ME 274 Lecture 6

Rigid Body Kinematics 1

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1/26/26

Housekeeping/Announcements

1. HW 5 is due tonight!
2. HW 6 got halved (only H2.B will be due) and postponed for Friday with HW 7's 2 problems (H2.C and H2.D)
3. I have set up Boilercast/Kaltura to give me a link to a recording of today's lecture. I will be uploading a recording of today's lecture on Brightspace and, hopefully, the course website.

    I have never done this before, so I am a little worried it could fail, but in case you are unable to come to the lecture because it's cold, I will provide this option.

4. Current plan for office hours: I will either be in the **tutorial room** or **ME2207** (one of the offices close to the ME clock/Railside)
5. There will be no quiz in today's lecture because of the cold.

***Reminder for Henny to wear a mic during the lecture.

New chapter! - *Chapter 2:* ***Planar Rigid Body*** *Kinematics*

- Review:
    - Definition of Kinematics: Branch of dynamics that describes **motion**
        - i.e. Position, velocity, and acceleration
    - Last chapter, *'Chapter 1: Particles Kinematics' we* talked about the **kinematics of particles**.

- For chapter 2, we will be expanding our understanding of kinematics to **rigid bodies.**
    - Thing to note about this chapter:
        - *All the problems in this chapter deal with points:*
            -
            -

[pg. 85 content]

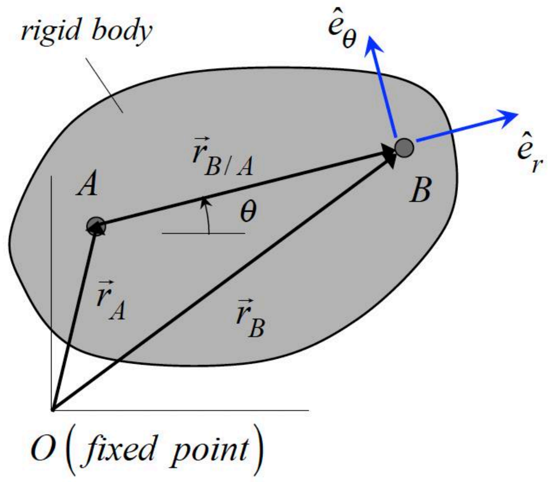
Goals for today:

1. Define what a rigid body is
2. Derive the **velocity and acceleration equations** for ***planar motion of a rigid body***

[pg. 85 content]

What is a rigid body?

- A rigid body is an object where the **distance between any two points on the object** [   ] regardless of the motion of the object.



- Although the distance between points A and B is constant, the vector's orientation angle (theta) changes with time [   ] **(reminds me of polar description)**

[pg. 86 content]

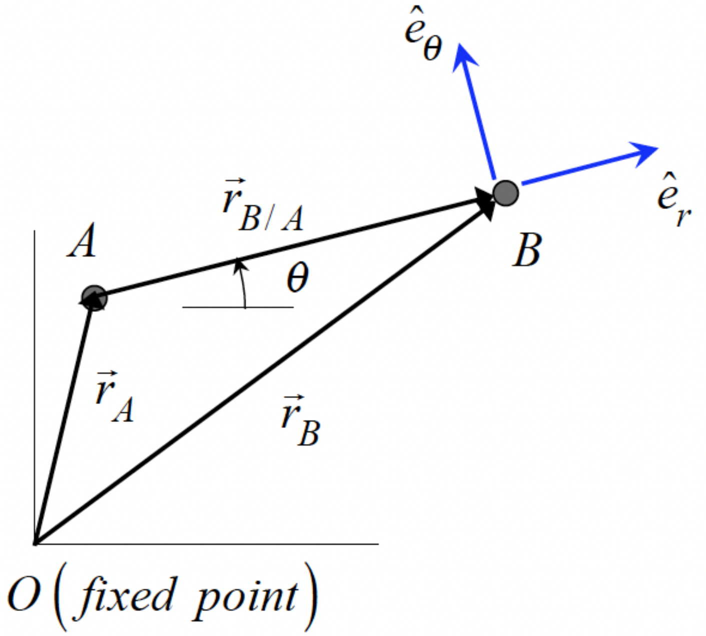
Last Lecture: Understand **Relative** and **Constrained Motion** Kinematics

