ME 274 Lecture 4

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1/21/26

Feel free to call me Eugenio, Henny, or Mr. Frias.
Whatever you feel most comfortable with!!

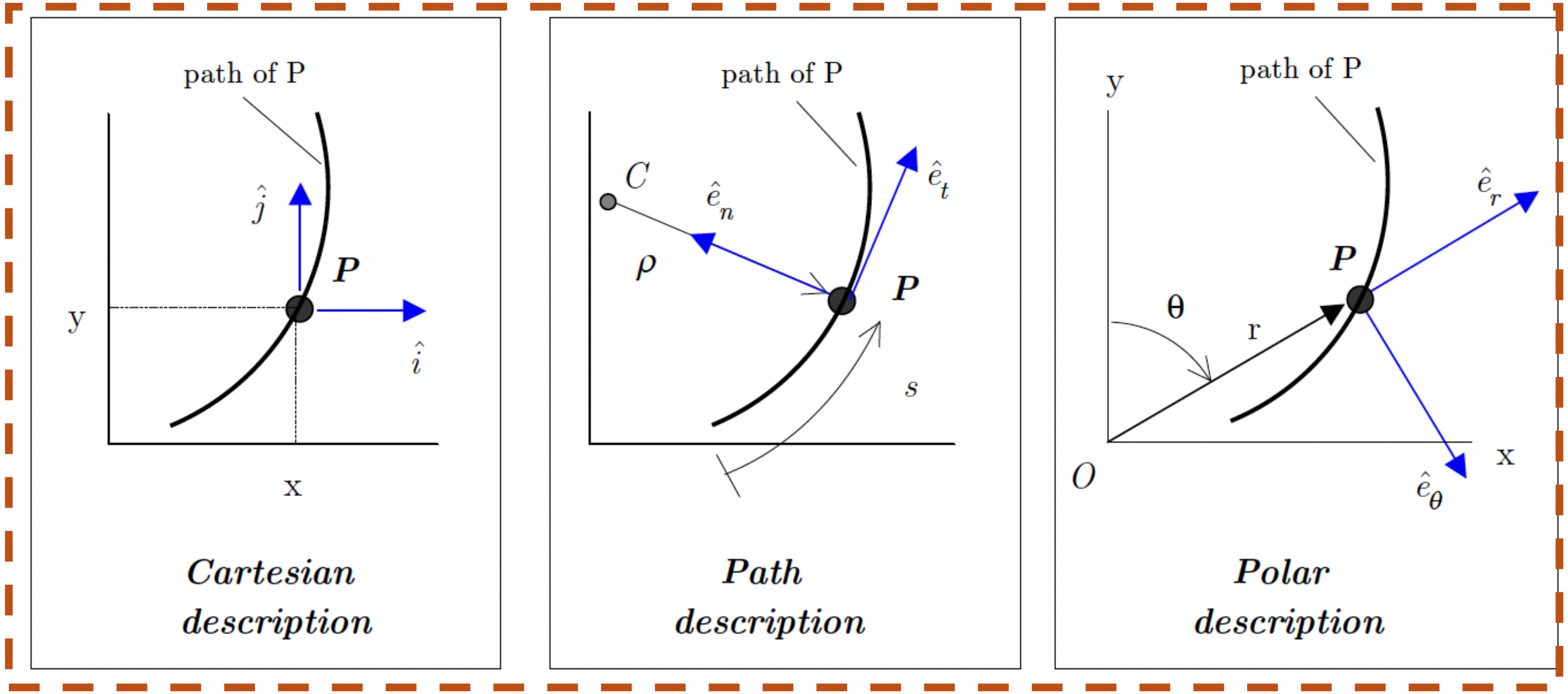
Housekeeping

- **Quiz 1 due tonight!!**
- **HW 3 due tonight!!**
- Other quizzes will be due at the end of class
- Tutorial room ME2142
- Office hours will be held after class in ME2142

(Joint as in combined/together)

High Level Overview to **Joint Description** – Combined Usage of the 3 Descriptions

Last week, we had a lecture explaining each. Today, we will be using all 3 and converting between them



[pg. 30]

The kinematics of velocity and acceleration may be described in Cartesian, path, or polar coordinates (figure above).

