Forced Vibrations Summary

GENERAL EOM: $M\ddot{x} + C\dot{x} + Kx = F(t)$, $F(t) \neq 0 \neq$ constant

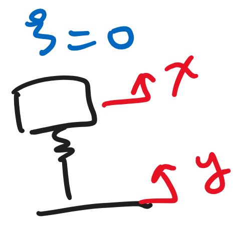
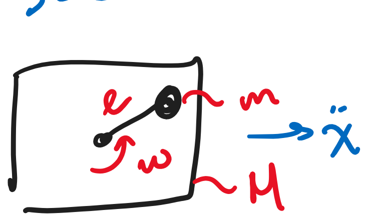
Full solution: $x(t) = x_c(t) + x_p(t)$

Forced vibration at steady state

SYSTEM	EOM	$F(t)$	$x_p(t)$ → Steady State Response
Undamped $\zeta = 0$	$M\ddot{x} + Kx = F(t)$	$F(t) = F_0 \sin\omega t$	$x_p(t) = A\sin\omega t + B\cos\omega t$; $A = \frac{F_0/K}{1-(\omega/\omega_n)^2}$, $B = 0$
		$F(t) = F_0\cos\omega t$	$A = 0$, $B = \frac{F_0/K}{1-(\omega/\omega_n)^2}$
Underdamped $0 < \zeta < 1$	$M\ddot{x} + C\dot{x} + Kx = F(t)$	$F(t) = F_0\sin\omega t$	$x_p(t) = A\sin\omega t + B\cos\omega t$, $A \neq B \neq 0$
		$F(t) = F_0\cos\omega t$	$x_p(t) = X\sin(\omega t - \phi)$; $X = \frac{F_0/K}{\left(\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2\right)^{1/2}}$; $\phi = \tan^{-1}\left(\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}\right)$; $X = \frac{F_0/K}{((K-M\omega^2)^2+(C\omega)^2)^{1/2}}$; $\phi = \tan^{-1}\left(\frac{C\omega}{K-M\omega^2}\right)$

Summary of Special Cases

$M\ddot{x} + C\dot{x} + Kx = F(t) = F_0\sin\omega t$ or $F_0\cos\omega t$

CASE	EOM	F_0
Base excitation (undamped) $\zeta = 0$ 	$M\ddot{x} + Kx = KY\sin\omega t$ or $KY\cos\omega t$	KY
Imbalanced Rotation (undamped) $\zeta = 0$ 	$(M+m)\ddot{x} + Kx = F_0\cos\omega t$	$m\omega^2 e$