- **Today, we will be building off these equations from last lecture**
- ***Relative motion kinematic equations*** of the *position, velocity, and acceleration* of two points. We can look at them in polar description too:

$$\vec{r}_{B/A} = r\ \hat{e}_r = \square$$

$$\vec{v}_{B/A} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = \square$$

$$\vec{a}_{B/A} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta = \square$$



[pg. 58 content]

Velocity equation derivation in a planar rigid body

- , since both points are on the same rigid body. With this we get:

$$\begin{aligned}\vec{v}_{B/A} &= \vec{v}_B - \vec{v}_A \\ &= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \\ &= r\dot{\theta}\hat{e}_\theta \qquad ; \\ &= r\dot{\theta}\left(\hat{k} \times \hat{e}_r\right) \qquad ; \quad \hat{e}_\theta = \hat{k} \times \hat{e}_r \quad (by\ the\ RIGHT\ HAND\ RULE) \\ &= \left(\dot{\theta}\hat{k}\right) \times (r\hat{e}_r) \\ &= \vec{\omega} \times \vec{r}_{B/A}\end{aligned}$$

- , where this is the *angular velocity* of the rigid body

[pg. 86 content]

Acceleration equation derivation in a planar rigid body

- Similar thing applies, , with acceleration…

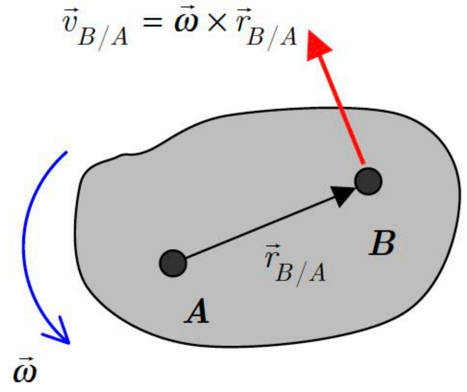
$$
\begin{aligned}
\vec{a}_{B/A} &= \vec{a}_B - \vec{a}_A \\
&= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta \\
&= \left(-r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta}\right)\hat{e}_\theta && ; \\
&= \left(r\dot{\theta}^2\right)\left[\hat{k} \times \left(\hat{k} \times \hat{e}_r\right)\right] + \left(r\ddot{\theta}\right)\left(\hat{k} \times \hat{e}_r\right) && ; \quad \hat{e}_r = -\hat{k} \times \left(\hat{k} \times \hat{e}_r\right) \\
&= \dot{\theta}\hat{k} \times \left[\left(\dot{\theta}\hat{k}\right) \times (r\hat{e}_r)\right] + \left(\ddot{\theta}\hat{k}\right) \times (r\hat{e}_r) \\
&= \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right] + \vec{\alpha} \times \vec{r}_{B/A}
\end{aligned}
$$

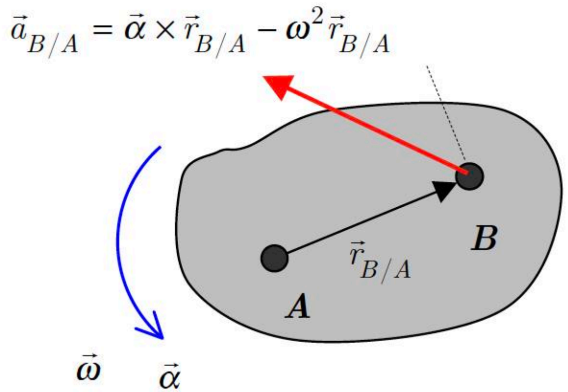
- , where this is the *angular acceleration* of the rigid body

[pg. 86 content]

Summary of things to note about *rigid body kinematic equations*

1. A rigid body has [ ] angular velocity and angular acceleration
2. The signs of w and alpha provide the sense of the rotational velocity and acceleration. Positive = [ ]; Negative = [ ] (right hand rule)
3. If A and B lie **in a plane perpendicular to the k direction** then acceleration equation simplifies to: [ ]
4. The relative velocity vector is [ ] perpendicular. The relative acceleration vector is [ ] perpendicular





