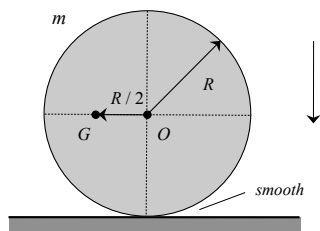


Given: An inhomogeneous disk (of mass m and outer radius R) has its geometric center at O and its center of mass at G (see figure below). The mass moment of inertia of the disk about G is known to be I_G . At the instant shown, the center of mass G is directly to the left of the disk geometric center O . The disk is placed on a smooth, horizontal surface.

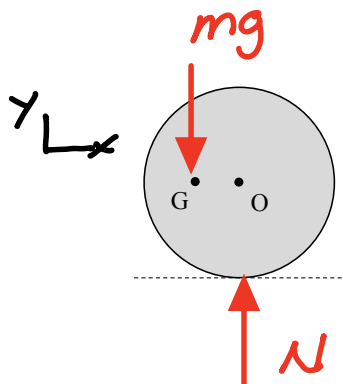


Find: If the disk is released from rest, determine the angular acceleration of the disk. Express your answer as a vector.

Leave your final answer in terms of, at most: m , g , R and I_G .

Solution:

STEP 1 – FBD



STEP 2 – Kinetics

$$(1) \quad \sum M_G = N \frac{R}{2} = I_G \alpha$$

$$(2) \quad \sum F_y = N - mg = m a_{Gy}$$

Combine (1) and (2):

$$\begin{cases} N = m(g + a_{Gy}) \\ N = \frac{2}{R} I_G \alpha \end{cases}$$

$$(3) \quad \frac{2}{R} I_G \alpha = m(g + a_{Gy})$$

STEP 3 – Kinematics

$$\begin{aligned} \vec{a}_G &= \vec{a}_O + \vec{\alpha} \times \vec{r}_{GO} - \omega^2 \vec{r}_{GO} \\ &= a_O \hat{i} + (\alpha \hat{k}) \times \left(-\frac{R}{2} \hat{i}\right) \\ &= a_O \hat{i} - \frac{\alpha R}{2} \hat{j} \end{aligned}$$

$$\therefore (4) \quad a_{Gy} = -\frac{\alpha R}{2}$$

STEP 4 – Solve

Substituting (3) into (4):

$$\frac{2}{R} I_G \alpha = m \left(g - \frac{\alpha R}{2} \right)$$

$$\hookrightarrow \vec{\alpha} = \left[\frac{mg}{\frac{2I_G}{R} + \frac{mR}{2}} \right] \hat{k}$$