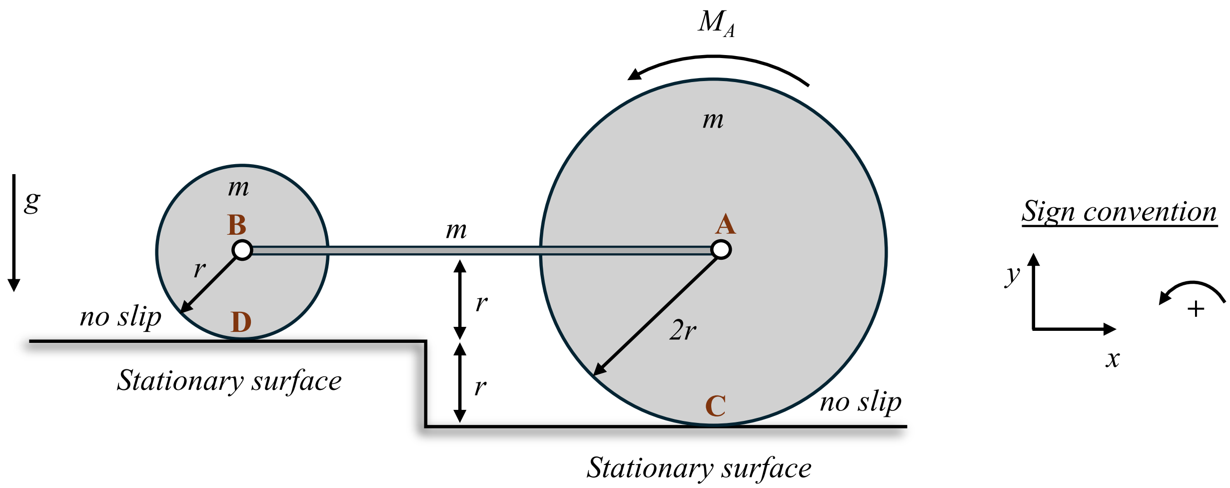
ME 274: Basic Mechanics II **Name** *(print)* SOLUTION

Final Exam - Spring 2025

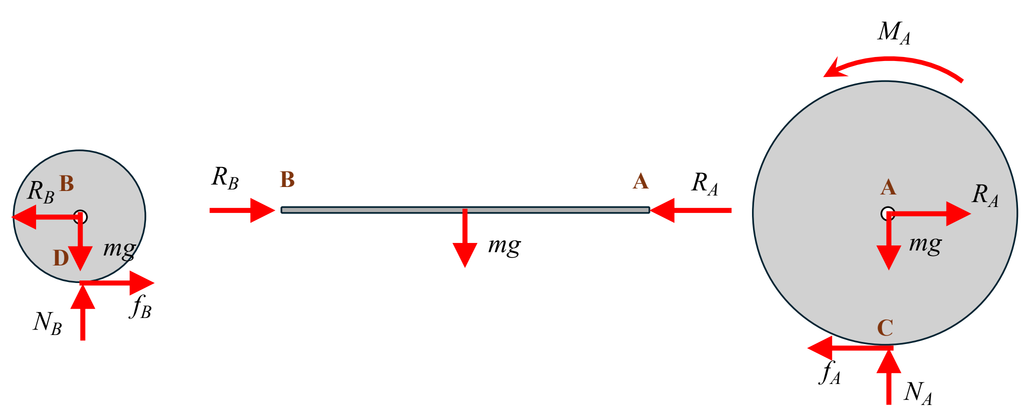
***PROBLEM NO. 1** (20 points) – PLEASE SCAN ALL PAGES*

A system consists of two homogeneous circular discs A and B. Disc A has mass m and radius $2r$. Disc B has mass m and radius r. A homogeneous, thin rigid bar of mass m connects the centers of discs A and B.