<u>The main difference between these is how we define our unit vectors</u>

Cartesian, Path, and Polar velocity and acceleration equations

velocity vector

$$\vec{v} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} \quad ; \quad Cartesian$$
$$= v\,\hat{e}_t \quad ; \quad path$$
$$= \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta \quad ; \quad polar$$

acceleration vector

$$\vec{a} = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} \quad ; \quad Cartesian$$
$$= \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\,\hat{e}_n \quad ; \quad path$$
$$= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta \quad ; \quad polar$$

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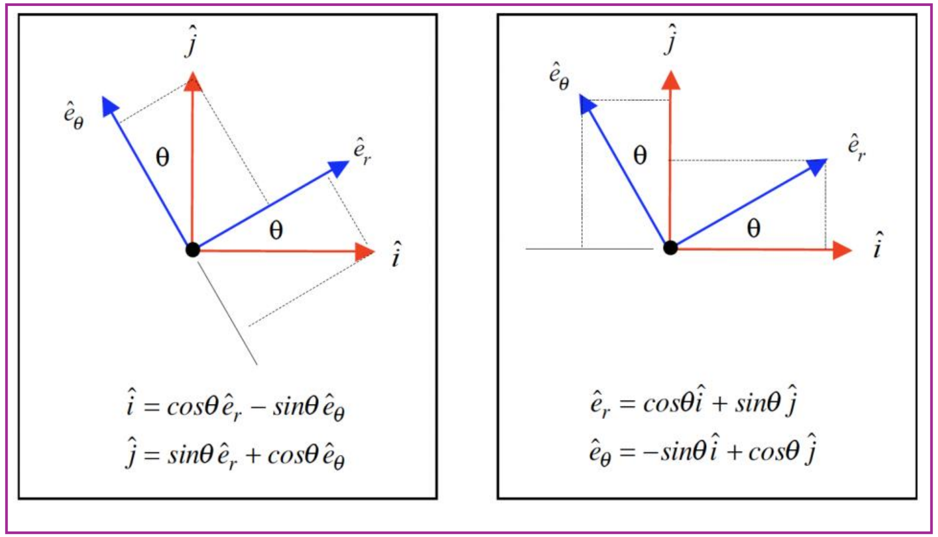
This slide is just serving as a reminder/quick reference.

Overview on how to convert between descriptions

- Two steps:
  1. Writing [   ] of *description X* in terms of *description Y* *(ie. [   ] step)*



[pg. 52]

2. Use either (i) [   ] or (ii) [   ] **methods** for *like kinematic descriptions (ie the two descriptions you just did trig for)* to solve and get the converted vector

- This is better shown in an example... next two slides

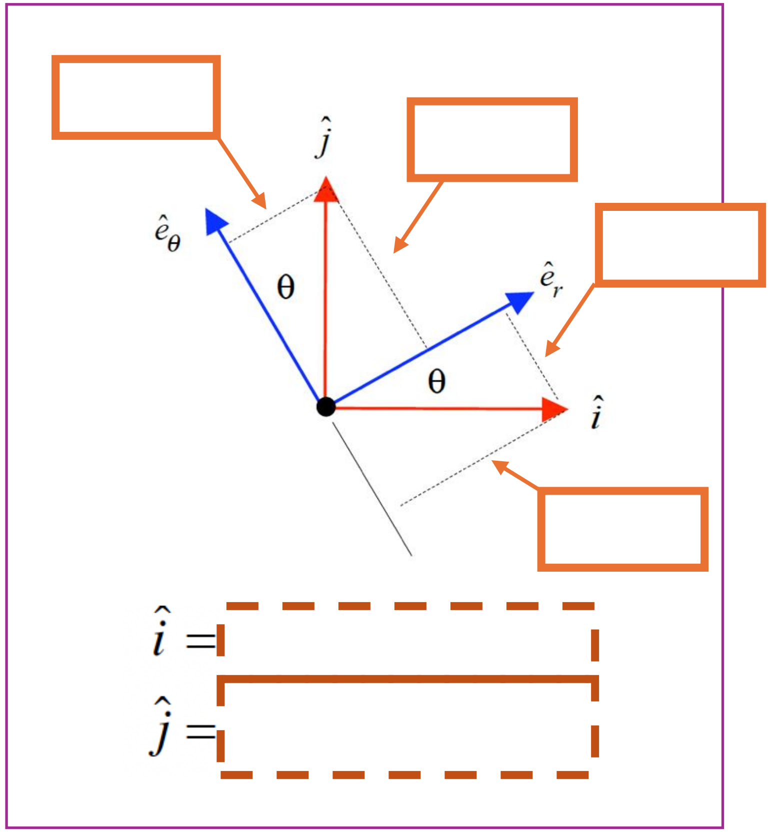
[Bottom half of pg. 51]