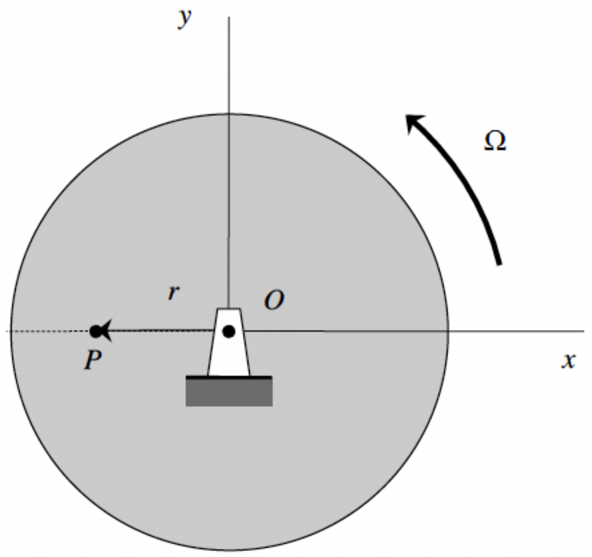
[pg. 89 for more details]

Example 2.A.1

Given: The disk shown is rotating at a non-constant rate of Ω about a fixed axis passing through its center O. At a particular instant, the acceleration vector of point P on the disk is $\vec{a}_P$.

Find: Determine:
(a) The angular velocity of the disk at this instant; and
(b) The angular acceleration of the disk at this instant.

Use the following parameters in your analysis: $\vec{a}_P = 3\hat{i} + 4\hat{j}$ m/s^2 and $r = 0.4$ m. Also, be sure to write your answers as vectors.



[pg. 93]

Example 2.A.1

p.93

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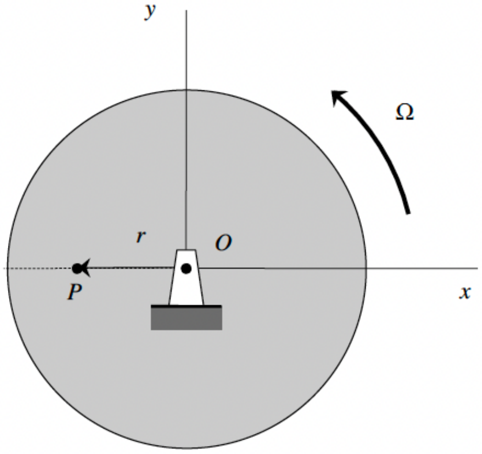


$\vec{a}_P = 3\hat{i} + 4\hat{j}\ m/s^2$

$r_{P/O} = 0.4m$

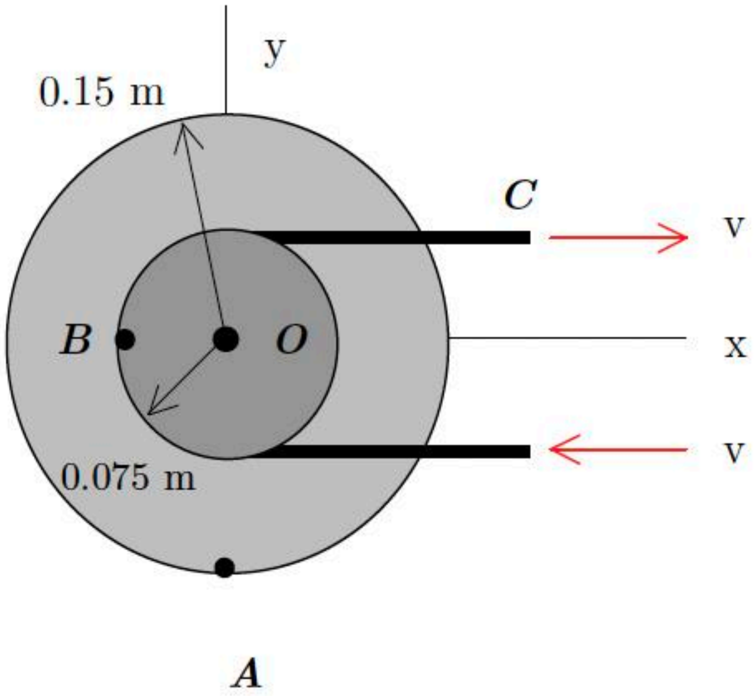
Find: (a) ω & (b) α ?

Example 2.A.2

Given: The belt-driven pulley and attached disk are rotating about a shaft passing through O. At a certain instant, the speed v of the belt is known to be 1.5 m/s and the MAGNITUDE of the acceleration of point A on the disk is 75 m/s^2. Assume that the belt does not slip on the disk.