Disc A is subjected to an external counterclockwise moment M_A, causing it to roll without slipping on a stationary surface. The system is released from rest. Determine the following at this instant:



(a) Draw FBDs of disc B, bar AB, and disc A. Forces should be indicated at the correct location. (*no partial credit within each FBD*)



(b) Solve the Newton-Euler equations and kinematics constraints for the entire system using given the sign convention and determine the following:

(i) Angular acceleration of A, α_A

(ii) Angular acceleration of B, α_B

To do this, provide your answers for the steps below. For full credit, <u>your answers must be written as vectors and in terms of at most m, r and M_A.</u>

Newton-Euler Equations for **disc A**:

$\Sigma M_c = I_c \alpha_A$

$M_A - R_A(2r) = \left(\frac{m(2r)^2}{2} + m(2r)^2\right)\alpha_A = 6mr^2\alpha_A$ … (1)

Newton-Euler Equations for **bar AB**:

$\Sigma F_x = ma_{AB}$

$R_B - R_A = ma_{AB}$ … (2)

Newton-Euler Equations for **disc B**:

$\Sigma M_D = I_D \alpha_B$

$R_B r = \left(\frac{mr^2}{2} + mr^2\right)\alpha_B = \frac{3}{2}mr^2\alpha_B$ … (3)

Kinematic Constraints:

Disc A

$\boldsymbol{a_A} = \boldsymbol{a_C} + \boldsymbol{\alpha_A} \times \boldsymbol{r_{AC}} - \omega^2 \boldsymbol{r_{AC}}$

$a_A \boldsymbol{i} = a_C\, \boldsymbol{j} + \alpha_A \boldsymbol{k} \times 2r\, \boldsymbol{j} = a_C\, \boldsymbol{j} - 2\alpha_A r\, \boldsymbol{i}$

Therefore, $\boxed{a_A = -2\alpha_A r}$ … (4)

Disc B

$\boldsymbol{a_B} = \boldsymbol{a_D} + \boldsymbol{\alpha_B} \times \boldsymbol{r_D} - \omega^2 \boldsymbol{r_{BD}}$

$a_B \boldsymbol{i} = a_D\, \boldsymbol{j} + \alpha_B \boldsymbol{k} \times r\, \boldsymbol{j} = a_D\, \boldsymbol{j} - \alpha_B r\, \boldsymbol{i}$

Therefore, $\boxed{a_B = -\alpha_B r}$ … (5)

Bar AB

$\boxed{a_A = a_B = a_{AB}}$… (6)

Solve the system of equations for α_A and α_B:

Solve (4)-(6): $\alpha_B = 2\alpha_A = -\frac{a_{AB}}{r}$

(1): $R_A = \frac{M_A}{2r} - 3mr\alpha_A$

(3): $R_B = \frac{3}{2}mr\alpha_B = 3mr\alpha_A$

(2): $R_B - R_A = 3mr\alpha_A - \left(\frac{M_A}{2r} - 3mr\alpha_A\right) = 6mr\alpha_A - \frac{M_A}{2r} = ma_{AB} = -2mr\alpha_A$

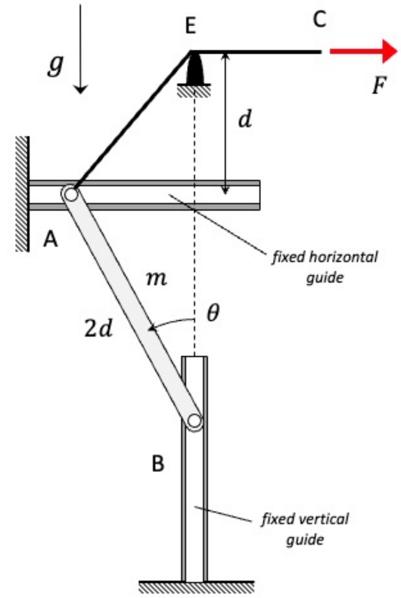
Simplifying, $8mr\alpha_A = \frac{M_A}{2r}$.

