

ME 274 Lecture 5

Particle kinematics: constrained and relative motion

Eugenio “Henny” Frias-Miranda

1/23/26

Housekeeping/Announcements

1. HW 4 due tonight!!
2. Quiz 2 today, last 10 min of class. Hand it in to me/set on the front desk, hope it goes well! :)
 - Feel free to check your answers with your peers. Please put away your laptops during quiz
3. Pi Tau Sigma ME 274 Exam review session dates:

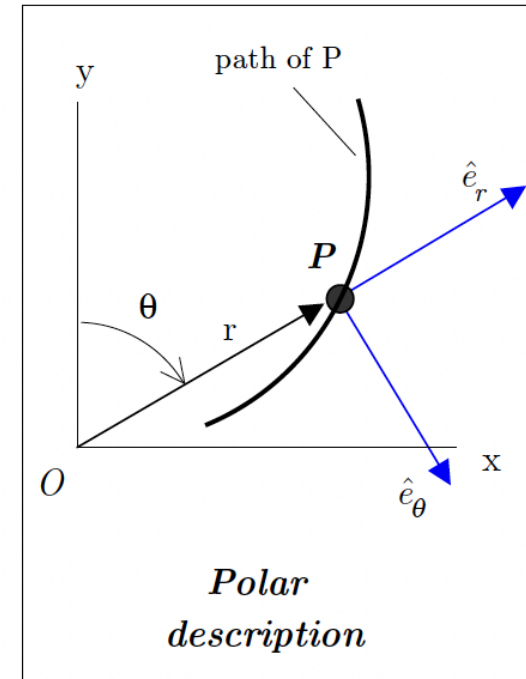
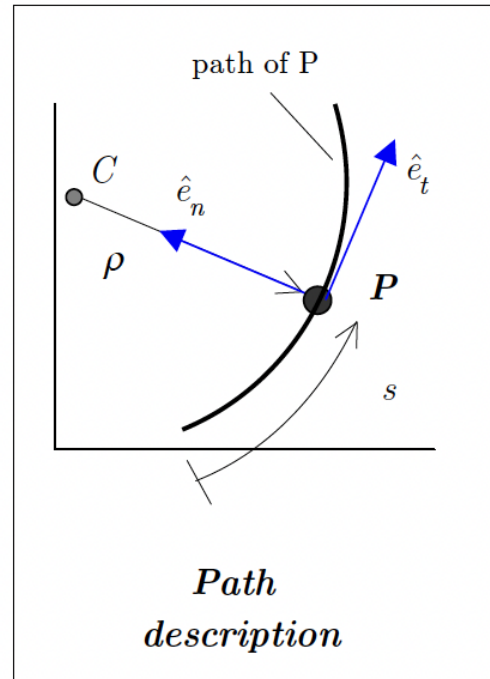
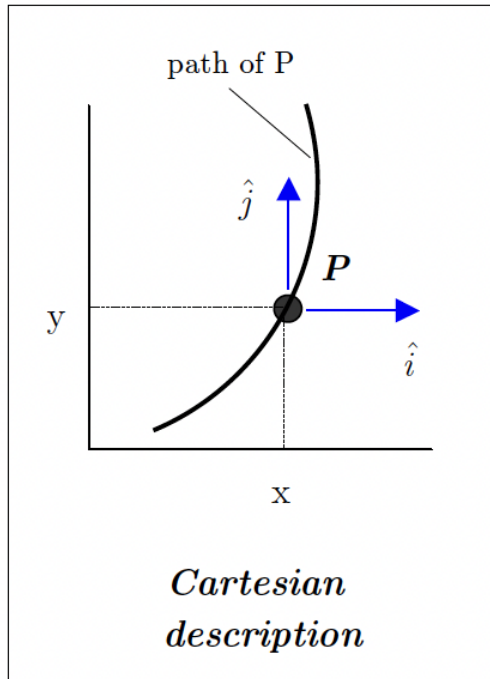
Here are the details for the review sessions we are hosting in West Lafayette for ME 274:

