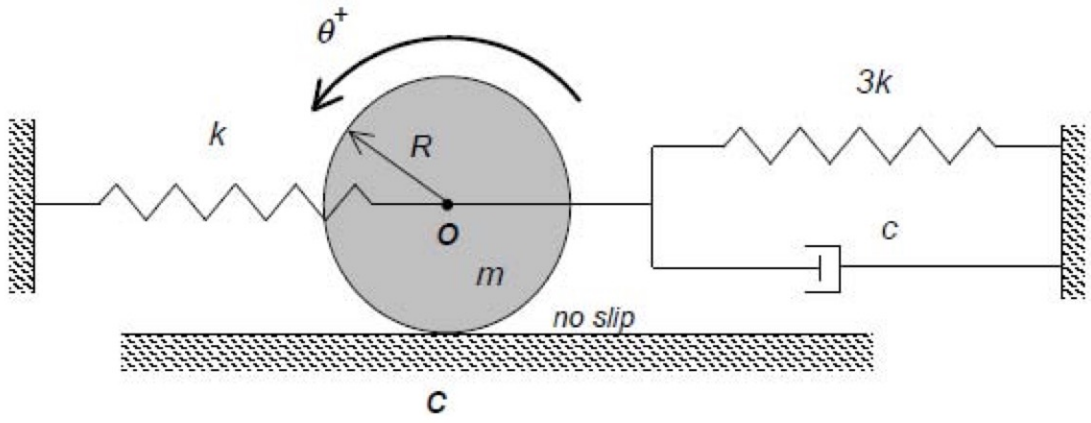
Homework 6.E

Given: A homogeneous disk having a mass of m and outer radius of R rolls without slipping on a rough, horizontal surface. A spring of stiffness k is connected between the center O of the disk and ground on the left side of the disk. A second spring of stiffness $3k$ and a dashpot with damping constant c is connected between O and ground on the right side of the disk, as shown in the figure below. Let θ represent the rotation of the disk measured positive counterclockwise, and let $\theta = 0°$ describe when the springs are unstretched.

Find: For this problem:
(a) Draw a free body diagram of the disk;
(b) Derive the single differential equation of motion for the system in terms of the coordinate θ, its time derivatives, and, at most, the following parameters: m, R, c, and k;
(c) Determine the value of the damping constant c required for the system to be underdamped with a damping ratio of $\zeta = 0.5$; and
(d) Suppose that the mass of the drum is doubled (to a mass of $2m$). Determine the new value of the damping constant c required for the system to be underdamped with a damping ratio of $\zeta = 0.5$.



Use the following parameters in your analysis: $m = 24$ kg, $k = 900$ N/m, and $R = 0.5$ m.