Sample final exam questions

Name SOLUTION

PROBLEM 3B

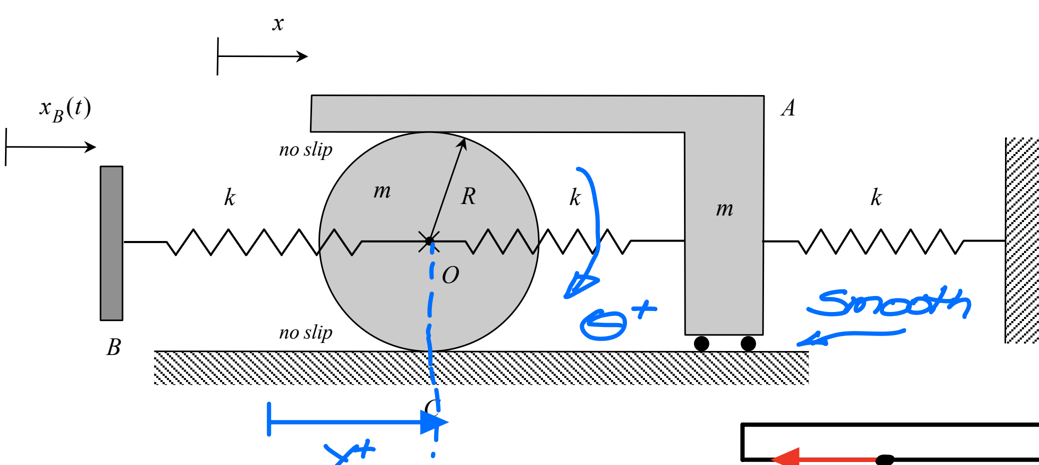
Given: A homogeneous disk (having a mass of m and outer radius R) is able to roll without slipping on a horizontal surface. A spring of stiffness k is attached between the center O of the disk and a moveable base B. A second spring (also of stiffness k) is attached between O and an L-shaped block A, with an arm of block A resting on the top surface of the disk. A third spring (also of stiffness k) is attached between A and a fixed wall on the right. As the system moves, the arm of block A does not slip on the top surface of the disk. Let x represent the motion of block A, and $x_B(t) = b\sin\omega t$ represent the motion of the base B. When $x = x_B = 0$, the springs are unstretched.

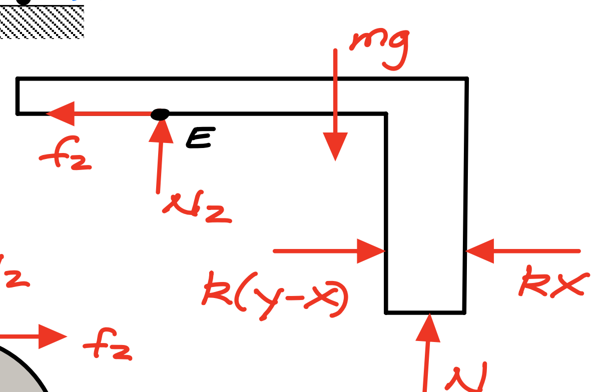
Find: For this problem:

a) Draw individual free body diagrams (FBDs) of the disk and of block A.

b) From your FBDs, derive the differential equation of motion (EOM) of the system in terms of the coordinate x.

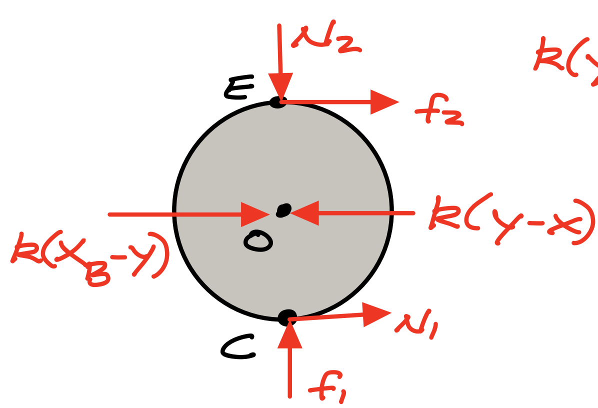
c) Determine the particular solution of the EOM, with this solution describing the steady-state motion of block A. Leave your answer in terms of the parameters of the problem.





1. FBDs

Since we will be using N/E equations, we need individual FBDs.



2. N/E

(1) **Disk:** $\sum M_c = k(x_B - y)R - k(y-x)R + f_2(2R)$

$$= I_c\ddot{\theta} = \left(\tfrac{3}{2}mR^2\right)\ddot{\theta}$$

(2) **Block:** $\sum F_x = -f_2 + k(y-x) - kx = m\ddot{x}$

$$(2) \Rightarrow f_2 = -m\ddot{x} - kx + k(y-x)$$

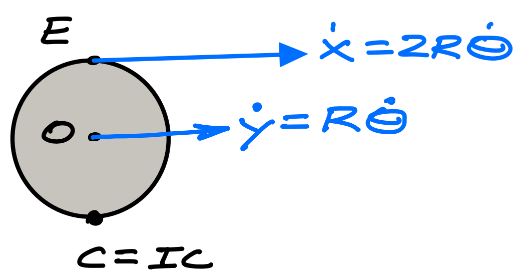
(3) $$(1) \Rightarrow \left(\tfrac{3}{2}mR^2\right)\ddot{\theta} = k(x_B - y)R - k(y-x)R + \left[-m\ddot{x} - kx + k(y-x)\right](2R)$$

3. Kinematics

(4) $\dot{x} = 2R\dot{\theta} \Rightarrow \ddot{\theta} = \frac{1}{2R}\ddot{x}$

$\dot{y} = R\dot{\theta} = \frac{R}{2R}\dot{x} = \frac{1}{2}\dot{x}$

(5) $\Rightarrow y = \frac{1}{2}x$



4. EOM

Substitute (4) and (5) into (3):

$$\left(\tfrac{3}{2}m\cancel{R}^2\right)\frac{\ddot{x}}{2\cancel{R}} = k\left(x_B - \tfrac{x}{2}\right)\cancel{R} - k\left(\tfrac{x}{2} - x\right)\cancel{R} + \left[-m\ddot{x} - kx + k\left(\tfrac{x}{2} - x\right)\right]2\cancel{R}$$

$$= -2m\ddot{x} - 3kx + kx_B$$

$\hookrightarrow \frac{11}{4}m\ddot{x} + 3kx = kb\sin\omega t \quad \longleftarrow$ EOM

$\div \frac{11}{4}m: \quad \ddot{x} + \underbrace{\boxed{\frac{12}{11}\frac{k}{m}}}_{\omega_n^2} x = \frac{4k}{11m} b\sin\omega t$

Particular solution

$$\begin{cases} x_p(t) = A\sin\omega t + B\cos\omega t \\ \ddot{x}_p(t) = -A\omega^2\sin\omega t - B\omega^2\cos\omega t \end{cases}$$

$\hookrightarrow$ Substitute into EOM:

$$(-\omega^2 + \omega_n^2)A\sin\omega t + (-\omega^2 + \omega_n^2)B\cos\omega t = \frac{4k}{11m}b\sin\omega t$$

$\hookrightarrow$ $\cos\omega t$: $(-\omega^2 + \omega_n^2)B = 0 \Rightarrow B = 0 \quad \longleftarrow$ B

$\sin\omega t$: $(-\omega^2 + \omega_n^2)A = \frac{4k}{11m}b$

$\hookrightarrow A = \frac{(4k/11m)}{-\omega^2 + \omega_n^2}b \quad \longleftarrow$ A