Motivating example 1: Polar to Cartesian

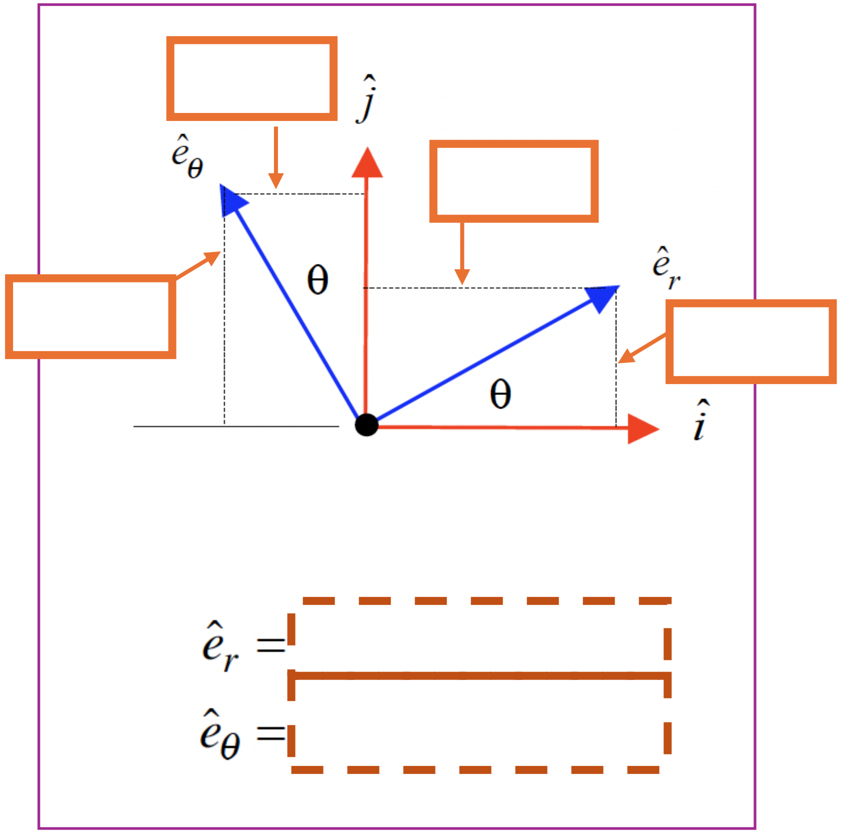
- Given: $\vec{v} = (10\hat{e}_r - 20\hat{e}_\theta)$ m/s and $\vec{a} = (3\hat{e}_r + 2\hat{e}_\theta)$ m/s^2
- Find: Cartesian components of velocity and acceleration when $\theta = 36.87°$

1. **Trigonometry Step:**

For *Projection* method:



For *Coefficient Balancing* method:



[pg. 52 content]

Motivating example 1: Polar to Cartesian (cont.)

***2. Projection* Method:**

2a. Using the equations we built from trig… :

$$\hat{i} = \cos\theta \hat{e}_r - \sin\theta \hat{e}_\theta = 0.8\hat{e}_r - 0.6\hat{e}_\theta$$

$$\hat{j} = \sin\theta \hat{e}_r + \cos\theta \hat{e}_\theta = 0.6\hat{e}_r + 0.8\hat{e}_\theta$$

2b. The vector projection of vector v onto a unit vector u is:

Where:

2c. Vector projection and solve:

$$\dot{x} = \quad = (0.8\hat{e}_r - 0.6\hat{e}_\theta) \bullet (10\hat{e}_r - 20\hat{e}_\theta) = (0.8)(10) + (-0.6)(-20) = 20 \text{ m/s}$$

$$\dot{y} = \quad = (0.6\hat{e}_r + 0.8\hat{e}_\theta) \bullet (10\hat{e}_r - 20\hat{e}_\theta) = (0.6)(10) + (0.8)(-20) = -10 \text{ m/s}$$

$$\ddot{x} = \quad = (0.8\hat{e}_r - 0.6\hat{e}_\theta) \bullet (3\hat{e}_r + 2\hat{e}_\theta) = (0.8)(3) + (-0.6)(2) = 1.2 \text{ m/s}^2$$

$$\ddot{y} = \quad = (0.6\hat{e}_r + 0.8\hat{e}_\theta) \bullet (3\hat{e}_r + 2\hat{e}_\theta) = (0.6)(3) + (0.8)(2) = 3.4 \text{ m/s}^2$$

Results:

$$\vec{v} = (20\hat{i} - 10\hat{j}) \text{ m/s}$$

$$\vec{a} = (1.2\hat{i} + 3.4\hat{j}) \text{ m/s}^2$$

[pg. 52 content]

Motivating example 1: Polar to Cartesian (cont.)

3. Coefficient Balancing **Method:**

3a. Using the equations we built from trig… :

$$\hat{e}_r = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath} = \boxed{}$$

$$\hat{e}_\theta = -\sin\theta\hat{\imath} + \cos\theta\hat{\jmath} = \boxed{}$$

3b. Balance of coefficients and solve:

$$\vec{v} = 10\hat{e}_r - 20\hat{e}_\theta = 10\boxed{} - 20\boxed{} = (20\hat{\imath} - 10\hat{\jmath})\ \mathrm{m/s}$$

$$\vec{a} = 3\hat{e}_r + 2\hat{e}_\theta = 3\boxed{} + 2\boxed{} = (1.2\hat{\imath} + 3.4\hat{\jmath})\ \mathrm{m/s^2}$$

Results: (same as projection method)

$$\vec{v} = (20\hat{\imath} - 10\hat{\jmath})\ \mathrm{m/s} \qquad \vec{a} = (1.2\hat{\imath} + 3.4\hat{\jmath})\ \mathrm{m/s^2}$$

[pg. 52 content]

Motivating example 2: Cartesian to Path

- *Given:* $\vec{v} = (30\hat{i} - 40\hat{j})$ m/s and $\vec{a} = (-10\hat{j})$ m/s^2
- *Find: speed [v] , rate of change of speed [v^dot] , radius of curvature [rho]*

1. Finding speed [v]:

$$v = \boxed{} = \sqrt{30^2 + 40^2} = 50 \text{ m/s}$$

2. Finding rate of change of speed [v^dot]:

$$\vec{v} = \boxed{} \Rightarrow \quad \hat{e}_t = \boxed{} = \frac{30\hat{i} - 40\hat{j}}{50} = 0.6\hat{i} - 0.8\hat{j}$$

$$\dot{v} = \boxed{} = (0.6\hat{i} - 0.8\hat{j}) \bullet (-10\hat{j}) = 8 \text{ m/s}^2$$

3. Finding radius of curvature:

$$\vec{a} = \boxed{} \quad \Rightarrow \quad |\vec{a}|^2 = \boxed{} \quad \Rightarrow \quad \rho = \frac{v^2}{\sqrt{|\vec{a}|^2 - \dot{v}^2}} = \frac{50^2}{\sqrt{10^2 - 8^2}} = \frac{2500}{6} \text{ m}$$

4. Takeaway:
Why are we highlighting this problem?

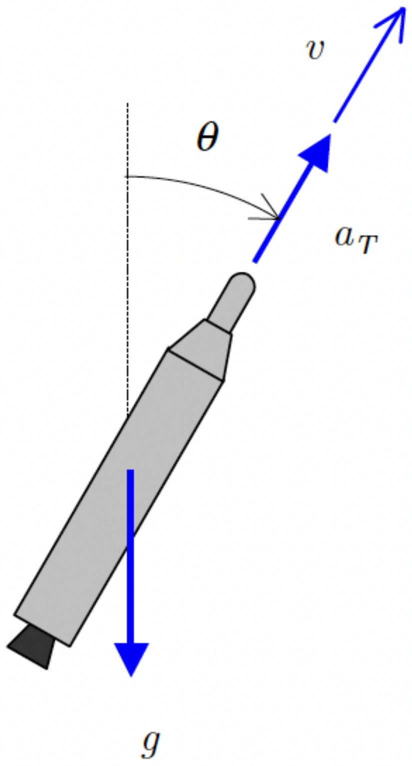
- This relationship is useful in problems involving path coordinates

