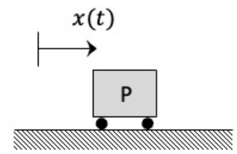
Parameters for describing harmonic motion

ME 274



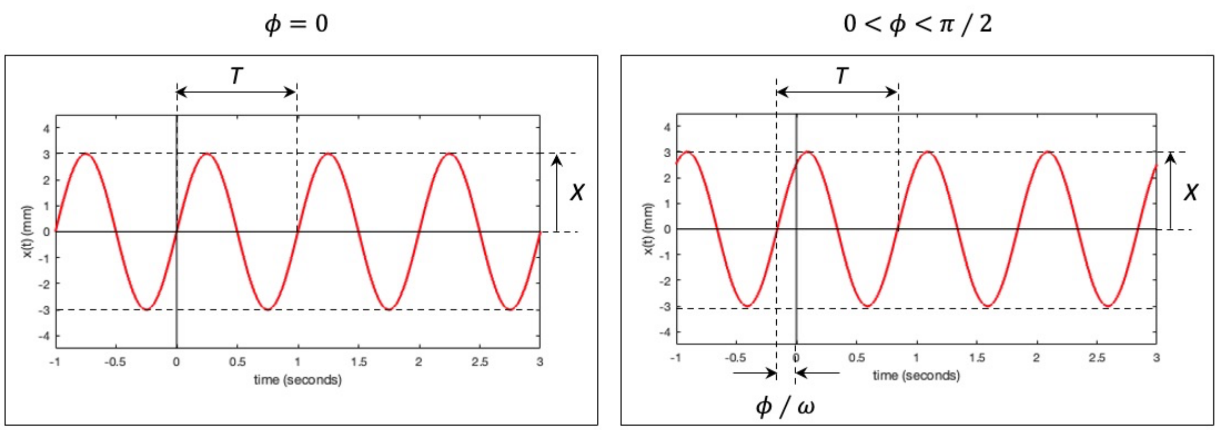
Suppose that the position "x" (in mm) of some point P is written in terms of time "t" (in seconds). If $x(t)$ represents "harmonic motion" of P, $x(t)$ can be written as:

$$x(t) = X sin(\omega t + \phi)$$

As seen in this equation, there are three parameters in this equation that are required to quantify this harmonic motion:

- X = the "amplitude" of the motion (in mm)
- ω = the "angular frequency" (in rad/s)
- ϕ = the "phase" (in rad)

Consider the two plots below of $x(t)$, one with $\phi = 0$, and one with $\phi > 0$.

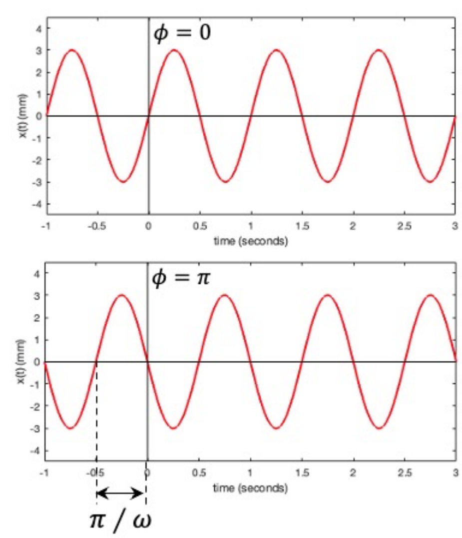


The time required for x(t) to complete one cycle of motion is called the "period", T. Considering the $\phi = 0$ plot on the left above, the period is the time T over which the argument of the sine function goes from zero to 2π; that is, $\omega T = 2\pi$, or:

$$\omega = \frac{2\pi}{T} = \text{angular frequency (rad/s)}$$

The frequency of the harmonic function is often described in terms of "cycles/second" (or, Hz) using the symbol of "f". Recall that there are 2π radians in one cycle of motion. Suppose that we know that $\omega = 10$ rad/s. From this, the frequency in Hz is given by: (10 rad/s)(cycle/2π rad) = $\frac{10}{2\pi}$ cycles/s = $\frac{5}{\pi}$ Hz. In general, we can write:

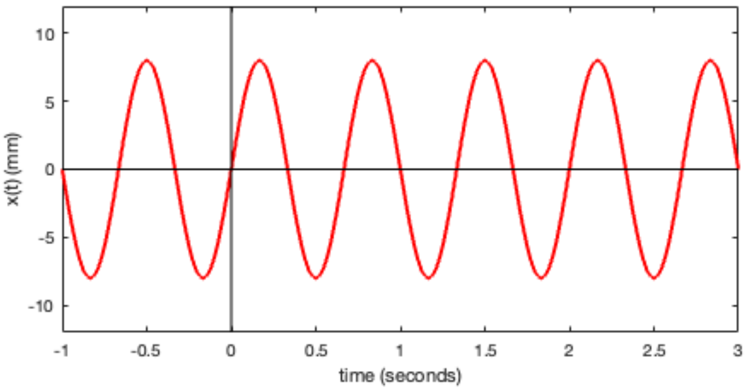
$$f = \frac{\omega}{2\pi} = \frac{1}{T} \text{ (Hz)}$$



In this course, we will often encounter a harmonic response that has a "180° phase shift". How does a harmonic function with $\phi = \pi$ look? See the plot to the right. You see that the $\phi = \pi$ phase-shifted harmonic function has its sign "flipped" over the entire motion with unchanged amplitude and frequency.

Get some practice! Consider the three examples on the backside of this page to be a quiz. From these plots, estimate the amplitude of response X, the angular frequency ω, the frequency in Hz f and the phase ϕ for each harmonic function.

Practice with harmonic functions

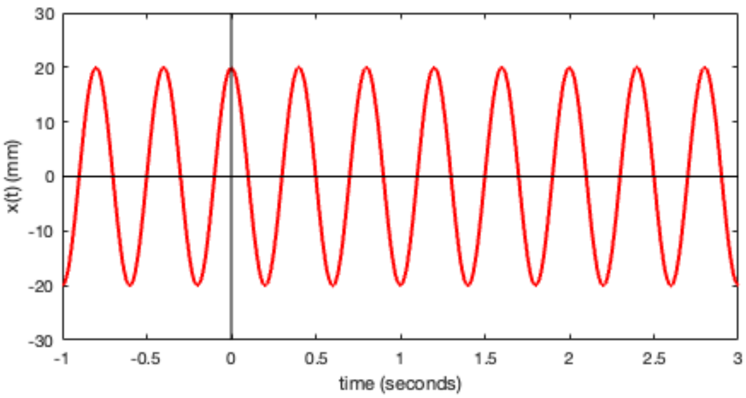


$X =$

$\omega =$

$f =$

$\phi =$

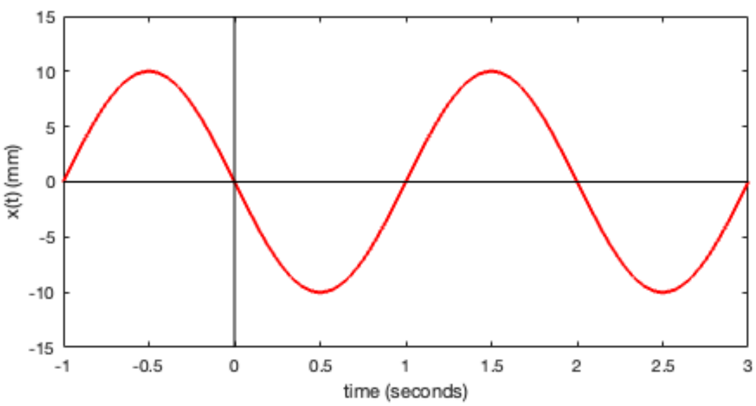


$X =$

$\omega =$

$f =$

$\phi =$



$X =$

$\omega =$

$f =$

$\phi =$