ME Class	Exam	Exam Date	Review Date	Review Time	Location
274	1	2/12	2/5	6:30-7:30p	WTHR 172
274	2	4/2	3/31	6:30-7:30p	WTHR 104

4. I will be sending an announcement on Brightspace/Email/Syllabus.
 - Current plan for office hours will be I will either be in the tutorial room or ME2207 (office close to ME clock)
 5. Students with accommodations, quiz policy, reach out to me after class.
 6. I usually upload my slides to the course website first, then Brightspace. I try to upload ASAP, but I usually upload 1 hour before the lecture since this gives me more time to prepare them.
 - <https://www.purdue.edu/freeform/me274/quizzes-2/>
- ***Reminder for Henny to wear a mic during the lecture.

Last lecture...

Joint Description – Combined Usage of the 3 Descriptions



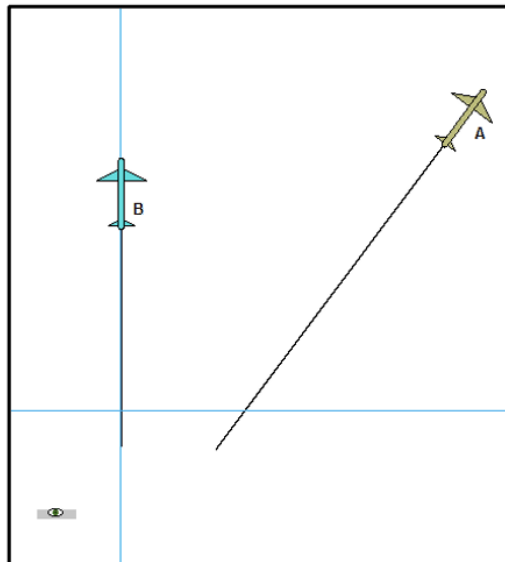
[pg. 30]

The kinematics of velocity and acceleration may be described in Cartesian, path, or polar coordinates (figure above).

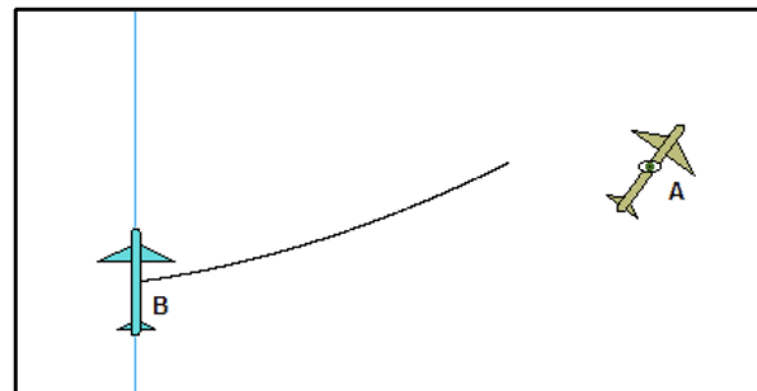
The main difference between these is how we define our unit vectors

Motivation to study Relative Motion Kinematics

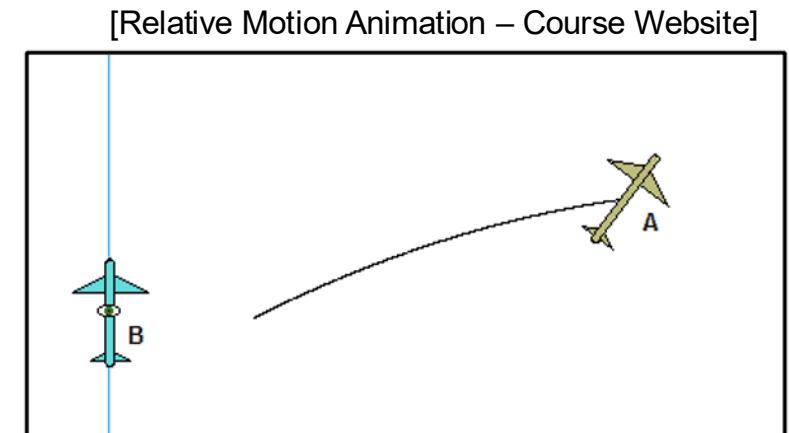
- Now that we can describe a particle's movement (i.e., $\vec{r}(t)$), a direction we can now go in is describing motion between multiple particles
 - This is known as $\vec{r}_B(t) - \vec{r}_A(t)$
 - *Definition of relative: 'considered in $\vec{r}_B(t) - \vec{r}_A(t)$ to something...'*
- In the animation below, you can see how depending on our point of view the observed path is different.
- Today, we will be deriving and using equations to relate the $\vec{r}_B(t) - \vec{r}_A(t)$ between two points in space