[pg. 53 content]

Example 1.C.1

Given: A rocket is traveling at an altitude at which the gravitational acceleration is known to be $g = 26.5\ \text{ft/s}^2$. The thrust force on the rocket produces an acceleration of $a_T = 29.3\ \text{ft/s}^2$ along the axis of the rocket. At the position shown ($\theta = 36.87°$) the speed of the rocket is known to be $v = 2800\ \text{ft/s}$.

Find: Determine:
(a) The rate of change of speed of the rocket at this instant; and
(b) The radius of curvature for the rocket's path at this instant.



[pg. 54]

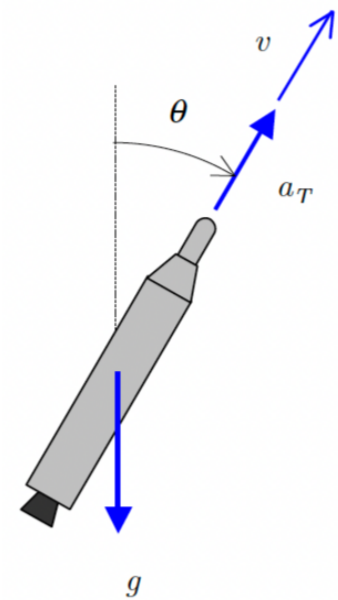
Example 1.C.1 p.54

Given: A rocket is traveling at an altitude at which the gravitational acceleration is known to be $g = 26.5$ ft/s^2. The thrust force on the rocket produces an acceleration of $a_T = 29.3$ ft/s^2 along the axis of the rocket. At the position shown ($\theta = 36.87°$) the speed of the rocket is known to be $v = 2800$ ft/s.

Find: Determine:
(a) The rate of change of speed of the rocket at this instant; and
(b) The radius of curvature for the rocket's path at this instant.

$a_T = 29.3$ ft/s^2 $\theta = 36.87°$

$g = 26.5$ ft/s^2 $v = 2800$ ft/s



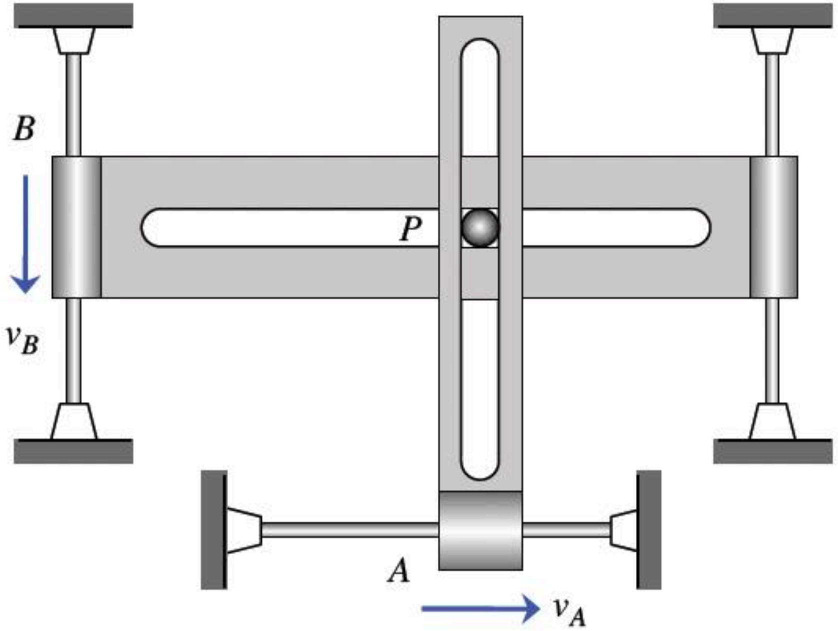
Example 1.C.2

Given: Pin P is constrained to move in the slotted guides that move at right angles to one another. At the instant shown, guide A moves to the right with a speed of v_A, a speed that is changing at a rate of $\dot{v}_A$. At the same time, B is moving downward with a speed of v_B with a rate of change of speed of $\dot{v}_B$.