Therefore, $\boxed{\boldsymbol{\alpha_A} = \frac{M_A}{16mr^2}\boldsymbol{k}}$.

and $\boxed{\boldsymbol{\alpha_B} = \frac{M_A}{8mr^2}\boldsymbol{k}}$

ME 274: Basic Mechanics II
Exam 2 - Spring 2025
***PROBLEM NO. 2** (20 points) – PLEASE SCAN ALL PAGES*

SOLUTION



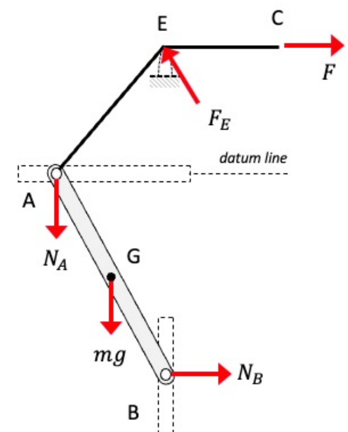
Given: A thin, rigid homogeneous bar AB has a mass of m and a length of $2d$. Ends A and B are constrained to move along *smooth* horizontal and vertical guides, as shown in the figure. An inextensible cable is attached to end A of the bar and is pulled over a smooth guide E by a *constant* force F acting in a horizontal direction to the right. At the initial state, $\theta = \theta_0 = 60°$, with the system being released from rest.

Find: It is desired to know the speed of A at its position when $\theta = 0$. To this end, please complete the following solution steps. *For full credit, you must provide the correct responses within the correct steps provided below.* In addition, you must justify all steps in your work.

Solution:

<u>*STEP 1*</u>: ***Free body diagram (FBD)***

Complete the FBD provided of the system made up of bar AB and the cable. Define the location of any gravitational datum lines that will be needed for the system.



<u>NOTE</u>: Only F does work on the system, since the other forces are perpendicular to the paths of points on which they act, or their work will be included in the potential energy.

<u>*STEP 2*</u>: ***Kinetics***

Write down the appropriate kinetics equation(s) and expressions for the terms in these equation(s). More space is provided on the next page. Do not do kinematics at this point…that is in the next step.

$$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$$

where:

$T_1 = 0$; I.A.R.

$T_2 = \frac{1}{2} m v_{G2}^2 + \frac{1}{2} I_G \omega_2^2$

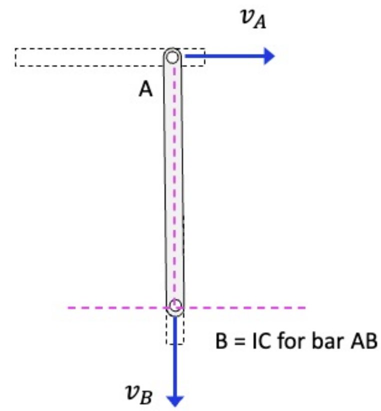
$V_1 = -mgh_{G1}$

$V_2 = -mgh_{G2}$

$U_{1\to2}^{(nc)} = \int_1^2 (\vec{F} \cdot \hat{e}_t) ds_C = F\Delta_C$

Therefore:

$$-mgh_{G1} + F\Delta_C = \frac{1}{2} m v_{G2}^2 + \frac{1}{2} I_G \omega_2^2 - mgh_{G2}$$



STEP 3: Kinematics
Write down the kinematics equations that will be needed to solve this problem.

At Position 2, the IC for AB is at point B. Therefore:

$$v_{B2} = 0$$
$$v_{G2} = v_{A2}/2$$
$$\omega_2 = v_{A2}/2d$$

Also:

$$h_{G1} = d\cos\theta_0 \ = d\ /\ 2$$
$$h_{G2} = d$$
$$\Delta_C = \sqrt{d^2 + (2d\sin\theta_0)^2} - d = d$$

STEP 4: Solve
Using the equations from STEPS 2 and 3, determine the speed of A for the position corresponding to $\theta = 0$. Write your final answer in terms of, *at most*: F, m, d and g.

