

ME 274 Lecture 7

Rigid Body Kinematics 2

Eugenio “Henny” Frias-Miranda

1/28/26

Housekeeping/Announcements

1. **HW 6 due friday!!**
2. Andres Arrieta's Singapore Maymester
3. Prof. Krousgrill Exam review session in a couple of weeks
4. ME2206 - Office Hours Location. Syllabus will be updated soon
5. Quiz 3 at the end of lecture, good luck!
 - Feel free to collaborate with your peers
 - I will go over the problem if I see people struggling

***Reminder for Henny to wear a mic during the lecture.



BIOINSPIRED MATERIALS AND STRUCTURES - SINGAPORE

TRANSLATING BIOLOGICAL PRINCIPLES INTO
ENGINEERING APPLICATIONS

May 15-30, 2026



The purpose of this course is to introduce bioinspired principles as a source of engineering design and functionality and to inspire multidisciplinary thinking and inter-cultural perspective as a tool for working in diverse and international settings.

In addition to course activities, lectures, and case studies students will experience cultural and fun events:

- Night Safari, Changi Jewel, Half-day City tour, ArtScience Museum, Clarke Quay, and much more!

Open to all engineering and biological science students, first year through seniors! Students must be in good academic standing.

Earn 3 credits for ME 49700. This is a Tech Elective for ME students. Students in other majors must check with their academic advisor to see how this course can be used in their plan of study.

Estimated cost is \$5,400 which includes: academic credits, all housing, ground transportation, all activities on the itinerary, daily breakfast, Welcome and Farewell meals, and international health insurance. **Additional costs:** passport, airfare, visa (if needed), and personal spending money.



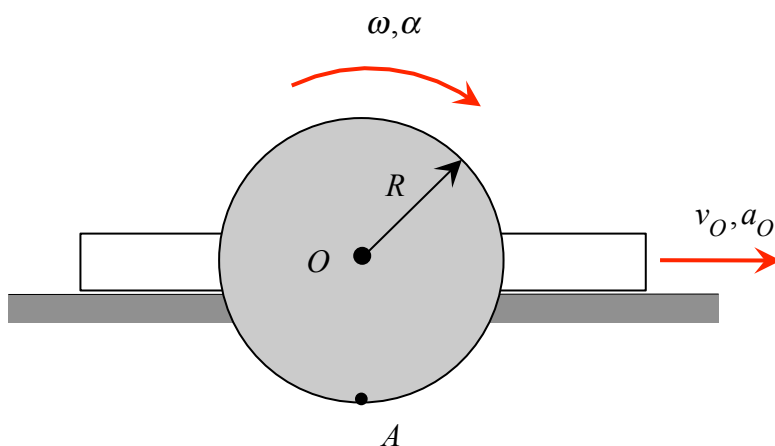
PURDUE
UNIVERSITY

Global Engineering Programs
and Partnerships

Homework H2.B

Given: A circular disk is pinned to a block at its center O , with the block being constrained to move along a horizontal surface. The angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$ of the disk are in the directions shown in the figure. The block is moving the right with a speed of v_O and an acceleration of a_O . At the position shown, point A on the perimeter of the disk is directly below O .

Find: For this position, determine the velocity and acceleration of point A . Express your answers as vectors.

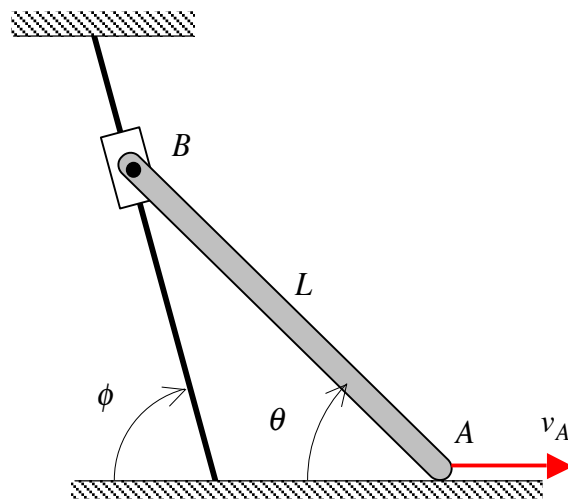


Use the following parameters in your analysis: $R = 0.75$ m, $\omega = 4$ rad/s, $\alpha = 2$ rad/s², $v_O = 3$ m/s and $a_O = 4$ m/s².