Find:

(a) The rate of change of speed of P at this instant; and
(b) The radius of curvature ρ of the path followed by P at this instant.

Use the following parameters: $v_A = 0.2$ m/s, $v_B = 0.15$ m/s, $\dot{v}_A = 0.75$ m/s^2 and $\dot{v}_B = 0$.



[pg. 55]

Example 1.C.2

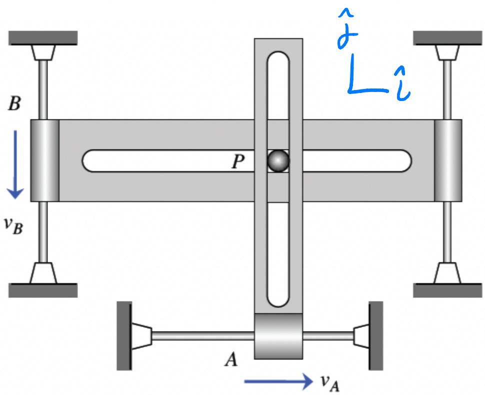
p.55

Given: Pin P is constrained to move in the slotted guides that move at right angles to one another. At the instant shown, guide A moves to the right with a speed of v_A, a speed that is changing at a rate of $\dot{v}_A$. At the same time, B is moving downward with a speed of v_B with a rate of change of speed of $\dot{v}_B$.

Find:
(a) The rate of change of speed of P at this instant; and $\dot{v}$
(b) The radius of curvature ρ of the path followed by P at this instant. ρ

Use the following parameters: $v_A = 0.2$ m/s, $v_B = 0.15$ m/s, $\dot{v}_A = 0.75$ m/s^2 and $\dot{v}_B = 0$.

$v_A = 0.2$ m/s, $\dot{v}_A = 0.75$ m/s^2
$v_B = 0.15$ m/s, $\dot{v}_B = 0$ m/s^2



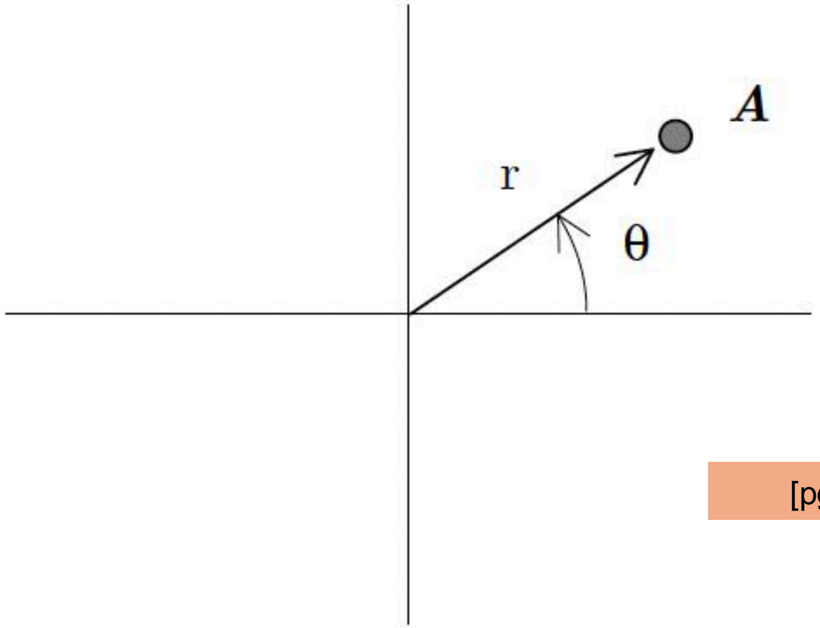
Example 1.C.3

Given: The path of point P is given in polar coordinates as: $r = 4 + 2\theta$, where r is given in meters and θ is given in radians. The angle θ is increasing at a constant rate of $\dot{\theta}$.

Find: Determine:

(a) The acceleration vector of P when $\theta = \pi$, if $\dot{\theta} = 3$ rad/s; and

(b) If the speed of P increasing, decreasing or constant?



