Example 1.A.1

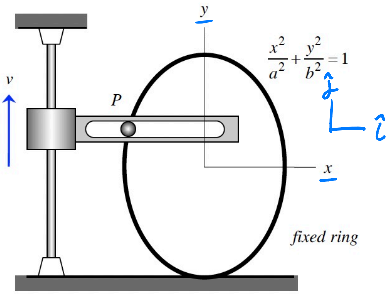
Given: Pin P is constrained to move along a elliptical ring whose shape is given by $x^2/a^2+y^2/b^2=1$ (where x and y are given in mm). The pin is also constrained to move within a horizontal slot that is moving upward at a constant speed of v.

Find: Determine:

(a) The velocity of pin P at the position corresponding to $y = 6$ mm; and

(b) The acceleration of pin P at the position corresponding to $y = 6$ mm.

Use the following parameters in your analysis: $a = 5$ mm, $b = 10$ mm, $v = 30$ mm/s.



fund. Equ. $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$; $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$

$\dot{y} = v$; $\ddot{y} = 0$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \rightarrow x$ solve for

$\Rightarrow \frac{d}{dt}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=1$

$\frac{dx^2}{dx}\frac{dx}{dt} + \frac{dy^2}{dy}\frac{dy}{dt} \leftarrow$ chain rule pg. 33

$\Rightarrow \frac{2x\dot{x}}{a^2}+\frac{2y\dot{y}}{b^2}=0$

$\Rightarrow 2x\dot{x}b^2 + 2y\dot{y}a^2 = 0$

$\Rightarrow \frac{d}{dt}(x\dot{x}b^2 + y\dot{y}a^2)=0 \Rightarrow \dot{x}$ solve for

product rule

$\dot{x}^2b^2 + x\ddot{x}b^2 + \underbrace{\dot{y}^2a^2 + \cancel{y\ddot{y}}a^2}_{\text{GIVEN}} = 0 \Rightarrow \ddot{x}$ solve for

Example 1.A.2

Given: A particle P moves on a path whose Cartesian components are given by the following functions of time (where both components are given in inches and time t is given in seconds):

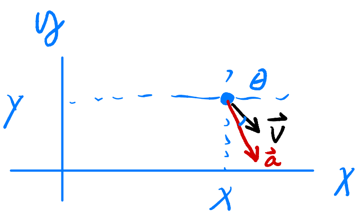
$$x(t) = t^3 + 10$$
$$y(t) = 2\cos 4t$$

Find: Determine at the time $t = 2$ s:
(a) The velocity vector of P;
(b) The acceleration of P; and
(c) The angle between the velocity and acceleration vectors of P.

$\boxed{\text{fundamental Eqns:} \quad \text{(a)}\ \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} \;;\quad \text{(b)}\ \vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}}$

$$x(t) = t^3 + 10$$
$$\dot{x} = 3t^2$$
$$\ddot{x} = 6t$$

$$y(t) = 2\cos 4t$$
$$\dot{y} = -2(4)\sin 4t = -8\sin 4t$$
$$\ddot{y} = -32\cos 4t$$



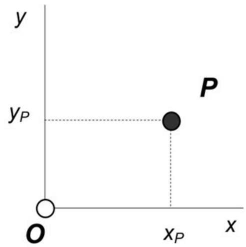
Remember... dot product:

$$\vec{v}\cdot\vec{a} = |\vec{v}|\,|\vec{a}|\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{a}}{|\vec{v}|\,|\vec{a}|}\right) = \cos^{-1}\left(\frac{\dot{x}\ddot{x} + \dot{y}\ddot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}\sqrt{\ddot{x}^2+\ddot{y}^2}}\right)$$

Question C1.2

Point P moves on a path described by $y_P = x_P^2/2$ with $x_P(t) = 3\sin \pi t$, where x_P and y_P have units of meters and t has units of seconds. Determine the acceleration vector of P at $t = 0$.



(chain rule)

$$\dot{y}_P = \frac{dy_P}{dt} = \frac{dy_P}{dx}\frac{dx}{dt} = \frac{dy_P}{dx}\dot{x}$$

(product rule)

$$\ddot{y}_P = \frac{d\dot{y}_P}{dt} = \ddot{x}\frac{dy_P}{dx} + \underbrace{\dot{x}}_{\frac{dx}{dt}}\underbrace{\frac{d}{dt}\left(\frac{dy_P}{dx}\right)}_{\frac{d^2y_P}{dx^2}\frac{dx}{dt}}$$

$$= \ddot{x}\frac{dy_P}{dx} + \dot{x}^2\frac{d^2y_P}{dx^2}$$

we take derivative of $\frac{dy_P}{dx}$ which is d^2y_P on numerator but we do "$\frac{\ }{dx^2}\,\frac{dx}{dy}$" to keep in terms of dt

$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$

For $\hat{j}$:
$\dot{y}_P = x_P$
$\ddot{y}_P = 1$

For $\hat{i}$:
$\dot{x} = 3\pi\cos(\pi t)$ $\quad \ddot{x} = -3\pi^2\sin(\pi t)$

Evaluate @ $t=0$:

$x(0)=0$, $\dot{x}(0) = 3\pi$, $\ddot{x}(0) = 0$

Evaluate $\ddot{y}(0)$:

$$\ddot{y}(0) = \ddot{x}(0)\,x(0) + \dot{x}(0)^2(1) = 0\cdot 0 + (3\pi)^2 = 9\pi^2$$

$$\boxed{\vec{a}(0) = 0\,\hat{i} + 9\pi^2\,\hat{j} \ \ \mathrm{m/s^2}}$$