Homework H2.C

Given: Thin bar AB (having a length of L) moves in a way that ends A and B slide along straight, fixed guides as shown in the figure. End A has a constant speed of v_A to the right as the bar moves.

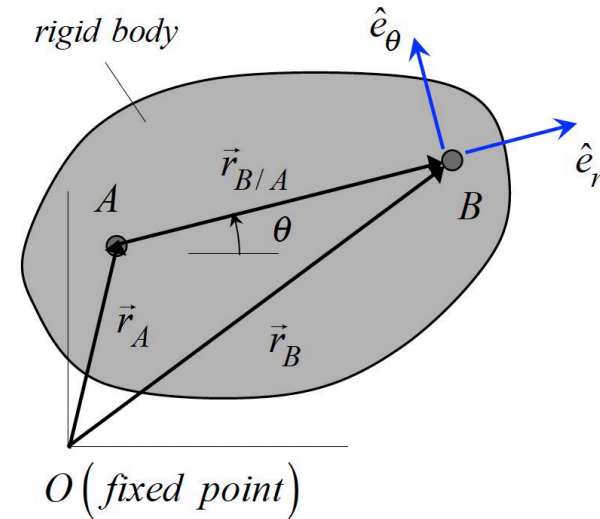
Find: Determine the velocity and acceleration of end B.



Use the following parameters in your analysis: $L = 2$ ft, $v_A = 4$ ft/s, $\theta = 36.87^\circ$ and $\phi = 45^\circ$.

Last Lecture...

1. Define what a rigid body is



2. Derive the **velocity and acceleration equations** for **planar motion of a rigid body**

$$\boxed{\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}}$$

$$\boxed{\vec{a}_B = \vec{a}_A + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}] + \vec{\alpha} \times \vec{r}_{B/A}}$$

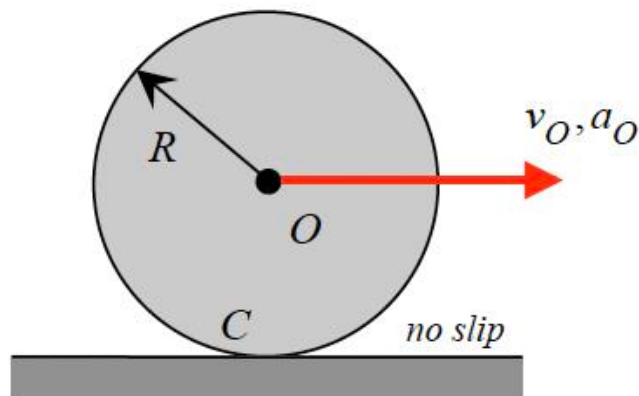
What is rolling without slipping?

- As the wheel moves, it is assumed that sufficient acts between the wheel and surface that the contact point C
- We will encounter many problems throughout the course...
- Rolling without slipping on the velocity and acceleration of other points on the body *can become quite complicated [video below]*.

- In the next slide, we will see why this happens:

$$v_{Cx} = \text{ } \quad v_{Cy} = \text{ }$$

$$a_{Cx} = \text{ } \quad a_{Cy} = \text{ }$$



- If x component is then the wheel slips

[Visualizing Mechanics: Path Tracing on an Object Rolling Without Slip]



If C is a no-slip point, what are the y-components of velocity and acceleration?

- Solving for y component of velocity at point C:

$$\vec{v}_C = \vec{v}_O + \vec{\omega} \times \vec{r}_{C/O}$$

$$v_{Cy}\hat{j} = v_O\hat{i} + (\omega\hat{k}) \times (-R\hat{j}) = (v_O + R\omega)\hat{i}$$

$$\Rightarrow \boxed{\phantom{v_{Cy}\hat{j} = (v_O + R\omega)\hat{i}}}$$

- Solving for y component of acceleration at point C:

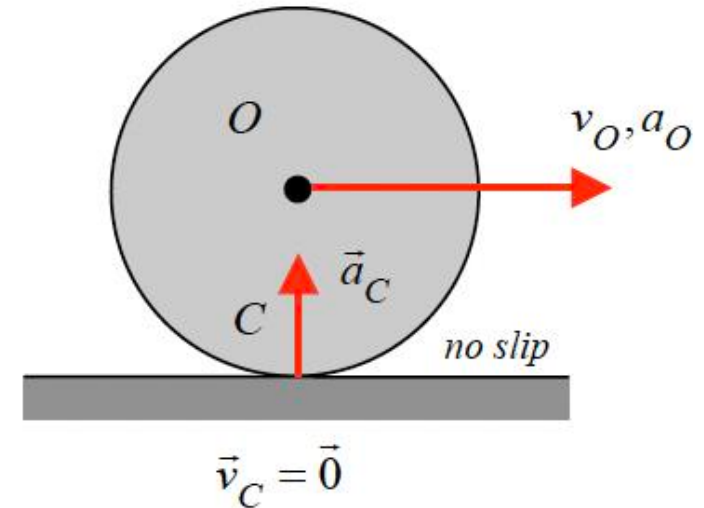
$$\vec{a}_C = \vec{a}_O + \vec{\alpha} \times \vec{r}_{C/O} - \omega^2\vec{r}_{C/O}$$

$$a_{Cy}\hat{j} = a_O\hat{i} + (\alpha\hat{k}) \times (-R\hat{j}) - \omega^2(-R\hat{j})$$

$$= (a_O + R\alpha)\hat{i} + R\omega^2\hat{j}$$

$$\Rightarrow \boxed{\phantom{a_{Cy}\hat{j} = (a_O + R\alpha)\hat{i} + R\omega^2\hat{j}}}$$

- **Velocity for a no-slip point is zero.**
- **Acceleration is not zero.**

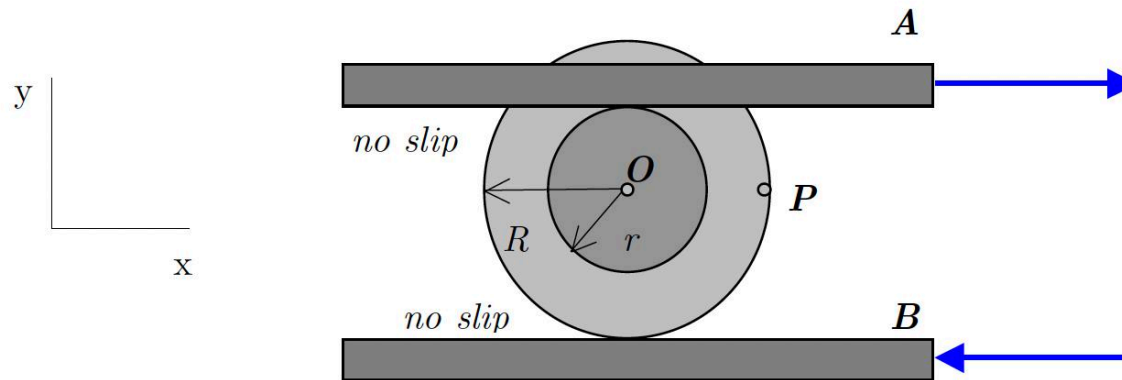


Example 2.A.4

Given: Rack A moves to the right with a speed of v_A and an acceleration of a_A . Rack B moves to the left with a constant speed of v_B . Assume $v_A = 0.8$ m/s, $a_A = 2$ m/s², $v_B = 0.6$ m/s, $r = 0.1$ m and $R = 0.16$ m.

Find: Determine:

- (a) The velocity of point P on the outer rim of the gear; and
- (b) The acceleration of point P on the outer rim of the gear.



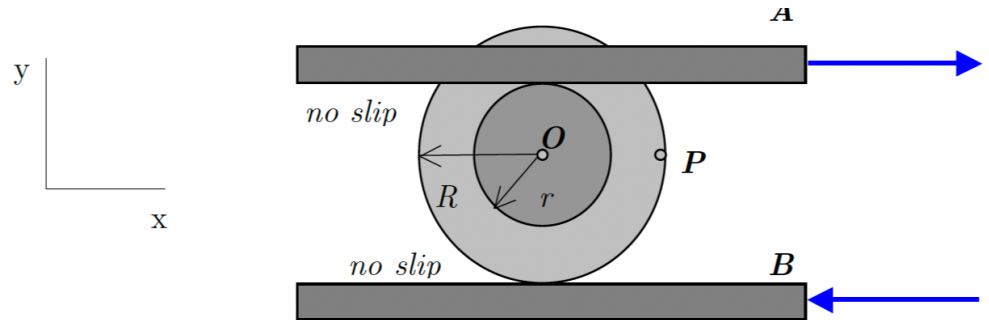
Example 2.A.4

Given: Rack A moves to the right with a speed of v_A and an acceleration of a_A . Rack B moves to the left with a constant speed of v_B . Assume $v_A = 0.8$ m/s, $a_A = 2$ m/s², $v_B = 0.6$ m/s, $r = 0.1$ m and $R = 0.16$ m.