[pg. 56]

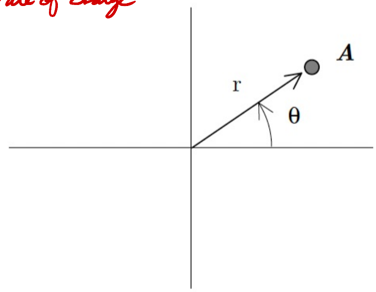
Example 1.C.3 p. 56

Given: The path of point P is given in polar coordinates as: $r = 4 + 2\theta$, where r is given in meters and θ is given in radians. The angle θ is increasing at a constant rate of $\dot{\theta}$.

Find: Determine:

(a) The acceleration vector of P when $\theta = \pi$, if $\dot{\theta} = 3$ rad/s; and

(b) If the speed of P increasing, decreasing or constant? ~ what is the sign of rate of change of speed (ie $\hat{e}_t$)?



$r = 4 + 2\theta$

$\theta = \pi$

$\dot{\theta} = 3$ rad/s (constant)

$\ddot{\theta} = 0$

$r = 4 + 2\theta = 4 + 2\pi$

chain rule

$\dot{r} = 2\dot{\theta} = (2)(3) = 6$

$\ddot{r} = 2\ddot{\theta} = 0$

Eqns:

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = v\hat{e}_t$$

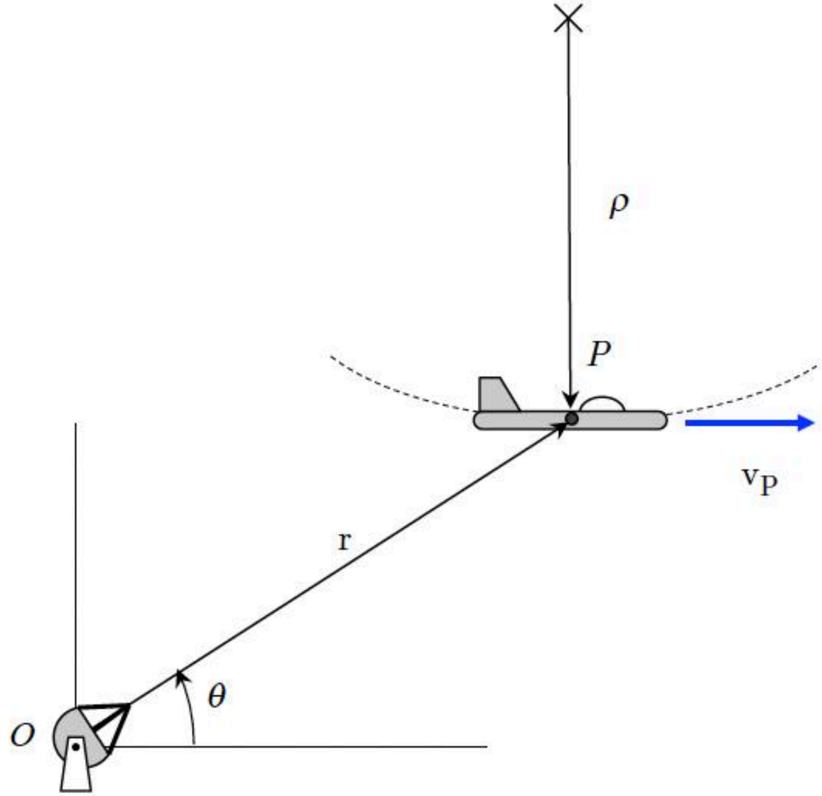
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Example 1.C.4

Given: At the bottom of a loop, an airplane P has a constant speed of v_P with the radius of curvature for the aircraft being ρ. The airplane is at a radial distance of r and at an angle of θ from a radar tracking station at O.

Find: Determine numerical values for $\ddot{r}$ and $\ddot{\theta}$ at this instant in time.

Use the following: $v_P = 75$ m/s, $\rho = 3000$ m, $r = 1000$ m and $\theta = 36.87°$.



[pg. 57]

Example 1.C.4

p. 57

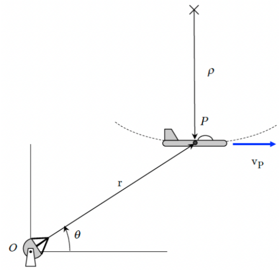
Given: At the bottom of a loop, an airplane P has a constant speed of v_P with the radius of curvature for the aircraft being ρ. The airplane is at a radial distance of r and at an angle of θ from a radar tracking station at O.

Find: Determine numerical values for $\ddot{r}$ and $\ddot{\theta}$ at this instant in time.

Use the following: $v_P = 75$ m/s, $\rho = 3000$ m, $r = 1000$ m and $\theta = 36.87°$.

$v = 75$ m/s
$\rho = 3000$ m
$r = 1000$ m
$\theta = 36.87°$

$\cos\theta = 0.8$
$\sin\theta = 0.6$

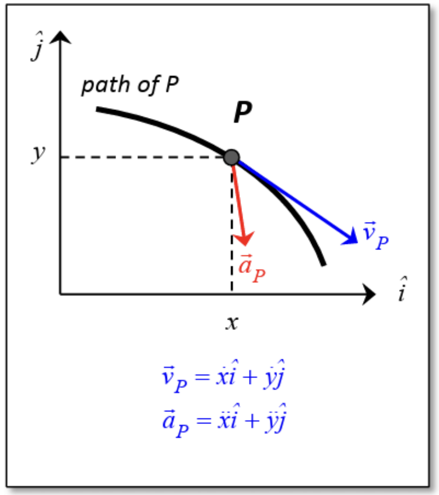


Summary: Particle Kinematics – Joint Description

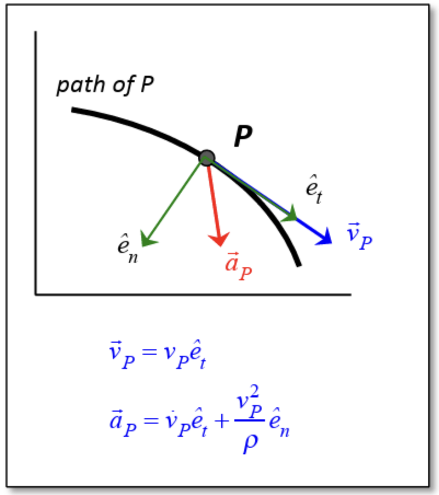
PROBLEM: Motion of a point described in a combination of descriptions.

[pg. 30 & 51]

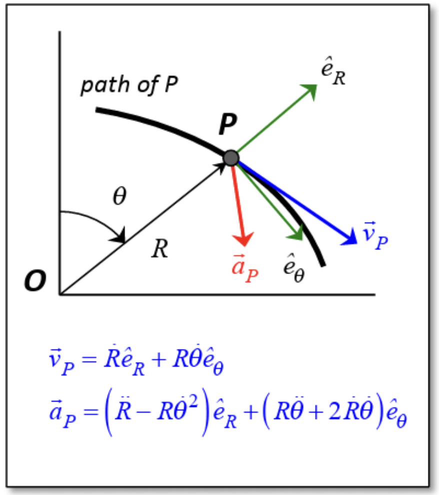
Cartesian



Path



Polar



Lec 4 Short Feedback Form:

SOLUTION: Projection of vector onto different set of unit vectors. Examples:

$$\dot{x} = \vec{v}_P \bullet \hat{i}$$

$$\ddot{x} = \vec{a}_P \bullet \hat{i}$$

[pg. 53, bottom half]

$$\dot{v}_P = \vec{a}_P \bullet \hat{e}_t = \vec{a}_P \bullet \left(\frac{\vec{v}_P}{\left|\vec{v}_P\right|}\right)$$

$$\frac{v_P^2}{\rho} = \vec{a}_P \bullet \hat{e}_n$$

[~pg. 53, motivating example]

$$\dot{R} = \vec{v}_P \bullet \hat{e}_R \qquad R\dot{\theta} = \vec{v}_P \bullet \hat{e}_\theta$$

$$\ddot{R} - R\dot{\theta}^2 = \vec{a}_P \bullet \hat{e}_R \qquad R\ddot{\theta} + 2\dot{R}\dot{\theta} = \vec{a}_P \bullet \hat{e}_\theta$$

[pg. 53, bottom half]