Substituting the kinematics into the W/E equation gives:

$$-\frac{mgd}{2} + Fd = \frac{1}{2}m\left(\frac{v_{A2}}{2}\right)^2 + \frac{1}{2}\left[\frac{m(2d)^2}{12}\right]\left(\frac{v_{A2}}{2d}\right)^2 - mgd = \left(\frac{1}{6}m\right)v_{A2}^2 - mgd \ \Rightarrow$$

$$v_{A2} = \sqrt{\frac{3d(2F + mg)}{m}}$$

ME 274: Basic Mechanics II **Name** *(print)* SOLUTION

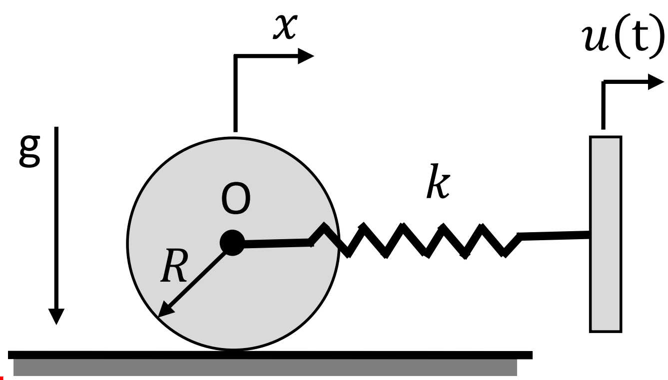
Final Exam - Spring 2025

***PROBLEM NO. 3** (20 points) – PLEASE SCAN ALL PAGES*

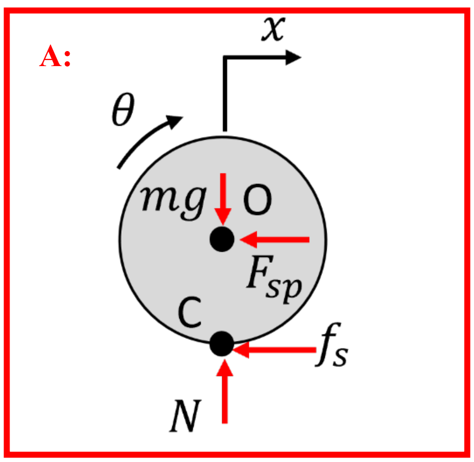
Given: A homogeneous disk of mass m and radius R rolls without slipping on a rough stationary surface. One end of a spring of stiffness k is attached to the center of the disk at O. The other end of the spring is attached to a lightweight plate. The plate moves with a set displacement of $u(t) = u_0 \sin(\omega t)$. Let x describe the position of the center of the disk and be positive to the right measured from the unstretched spring position at time $t = 0$. Answer the following questions in terms of at most $\ddot{x}, x, t, m, k, R, u_0$, and ω. For full credit you must clearly show your work starting from the relevant equations given in the equation sheet.

Find:

A. Draw a FBD of disk.
B. The equation of motion of (EOM) system in terms of x.
C. The natural frequency of the system.
D. A particular solution $x_p(t)$ of the EOM due to the base motion $u(t)$.



1. FBD



2. Kinetics

$\sum M_C = I_C \alpha = -F_{sp} R$ $\quad$ $I_C = \frac{3}{2} m R^2$ $\quad$ $F_{sp} = k(x - u(t))$

3. Kinematics

$x = R\theta$ $\quad$ $\ddot{x} = R\,\ddot{\theta}$ $\quad$ $\alpha = \ddot{\theta}$

4. Solve

$\frac{3}{2}mR^2\left(\frac{\ddot{x}}{R}\right) = -k(x-u(t))R$

B: $3m\ddot{x} + 2kx = 2ku_0\sin(\omega t)$

C: $\omega_n = \sqrt{\frac{2k}{3m}}$

$x_p(t) = A\sin(\omega t)$

$\ddot{x}_p(t) = -\omega^2 A\sin(\omega t) = -\omega^2 x_p(t)$

Plug guess for $x_p(t)$ into EOM:

$-3m\omega^2 x_p + 2kx_p = 2ku_0\sin(\omega t)$

D: $x_p(t) = \frac{2ku_0}{2k-3m\omega^2}\sin(\omega t)$

ME 274: Basic Mechanics II **Name** *(print)* SOLUTION

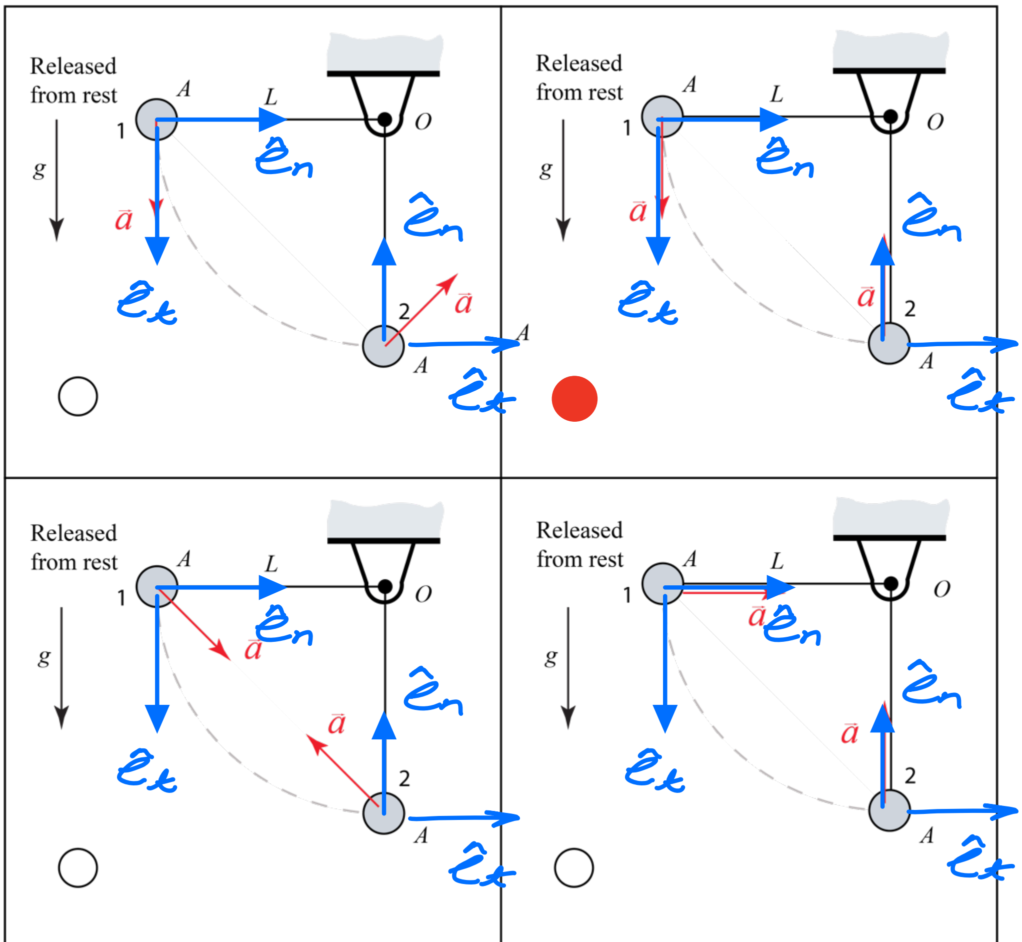
Final Exam - Spring 2025

***PROBLEM NO. 4** (20 points) – PLEASE SCAN ALL PAGES*

***Part 4A** (4 points) NO PARTIAL CREDIT*

Given: A particle A, having mass m, is attached to a massless bar of length L, with the bar pinned to ground at O. The system is released from rest.

Find: Consider each scenario below. Fill in the circle next to the answer that gives the correct direction for the acceleration vectors associated with particle A at positions 1 and 2.