Find: For this instant:

(a) Determine the angular acceleration of the pulley and disk;
(b) Determine the acceleration vector of point B on the disk; and
(c) Determine the acceleration of point C on the belt.



[pg. 94]

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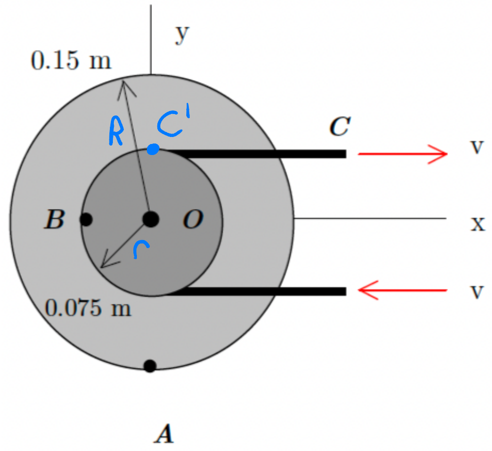
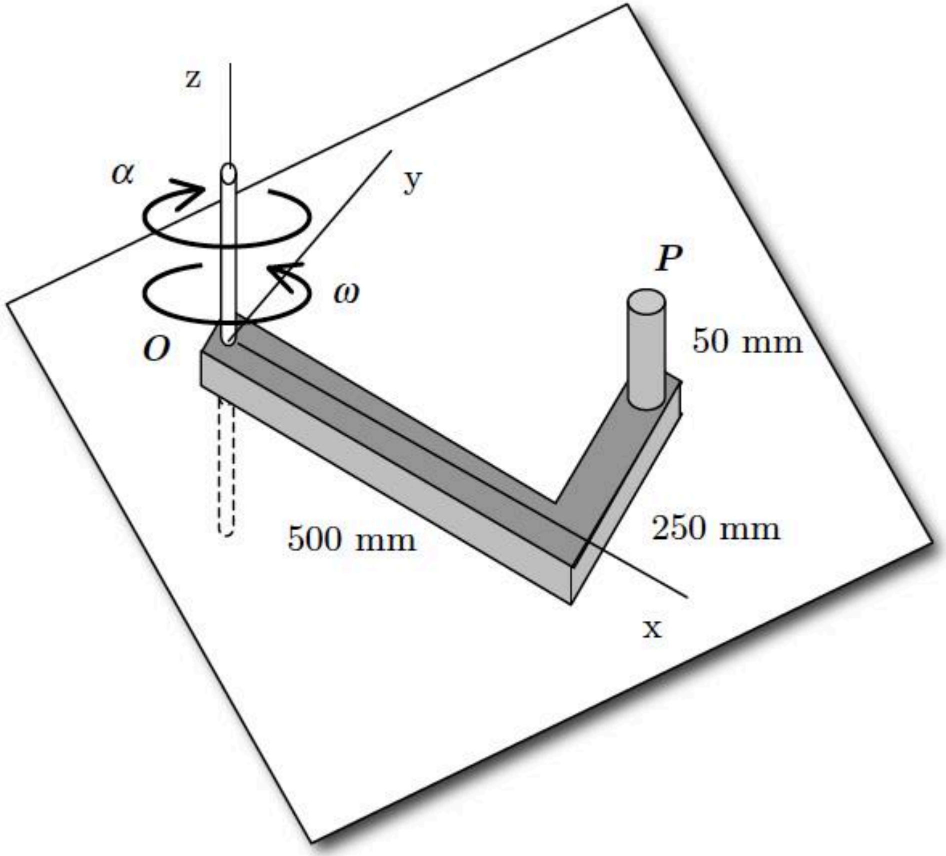


Example 2.A.3

Given: The system shown below rotates about a vertical shaft at point O, such that $\omega = 2$ rad/s and $\alpha = 3$ rad/s^2.

Find: Determine:

(a) The velocity of point P; and
(b) The acceleration of point P.



[pg. 95]

Example 2.A.3 p.95

Given: The system shown below rotates about a vertical shaft at point O, such that $\omega = 2$ rad/s and $\alpha = 3$ rad/s^2.