Find: Determine:

- (a) The velocity of point P on the outer rim of the gear; and
- (b) The acceleration of point P on the outer rim of the gear.

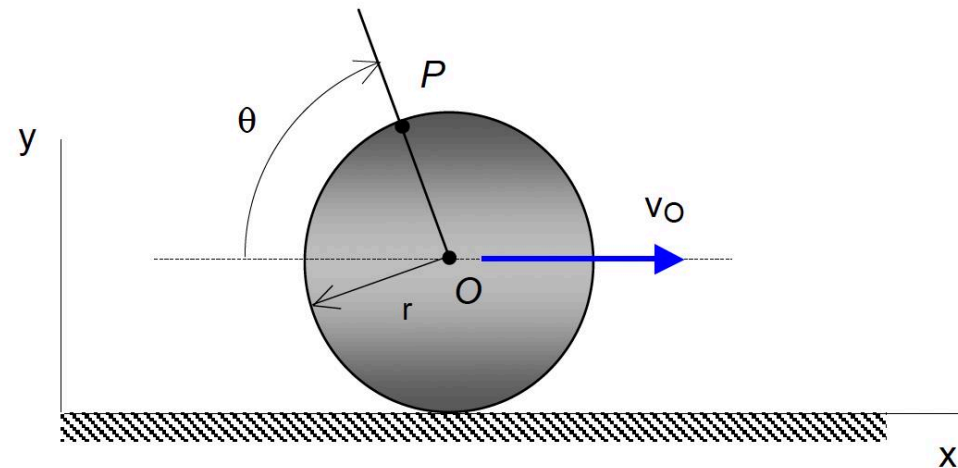
$\vec{v}_P ?$
 $\vec{a}_P ?$



Example 2.A.5

Given: A wheel rolls without slipping on a rough horizontal surface. At one instant, when $\theta = 90^\circ$, the center of the wheel O is moving to the right with a speed of $v_O = 5 \text{ ft/s}$ with this speed decreasing at a rate of 3 ft/s^2 .

Find: Determine the acceleration of point P on the circumference of the wheel at this instant, if $r = 2 \text{ ft}$. Make a sketch of this acceleration vector at P .

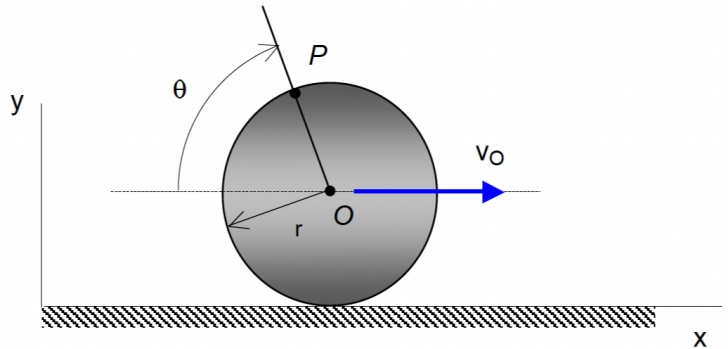


Example 2.A.5

Given: A wheel rolls without slipping on a rough horizontal surface. At one instant, when $\theta = 90^\circ$, the center of the wheel O is moving to the right with a speed of $v_O = 5 \text{ ft/s}$ with this speed decreasing at a rate of 3 ft/s^2 .

$\vec{a}_P?$

Find: Determine the acceleration of point P on the circumference of the wheel at this instant, if $r = 2 \text{ ft}$. Make a sketch of this acceleration vector at P .