None of these scenarios are correct

Position 1: $\vec{a} = \dot{v}\hat{e}_t + \cancel{\frac{v^2}{\rho}}^{0}\hat{e}_n \Rightarrow$ in $\hat{e}_t$ direction

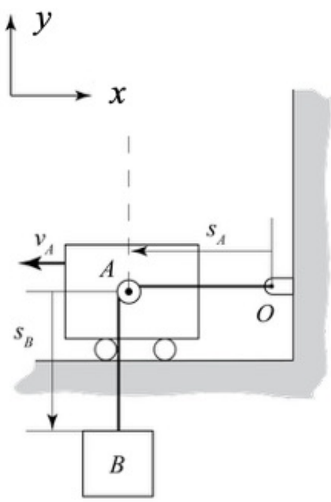
Position 2: $\vec{a} = \cancel{\dot{v}}^{0}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n \Rightarrow$ in $\hat{e}_n$ direction

Part 4B *(2 points) NO PARTIAL CREDIT*

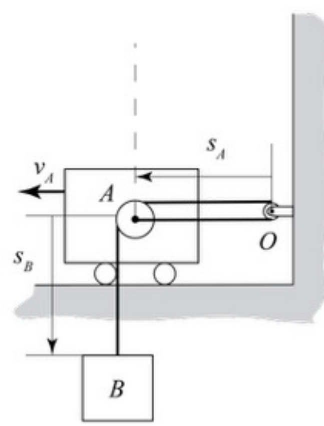
Given: Consider the two systems described below, each featuring a cart labeled A and a block labeled B. Cart A is equipped with a small pulley. In System I, a rope extends from block B, wraps around the pulley on cart A, and is anchored to the wall at point O.

In System II, a rope extends from block B, wraps around the pulley on cart A, then passes around another pulley mounted on the wall at point O, and finally connects back to cart A. In both systems, cart A moves to the left at a speed v_A. **<u>In both systems I and II, the pulleys' radii are considered negligible</u>**.

Find: Is the absolute value of the *y* component of the velocity of block *B* (i.e., $|\dot{s}_B|$) greater in System I or II? Fill in the circle below the system with the larger $|\dot{s}_B|$.



System I



System II

$s_A + s_B = \text{constant}$

$\hookrightarrow \dot{s}_A + \dot{s}_B = 0$

$\hookrightarrow |\dot{s}_B| = |\dot{s}_A| = V_A$

$2 s_A + s_B = \text{constant}$

$\hookrightarrow 2\dot{s}_A + \dot{s}_B = 0$

$\hookrightarrow |\dot{s}_B| = 2|\dot{s}_A| = 2 V_A$

ME 274: Basic Mechanics II — **Name** *(print)* SOLUTION

Final Exam - Spring 2025

PROBLEM NO. 4 **(***continued) – PLEASE SCAN ALL PAGES*

Part 4C *(2 points) NO PARTIAL CREDIT*

Given: In System I, a particle P of mass m is free to slide without friction along a lightweight bar that rotates in a **horizontal plane** about a vertical shaft at point O. A spring of stiffness k and unstretched length R_O connects P to O. Initially, the spring is compressed to half its natural length, and the bar rotates with an angular speed ω_1. Upon release, the particle moves outward, and when the spring reaches its unstretched length, the bar's rotational speed is ω_2.

In System II, the initial conditions are the same, with the bar rotating at the same initial angular speed ω_1. However, two springs are attached to the particle: one compressed to half its natural length and the other stretched to 1.5 times its natural length. After release, the particle moves until both springs reach their unstretched lengths.