Find: Determine:
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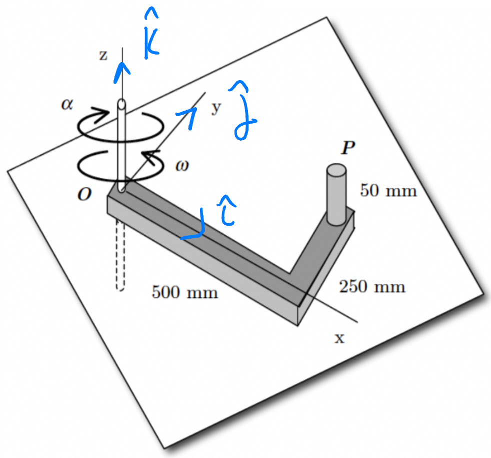
(b) The acceleration of point P.

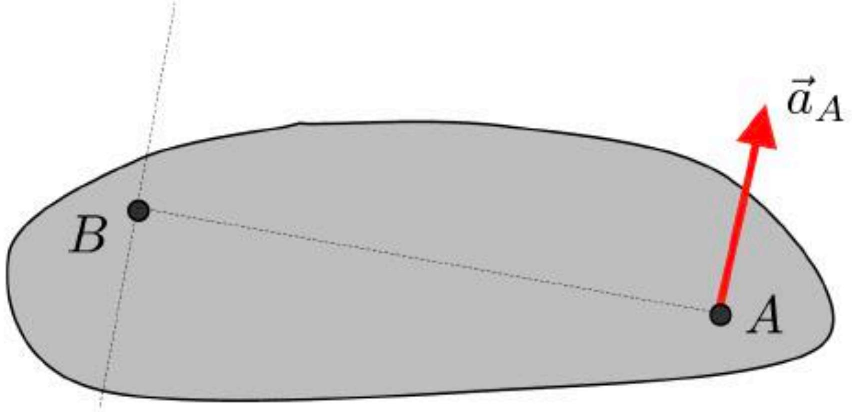


$\omega = 2$ rad/s , $\alpha = 3$ rad/s^2

relative position vector:
$\vec{r} = 0.5\hat{i} + 0.25\hat{j} + 0.05\hat{k}$ [m]

Question C2.1

The rigid body shown below has a counterclockwise angular velocity ω and a clockwise angular acceleration α. The direction of the acceleration of point A, $\vec{a}_A$, is shown in the figure with $\vec{a}_A$ being perpendicular to line AB. Make a sketch showing the direction of the acceleration vector for point B. This sketch does not need to be accurate; simply show its direction relative to the lines in the figure, where these two lines are perpendicular and parallel to $\vec{a}_A$.



[pg. 126]

Question C2.1 p.126

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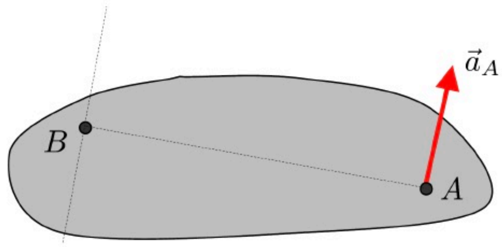


Goals:

Goals for chapter (5 lectures):

1. Velocity and acceleration equations for planar motion of a rigid body (Lectures 6-8)
2. Use instant centers to study velocity of points on a rigid body moving in a plane (Lectures 9 and 10)
    - Lecture 10 is chapter 2’s last lecture

Goal for Lecture 6 (today):

- Define what a rigid body is
- Derive the velocity and acceleration equations for planar motion of a rigid body

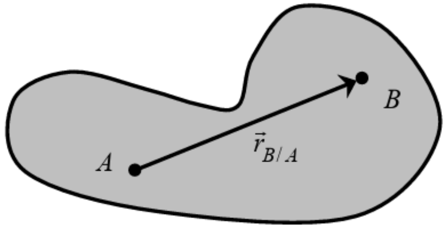
[pg. 85 content]

Summary: Rigid Body Kinematics 1

PROBLEM: Two points A and B on the same rigid body undergoing planar motion.

[top of pg. 89]

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A} \right)$$



Lec 6 Short Feedback Form:

COMMENTS:

[pg. 88]

- $\vec{\omega}$ and $\vec{\alpha}$ are the *angular velocity* and *angular acceleration* vectors of the body. These are the same for ANY two points A and B.

[pg. 58]

- $\vec{r}_{B/A}$ points *FROM* point A *TO* point B.

[pg. 85]

- If A and B lie in the same plane, then: $\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$

[pg. 85-86]

- From where did these equations come? From the general motion of two points (Chapter 1) with the constraint that $\left| \vec{r}_{B/A} \right|$.