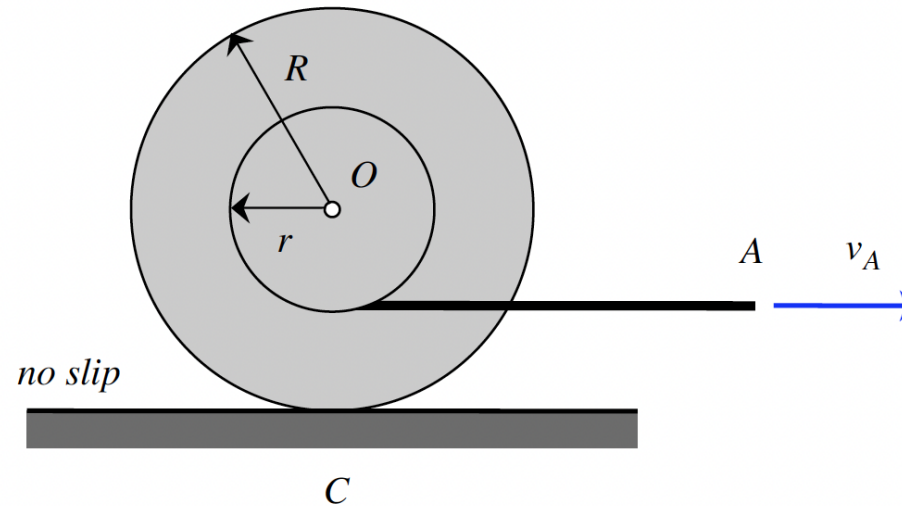
Example 2.A.6

Given: A cable is wrapped around the inner radius of a spool. End A of the cable is moving to the right with a speed of v_A . The spool is able to roll without slipping on a rough horizontal surface.

Find: Determine:

- (a) The velocity of the center O of the spool; and
- (b) The angular velocity of the spool.

Use the following parameters in your analysis: $v_A = 2$ m/s, $r = 0.4$ m and $R = 0.8$ m. Also, be sure to express your answers as vectors.



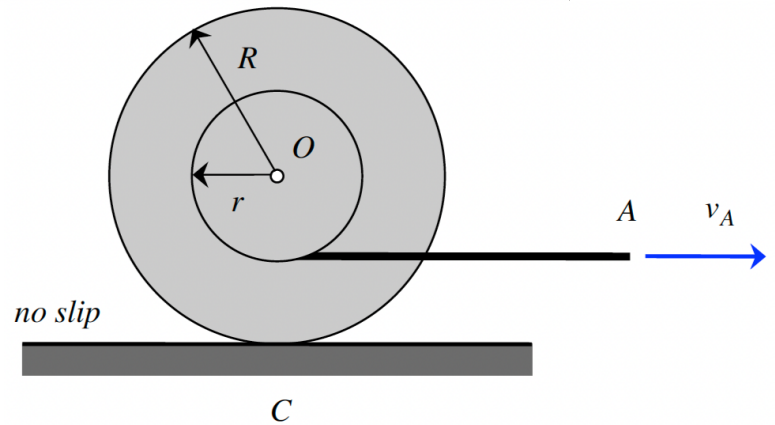
Example 2.A.6

Given: A cable is wrapped around the inner radius of a spool. End A of the cable is moving to the right with a speed of v_A . The spool is able to roll without slipping on a rough horizontal surface.

Find: Determine:

- (a) The velocity of the center O of the spool; and $v_o?$
- (b) The angular velocity of the spool. $\omega?$

Use the following parameters in your analysis: $v_A = 2$ m/s, $r = 0.4$ m and $R = 0.8$ m. Also, be sure to express your answers as vectors.



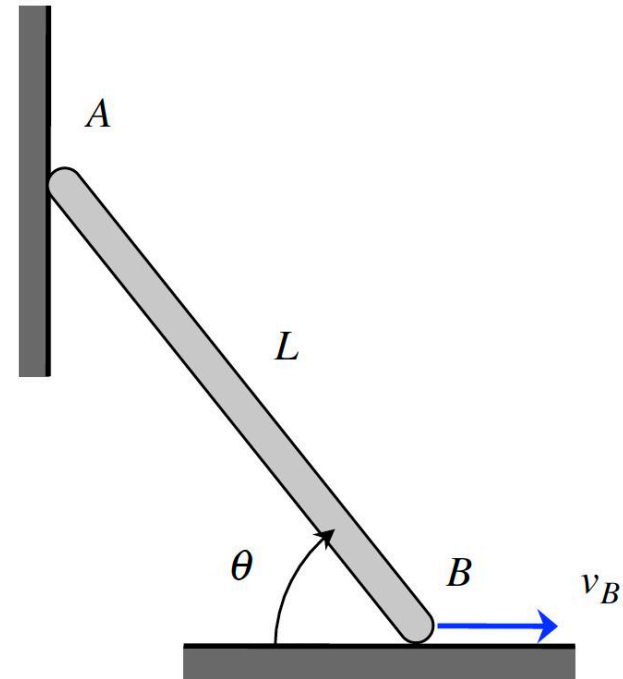
Example 2.A.7

Given: End B of the link moves to the right with a constant speed v_B .