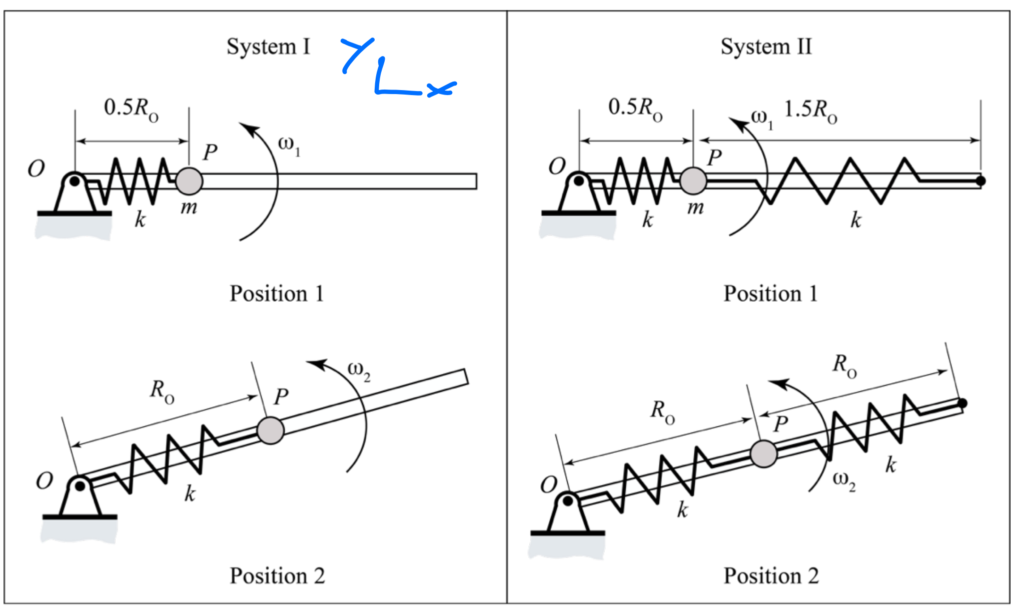
Find: Let $(\omega_2)_I$ and $(\omega_2)_{II}$ represent the final rotational speeds in Systems I and II, respectively. Circle the correct response:

a) $(\omega_2)_I < (\omega_2)_{II}$

b) $\boxed{(\omega_2)_I = (\omega_2)_{II}}$

c) $(\omega_2)_I > (\omega_2)_{II}$

d) More information is needed on the two systems to answer this question.



Horizontal Plane

Angular momentum about O is conserved in each case. Since $\vec{H}_O = mR_O^2\,\hat{k}$, we see that for a given R_O, $\vec{H}_O$ does not depend on spring stiffness $\Rightarrow (\omega_2)_I = (\omega_2)_{II}$

ME 274: Basic Mechanics II **Name** *(print)* SOLUTION
Final Exam - Spring 2025
PROBLEM NO. 4 **(***continued***)** *– PLEASE SCAN ALL PAGES*

Part 4D *(4 points) NO PARTIAL CREDIT*

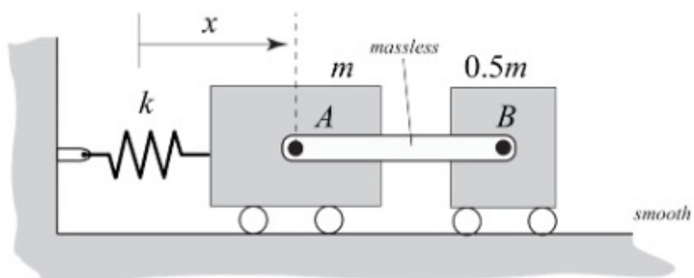
Consider Systems 1 and 2 shown below. System 1 is made up of a spring and block A with mass m rigidly connected to another block B with mass $0.5m$, and the blocks are moving in pure translation along a smooth horizontal surface. System 2 is made up of a spring and a homogeneous disk of mass m and outer radius R, with the center of the disk at O, and rolls without slipping on a horizontal surface. Each system has the same total mass m and same spring stiffness k. Let ω_{n1} and ω_{n2} represent the natural frequencies of Systems 1 and 2, respectively. Circle the answer below that most accurately represents the natural frequencies for the two systems:

a) $\omega_{n1} > \omega_{n2}$

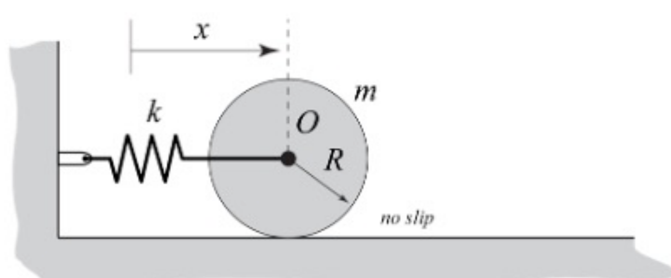
b) $\boxed{\omega_{n1} = \omega_{n2}}$

c) $\omega_{n1} < \omega_{n2}$

d) More information is needed on the two systems to answer this question.

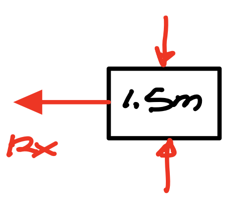


System 1



System 2

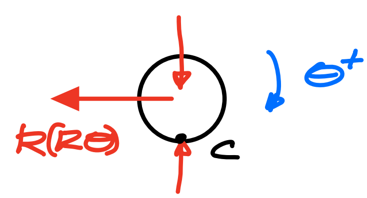
System 1



$\Sigma F_x = -kx = \frac{3}{2}m\ddot{x}$

$\hookrightarrow \ddot{x} + \underbrace{\boxed{\frac{2k}{3m}}}_{\omega_{n1}} x = 0$

System 2



$\Sigma M_c = -(kR\theta)R = I_c\ddot{\theta}$

$= \left(\frac{3}{2}mR^2\right)\frac{\ddot{x}}{2R}$

$\hookrightarrow \ddot{x} + \underbrace{\boxed{\frac{2}{3}\frac{k}{m}}}_{\omega_{n2}} x = 0$

ME 274: Basic Mechanics II **Name** *(print)* SOLUTION
Final Exam - Spring 2025
PROBLEM NO. 4 **(***continued***)** *– PLEASE SCAN ALL PAGES*

Part 4E *(8 points) NO PARTIAL CREDIT FOR EACH PART*

The following equation of motion (EOM) has been derived for a single-degree-of -freedom system:

$$m\ddot{x} + c\dot{x} + kx = F_o,$$

where $m = 4\text{kg}$, $c = 16\ \text{kg/s}$, $k = 100\ \text{N/m}$, and $F_o = 50$ N. Circle the correct answers below.

I. (2 points) What is the undamped natural frequency ω_n of the system?

- a. 5 rad/s
- b. 6 rad/s
- c. 10 rad/s
- d. 25 rad/s
- e. None of the above

$$\ddot{x} + \frac{c}{m}x + \frac{k}{m}x = \frac{F_o}{m}$$

$$\therefore\ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{4}\,\frac{N/m}{kg}} = 5\,\frac{rad}{s}$$

II. (2 points) What is the damping ratio ζ of the system?

- a. $\frac{1}{4}$
- b. $\frac{2}{5}$
- c. 1
- d. $\frac{5}{4}$
- e. None of the above

$$2\zeta\omega_n = \frac{c}{m}$$

$$\hookrightarrow\ \zeta = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}}$$

$$= \frac{16}{(2)\sqrt{(100)(4)}}\ \frac{kg/s}{\sqrt{(N/m)\,kg}}$$

$$= \frac{4}{5}$$

III. (2 points) How would you classify damping in the system?

- a. Undamped
- b. Underdamped
- c. Overdamped
- d. Critically damped

IV. (2 points) What is the static deformation x_{ST} of the system?

- a. $\frac{1}{2}$ m
- b. $\frac{2}{3}$ m
- c. $\frac{3}{4}$ m
- d. $\frac{4}{5}$ m
- e. None of the above

$$\frac{k}{m}x_{st} = \frac{F_o}{m} \Rightarrow x_{st} = \frac{F_o}{k}$$

$$= \frac{50}{100}\ \frac{N}{N/m}$$

$$= \frac{1}{2}N$$