ME 274: Basic Mechanics II

Lecture 1: Course Intro & Point Kinematics, Cartesian Description

PURDUE UNIVERSITY | School of Mechanical Engineering

ME 274 Course Structure

- Textbook: Dynamics – A Lecturebook
  - https://www.purdue.edu/freeform/me274/daily-schedule-sp-2026/
  - Will be **used daily during lecture**
  - Can be purchased for $90 from the University Book Store

- Course website: **https://www.purdue.edu/freeform/me274**
  - Homework assignments, solution videos, and discussion posts
  - Additional example problems
  - Section specific announcements on instructor page

ME 274 Course Structure

Assignments and Grading

Grade scale: 97-100% A+, 93-97% A, 90-93% A-, 87-90% B+, etc.....

Homework and Quizzes: 25%

Homework

- Two homework problems per lecture, submitted to Gradescope by **11:59 PM the day of the next lecture.**
- One problem graded for correctness, one problem graded for completion.
- Three dropped assignments.

Quizzes

- In-class quizzes will be given throughout the semester and will be **unannounced.**
- Quizzes serve as practice for you and an assessment of class understanding for me.
- Grading will be based off completion/demonstrated effort.
- If you need to miss a lecture email me **before the start of class** with the subject line **ME 274 Absence** (no body text needed).

Exams: 75%

- Two 1.5 hour evening midterms, Th 2/12 & Th 4/2
- 2 hour final exam, date TBD during finals week
- Weighting of midterm average and final: **Higher score → 50%** of course grade, **Lower score → 25%** of course grade

ME 274 Course Structure

Teaching Team and Help Resources

- Dr. Andress office hours: T 9:30-10:30 & W 3:30-4:30 in ME 2008A, (or by appointment)
- See syllabus for additional faculty office hours
- ME tutorial rooms – staffed by TAs during normal business hours
- Discussion thread on course website
- 800+ of your peers!

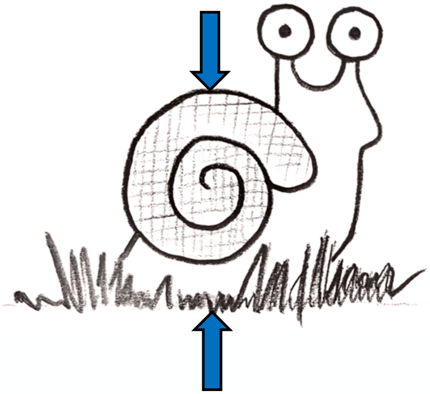
Moving from statics to dynamics

Statics (ME 270):

- Systems in equilibrium

$$\sum F = 0, \sum M = 0$$

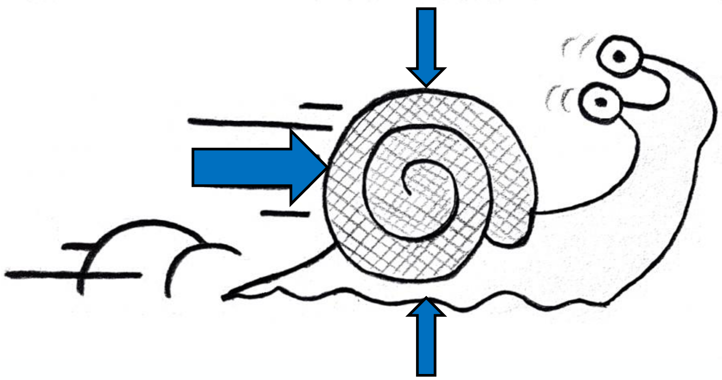
- No acceleration



Dynamics (ME 274):

- Net forces result in acceleration

$$\sum F = m\,a$$



Course Objectives:

1. Describe motion precisely
2. Relate motion to forces
3. Predict system behavior over time

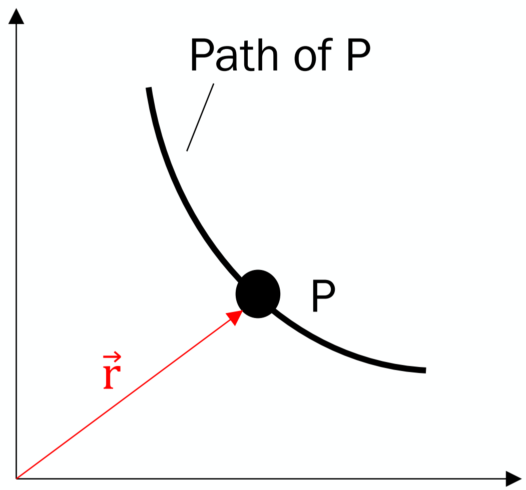
Image: https://enterfea.com/difference-between-static-and-dynamic-analysis/

Describing motion: particle kinematics

Kinematics describe particle motion without reference to forces – **how the system moves** with respect to a chosen coordinate system.

Describes:

- Position, $\vec{r}(t)$
- Velocity, $\vec{v}(t)$
- Acceleration, $\vec{a}(t)$



For all coordinate systems:

$$\vec{v} = \frac{d\vec{r}}{dt},$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2}$$

Particle Kinematics – Cartesian Description

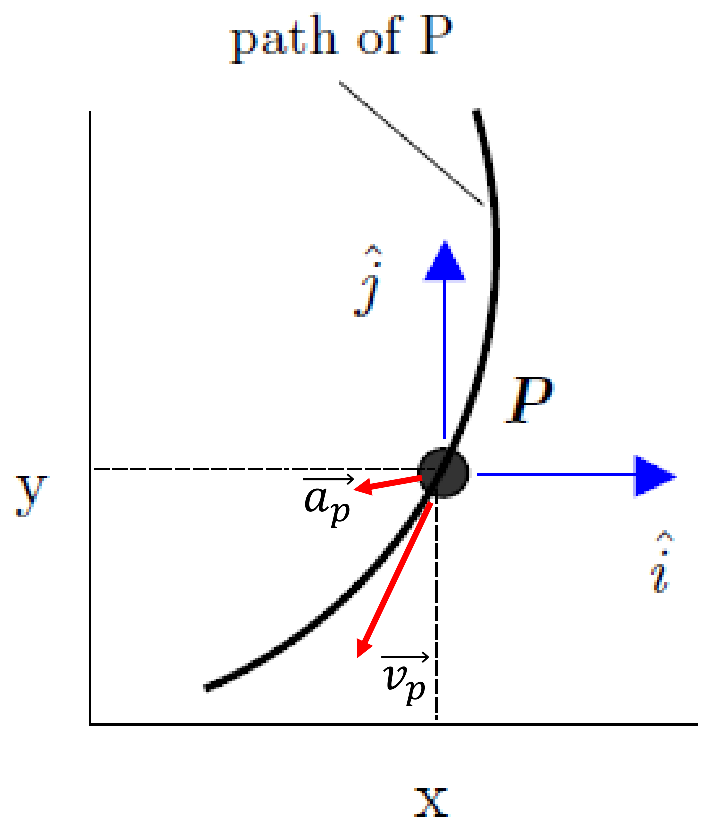
Cartesian Kinematics

- The path of particle P is expressed in terms of ***x*** and ***y*** components

$$\vec{r} = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

- All vectors are represented as linear combinations of the **unit vectors** $\hat{\imath}$ **and** $\hat{\jmath}$

$$\frac{d\hat{\imath}}{dt} = 0, \frac{d\hat{\jmath}}{dt} = 0$$



Velocity components come from **time derivatives of position components**

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$$

Acceleration components come from **second time derivatives of position components**

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

Explicit vs. Implicit Time Dependence

In Cartesian kinematics, position can be defined in two ways:

Explicit time dependence

- Position components given directly as a function of time:

$$x = x(t), \qquad y = y(t)$$

- Velocity and acceleration found by direct differentiation

$$\dot{x} = \frac{dx}{dt}, \dot{y} = \frac{dy}{dt}$$

$$\ddot{x} = \frac{d^2x}{dt^2}, \ddot{y} = \frac{d^2y}{dt^2}$$

Implicit time dependence

- Geometry defines the path and one coordinate depends on time **indirectly**

$$x = x(t), \qquad y = f(x) \quad \textbf{or}$$

$$y = y(t), \qquad x = f(y)$$

- Differentiation requires the **chain rule:**

  if $x = x(t)$, $y = f(x)$,

$$\dot{x} = \frac{dx}{dt},$$

$$\dot{y} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \dot{x}\frac{dy}{dx}$$

Example: Implicit time dependence

Problem Statement: Suppose that $y = \sin(x)$ and $\dot{x} = 3$ m/s $= constant$, and we want to know the velocity and acceleration when $x = \pi/2$.

Find: Cartesian components of velocity and acceleration

Solution:

x components of velocity and acceleration are known:

$$x = \frac{\pi}{2},\ \dot{x} = 3\ \text{m/s} = \text{constant} \quad \rightarrow \ \ddot{x} = 0$$

To find the y component of velocity, differentiate using the chain rule:

$$y = \sin(x) \ \rightarrow \ \dot{y} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x,\quad \frac{dx}{dt} = \dot{x} \qquad \Longrightarrow \boldsymbol{\dot{y} = \dot{x}\cos x = 3\cos\frac{\pi}{2} = 0\ \text{m/s}}$$

To find the y component of acceleration, differentiate velocity with respect to time:

$$\ddot{y} = \frac{d}{dt}(\dot{y}) = \frac{d}{dt}(\dot{x}\cos x)$$ ← Use chain **and** product rule

$$\ddot{y} = \ddot{x}\cos x - \dot{x}^2 \sin x \Rightarrow \boldsymbol{\ddot{y} = (0)\cos\frac{\pi}{2} - 3^2\sin\frac{\pi}{2} = -9\ \text{m/s}}$$

Answer: $\boldsymbol{\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} = 3\hat{\imath} + 0\hat{\jmath}\ \text{m/s}}$, $\boldsymbol{\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = 0\hat{\imath} - 9\hat{\jmath}\ \text{m/s}^2}$

Lecturebook example: 1.A.2

Given: A particle P moves on a path whose Cartesian components are given by the following functions of time (where both components are given in inches and time t is given in seconds):

$$x(t) = t^3 + 10$$
$$y(t) = 2\cos 4t$$

Find: Determine at the time $t = 2$ s:
(a) The velocity vector of P;
(b) The acceleration of P; and
(c) The angle between the velocity and acceleration vectors of P.

Solution:

a) Velocity components can be found directly by differentiating wrt t: $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$

$$\dot{x} = 3t^2, \qquad \dot{y} = -8\sin 4t$$

$$\vec{v} = 3t^2\hat{\imath} - 8\sin 4t\,\hat{\jmath}$$ ⇒ plug in t = 2 and solve

b) Acceleration components can be found by taking the second derivative wrt t: $\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$

$$\ddot{x} = 6t, \qquad \ddot{y} = -32\cos 4t$$

$$\vec{a} = 6t\hat{\imath} - 32\cos 4t\,\hat{\jmath}$$ ⇒ plug in t = 2 and solve

c) From linear algebra remember: $\vec{v} \cdot \vec{a} = |\vec{v}||\vec{a}|\cos(\theta)$

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{a}}{|\vec{v}||\vec{a}|}\right) = \cos^{-1}\left(\frac{\dot{x}\ddot{x}+\dot{y}\ddot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}\sqrt{\ddot{x}^2+\ddot{y}^2}}\right)$$ ⇒ plug in t = 2 and solve

Lecturebook example: 1.A.1

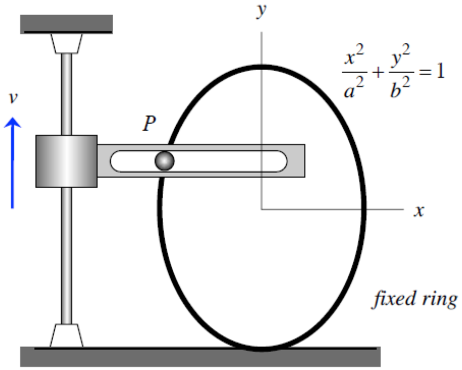
Given: Pin P is constrained to move along a elliptical ring whose shape is given by $x^2/a^2+y^2/b^2 = 1$ (where x and y are given in mm). The pin is also constrained to move within a horizontal slot that is moving upward at a constant speed of v.

Find: Determine:

(a) The velocity of pin P at the position corresponding to $y = 6$ mm; and

(b) The acceleration of pin P at the position corresponding to $y = 6$ mm.

Use the following parameters in your analysis: $a = 5$ mm, $b = 10$ mm, $v = 30$ mm/s.



Solution:

a) Velocity components are defined as: $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$

Known: vertical motion of P constrained by slot → $\dot{y} = v$

Known: the slot moves upward at a constant speed → $\ddot{y} = 0$

Known: x position constrained with $\frac{x^2}{a} + \frac{y^2}{b} = 1$ → implicitly differentiate and solve for $\dot{x}$

$$\frac{2x\dot{x}}{a^2} + \frac{2y\dot{y}}{b^2} = 0 \Rightarrow b^2x\dot{x} + a^2y\dot{y} = 0$$

Solve for x from original constraint eqn: $x = \pm a\sqrt{(1 - \frac{y^2}{b^2})}$

Solve for $\dot{x}$: $\dot{x} = -\frac{a^2y\dot{y}}{b^2x} = -\frac{ay\dot{y}}{b^2\sqrt{(1-\frac{y^2}{b^2})}}$ ←Plug in known numerical values and solve

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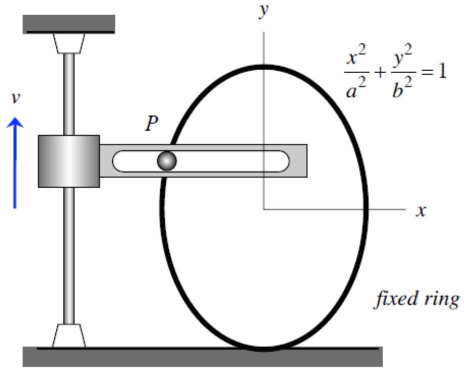


Solution:

a) Acceleration components defined as: $\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$

Known from part (a): $b^2x\dot{x} + a^2y\dot{y} = 0$ → implicitly differentiate and solve for $\ddot{x}$

$$\dot{x}^2b^2 + x\ddot{x}b^2 + \dot{y}^2a^2 + y\ddot{y}a^2 = 0$$

Known from part (a): $\ddot{y} = 0$ → $\dot{x}^2b^2 + x\ddot{x}b^2 + \dot{y}^2a^2 = 0$

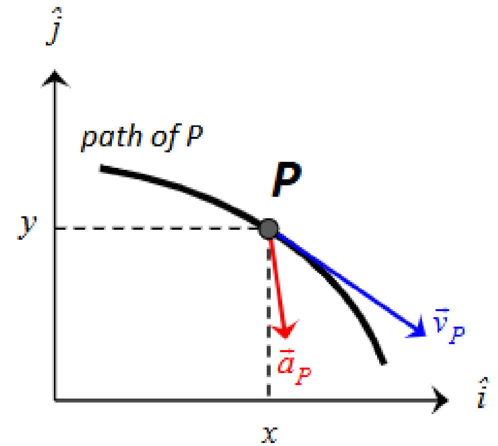
Plug in known numerical values and solve for $\ddot{x}$

Note: once you solve for $\dot{x}, \ddot{x}, \dot{y}, \ddot{y}$, remember to plug back into the vector equations for $\vec{a}, \vec{v}$

Summary: Particle Kinematics – Cartesian Description

1. *PROBLEM*: Motion of a point is described in Cartesian *xy*-coordinates.

2. *FUNDAMENTAL EQUATIONS*:



$$\vec{v}_P = \dot{x}\hat{i} + \dot{y}\hat{j} = velocity\ of\ P$$

$$\vec{a}_P = \ddot{x}\hat{i} + \ddot{y}\hat{j} = acceleration\ of\ P$$

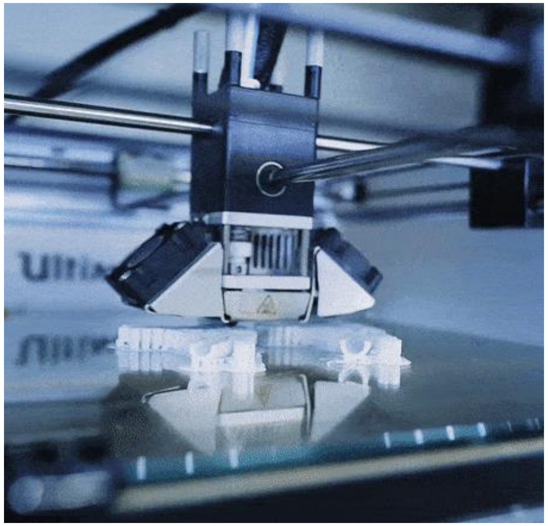
with $\dot{x} = \dfrac{dx}{dt}$, etc.

3. *CHAIN RULE OF DIFFERENTIATION*: Suppose that *y* is given in terms of *x* (instead of time *t*) – how do you find $\dot{y} = dy/dt$??

The chain rule!! $\dot{y} = \dfrac{dy}{dt} = \dfrac{dy}{dx}\dfrac{dx}{dt} = \dot{x}\dfrac{dy}{dx}$ (🡰 remember this!)

4. *COMMENT*: The Cartesian description is easy to use, but not as useful as other descriptions. More later…

Real-World Examples of Cartesian Kinematics



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CNC milling

Gantry vending machine