Find: Determine:

- (a) The angular velocity of link AB; and
- (b) The angular acceleration of link AB.

Use the following parameters in your analysis: $v_B = 3 \text{ m/s}$, $L = 0.5 \text{ m}$ and $\theta = 36.87^\circ$. Also, be sure to express your answers as vectors.



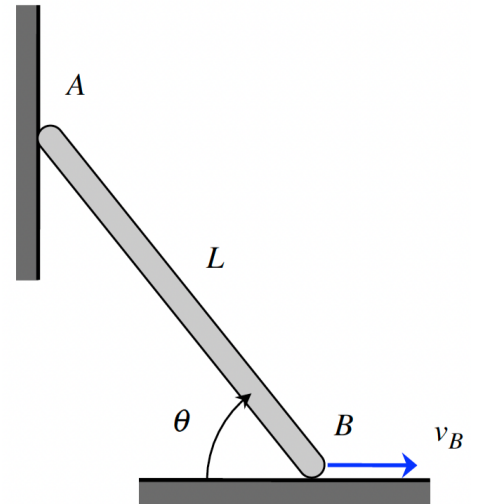
Example 2.A.7

Given: End B of the link moves to the right with a constant speed v_B .

Find: Determine:

- (a) The angular velocity of link AB; and $\omega_{AB}?$
- (b) The angular acceleration of link AB. $\alpha_{AB}?$

Use the following parameters in your analysis: $v_B = 3 \text{ m/s}$, $L = 0.5 \text{ m}$ and $\theta = 36.87^\circ$. Also, be sure to express your answers as vectors.



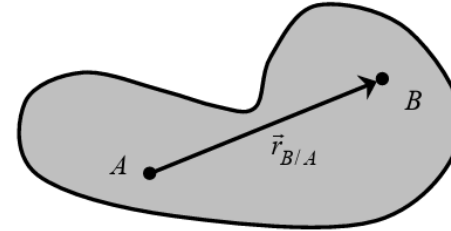
Summary: Rigid Body Kinematics 2

PROBLEM: Two points A and B on the same rigid body undergoing planar motion.

[pg. 86-87]

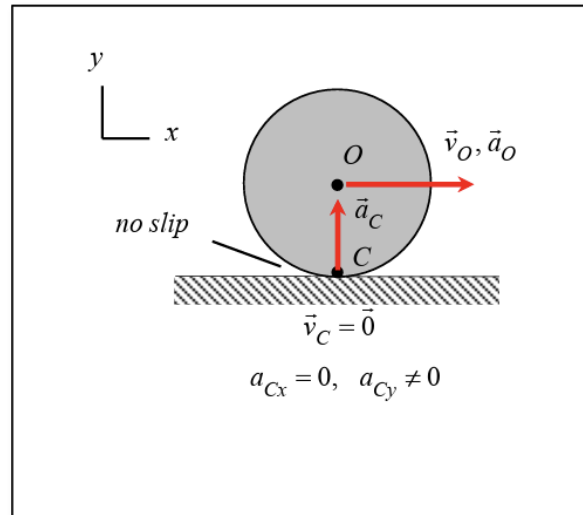
$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$



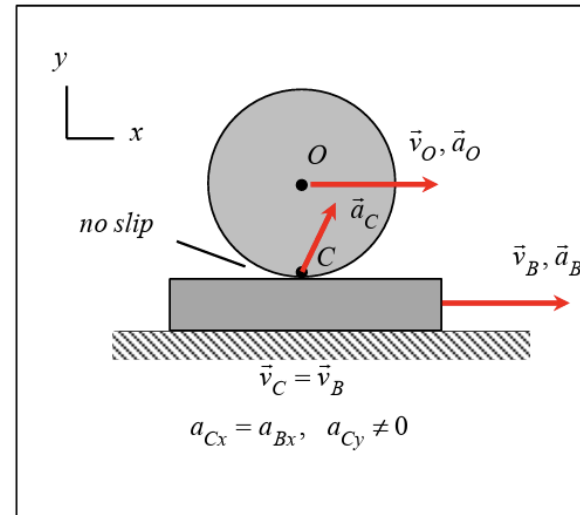
SPECIAL TOPIC: Rolling without slipping

rolling on fixed surface



[pg. 91-92]

rolling on moving surface



[pg. 92]

Lec 7 Short Feedback Form:

