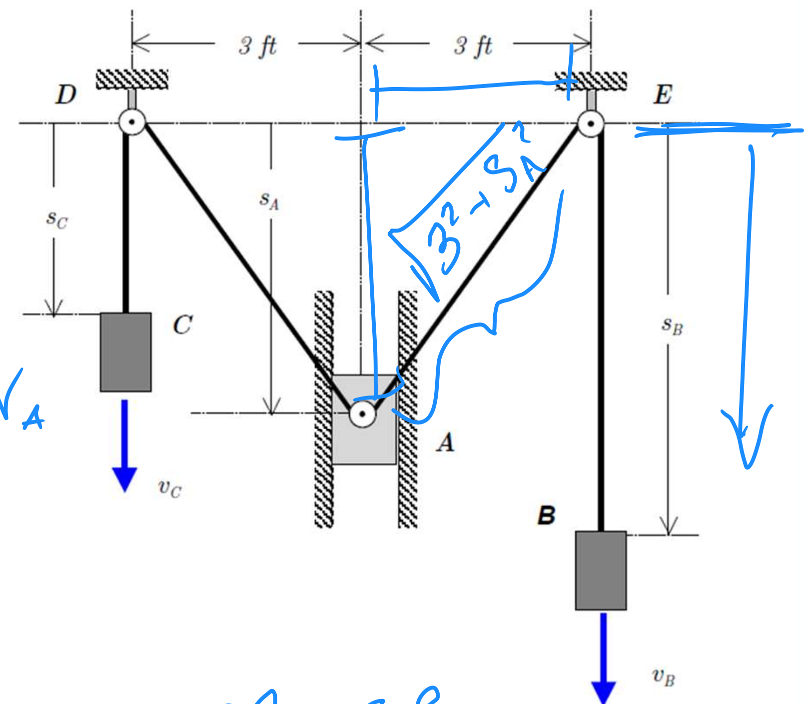
4) Differentiate again to determine acceleration constraint

$$0 = 4a_A + 3a_B \rightarrow a_A = -\frac{3a_B}{4}$$

Example 1.D.4

Given: Blocks B and C are connected by a single inextensible cable, with this cable being wrapped around pulleys at D and E. In addition, the cable is wrapped around a pulley attached to block A as shown. Assume the radii of the pulleys to be small. Blocks B and C move downward with speeds of $v_B = 6$ ft/s and $v_C = 18$ ft/s, respectively.

Find: Determine the velocity of block A when $s_A = 4$ ft.



Constraint eqn:

$$L = S_C + 2\sqrt{3^2 + S_A^2} + S_B$$

$$\frac{dL}{dt} = 0 = V_C + 2\left(\tfrac{1}{2}\right)\left(9 + S_A^2\right)^{-1/2}(2S_A)V_A + V_B$$

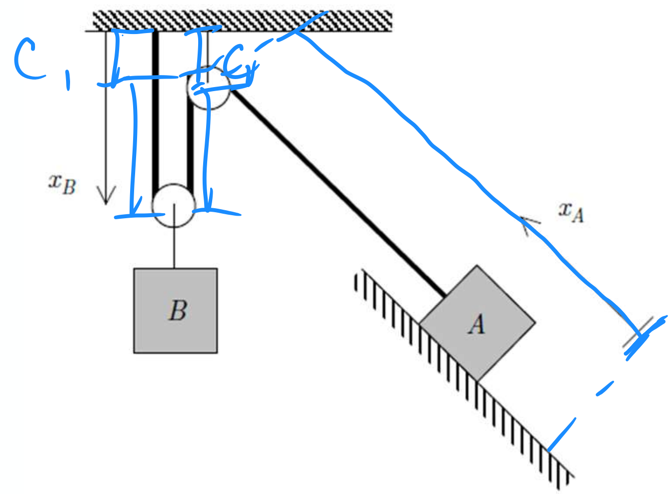
$$0 = V_C + \frac{2S_A V_A}{\sqrt{9 + S_A^2}} + V_B \rightarrow$$

rearrange + solve for V_A

Example 1.D.4

Given: Block A moves with an acceleration of $\ddot{x}_A = a_A = 0.44\ \text{m/s}^2$.

Find: Determine the acceleration of block B.



$$L = 2x_B + (\text{const} - x_A)$$

$$0 \qquad 0$$

$$2x_B + \cancel{\text{const}} - x_A + \cancel{2c_1}$$

$$0 = 2v_B - v_A$$

$$0 = 2a_B - a_A$$

sub in & solve