Stationary Observer






View of B as seen by
an observer on A



View of A as seen by
an observer on B

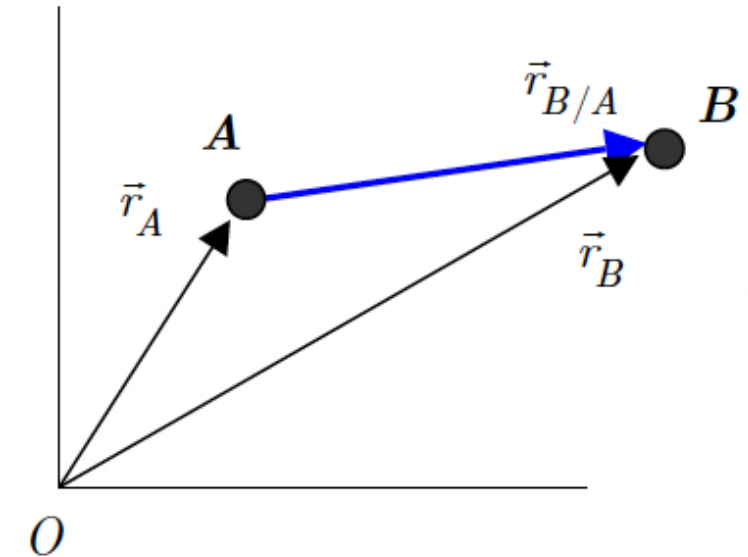


Relating one vector to another

- The vector  is the vector which points  point A  point B.
- Looking at the diagram to the right we can see that:



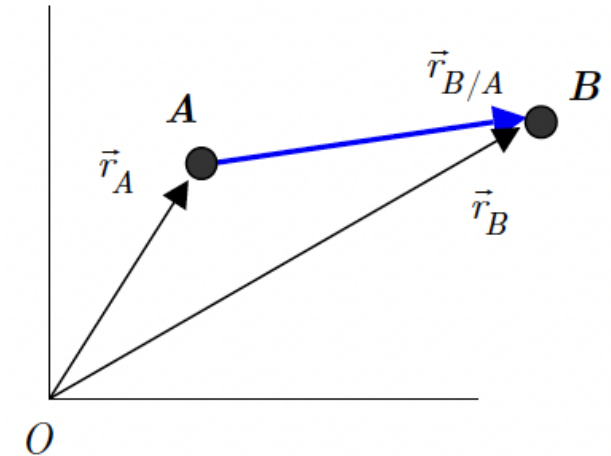
- Taking the derivative of this vector above will give us relative velocity and relative acceleration vectors of **B with respect to A**



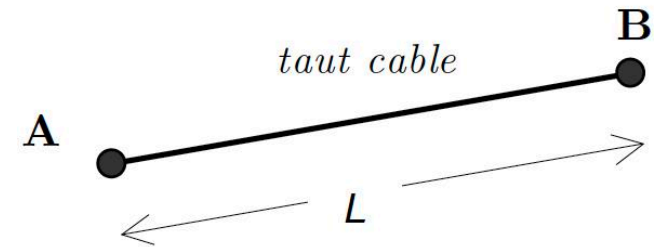
Goals of today: Understand **Relative** and **Constrained Motion** Kinematics

1. Develop the *relative motion kinematic equations* relating the *velocity* and *acceleration* of two points.

- We are going to take the derivative (twice) of the equation on the previous slide



2. Study the relative kinematics of two points whose motion are constrained by connection of *taut* (i.e. ~tight) *inextensible* cables.



Derivation of Relative Motion Kinematic Equations

1. Equation from the previous slide...



- Represents position of point B **relative to** the position of point A

2. We take the time derivative of above equation to get the velocity equation.

$$\frac{d}{dt}\vec{r}_{B/A} = \frac{d}{dt}\vec{r}_B - \frac{d}{dt}\vec{r}_A \Rightarrow$$

- Represents the velocity of B **with respect to** A

3. Again, for the acceleration equation.

$$\frac{d^2}{dt^2}\vec{r}_{B/A} = \frac{d^2}{dt^2}\vec{r}_B - \frac{d^2}{dt^2}\vec{r}_A \Rightarrow$$

- Represents the acceleration of B **with respect to** A

Tip: whenever we say with “Y with respect to X”.

X is our:

-

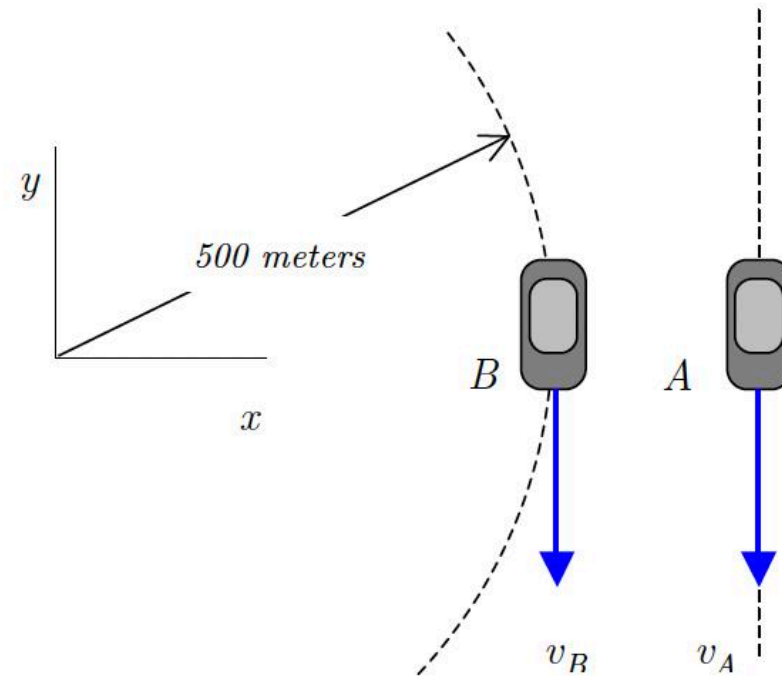
General steps for solving problems with *taut, inextensible cables*...

1. \vec{r}_i to describe motion of the various particles
2. For each cable, write an \vec{r}_i
3. Differentiate (d/dt) the above expression to get the **velocity equation**
4. Differentiate again to determine the **acceleration equation**
5. *Repeat* steps 2 through 4 **for each cable** in the system

Example 1.D.1

Given: At the instant shown, car B is traveling with a speed of 50 km/hr and is slowing down at a rate of 10 km/hr². Car A is moving with a speed of 80 km/hr, a speed that is increasing at a rate of 10 km/hr². At this instant, A and B are traveling in the same direction.

Find: What acceleration does a passenger in car A observe for car B?



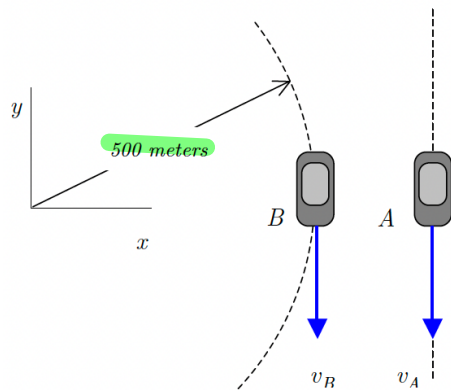
Example 1.D.1

p.61

Given: At the instant shown, car B is traveling with a speed of 50 km/hr and is slowing down at a rate of 10 km/hr². Car A is moving with a speed of 80 km/hr, a speed that is increasing at a rate of 10 km/hr². At this instant, A and B are traveling in the same direction.

Find: What acceleration does a passenger in car A observe for car B?

$$\begin{aligned} \vec{v}_B &= 50 \text{ km/hr} & \dot{v}_B &= \vec{a}_B = -10 \text{ km/hr}^2 \\ \vec{v}_A &= 80 \text{ km/hr} & \dot{v}_A &= \vec{a}_A = 10 \text{ km/hr}^2 \\ P &= 500 \text{ m} \end{aligned}$$



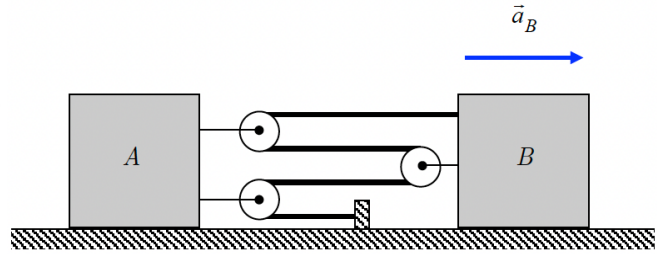
Example 1.D.3

P.63

Given: Block B has a constant acceleration of $a_B = 3 \text{ m/s}^2$ to the right. At the instant shown, B has a velocity of 2 m/s to the right.

Find: Determine:

- (a) The velocity of block A; and
- (b) The acceleration of block A.



$$v_B = 2 \text{ m/s}$$

$$a_B = 3 \text{ m/s}^2$$

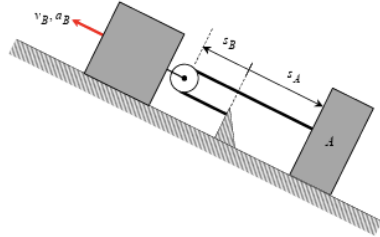
$L =$ total length of the rope i.e. constant value

s_A & $s_B =$ position/distance of A & B

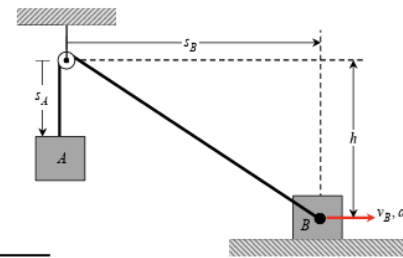
Notes
wrapped piece
around pulley
is a constant
 C_1, C_2, C_3

Summary: Particle Kinematics – Constrained and Relative Motion

PROBLEM: Two bodies connected by inextensible cable.
Write down the length of the cable in terms of motion variables and differentiate.



$$L = s_A + 2s_B + const = const \Rightarrow \dot{s}_A = -2\dot{s}_B$$



$$L = s_A + \sqrt{h^2 + s_B^2} + const = const \Rightarrow \dot{s}_A = -\frac{s_B \dot{s}_B}{\sqrt{h^2 + s_B^2}}$$

Lec 5 Short Feedback Form:



PROBLEM: The motion of one point relative to another point.

[pg. 58]

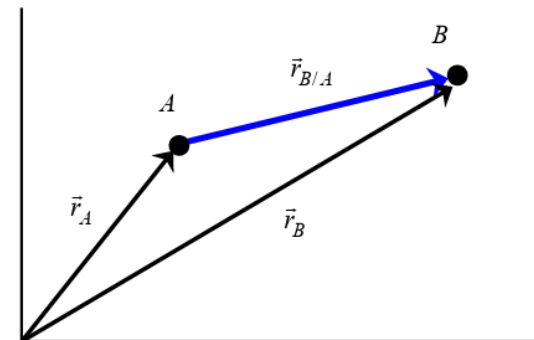
$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = \text{position of } B \text{ with respect to } A$$

[pg. 59]

$$\vec{v}_{B/A} = \frac{d\vec{r}_{B/A}}{dt} = \vec{v}_B - \vec{v}_A = \text{velocity of } B \text{ with respect to } A$$

[pg. 59]

$$\vec{a}_{B/A} = \frac{d\vec{v}_{B/A}}{dt} = \vec{a}_B - \vec{a}_A = \text{acceleration of } B \text{ with respect to } A$$



[example problems....]