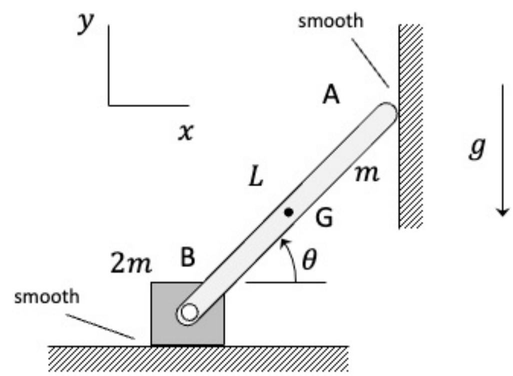
Problem 1 *(20 points)*

Given: A block having a mass of 2*m* is able to slide on a smooth, horizontal surface, as shown to the right. A thin, homogeneous bar having a mass of *m* and length *L* is pinned to the block at end O of the bar. End A of the bar is in contact with a smooth, vertical wall. The system is released from rest with bar OA being at an angle of θ from the horizontal.

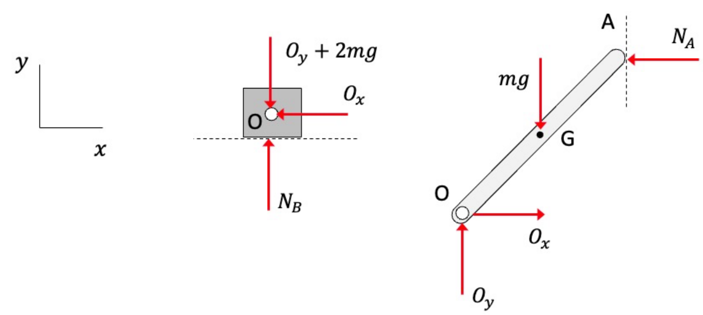


Find: In this problem, you are asked to determine angular acceleration of bar OA. To this end, you need to respond to the following *Steps 1-4* of the solution. *Please provide your responses in the specified locations - otherwise, your work will be missed and not graded.*

Solution

STEP 1: FBDs

Complete the individual free body diagrams (FBDs) of the block and of bar OA below.



STEP 2: Kinetics

STEP 2A: Using the *xy*-coordinate axes provided, write down the three Newton/Euler equations *for bar OA* from the FBD of the bar in *STEP 1*.

(1) $\sum F_x = -N_A + O_x = ma_{Gx} \quad \Rightarrow \quad N_A = O_x - ma_{Gx}$
(2) $\sum F_x = -mg + O_y = ma_{Gy} \quad \Rightarrow \quad O_y = mg + ma_{Gy}$
(3) $\sum M_G = -O_y(L/2)cos\theta + O_x(L/2)sin\theta + N_A(L/2)sin\theta = I_G\alpha \quad ; \quad I_G = (1/12)mL^2$

STEP 2B: Using the *xy*-coordinate axes provided, write down the two Newton's 2nd Law equations *for the block* from the FBD of the block in *STEP 1*.

(4) $\sum F_x = -O_x = 2ma_O \quad \Rightarrow \quad O_x = -2ma_O$
(5) $\sum F_y = -2mg - O_y + N_B = 0$

Problem 1 (continued)

Step 3: Kinematics

Write the kinematics equations that are needed to solve this problem.

$\vec{a}_O = \vec{a}_A + \vec{\alpha} \times \vec{r}_{O/A} - \omega^2 \vec{r}_{O/A}$

$a_O \hat{\imath} = a_A \hat{\jmath} + (\alpha \hat{k}) \times (-Lcos\theta\hat{\imath} - Lsin\theta\hat{\jmath})$

$= (L\alpha sin\theta)\hat{\imath} + (a_A - L\alpha cos\theta)\hat{\jmath}$ $\Rightarrow$ $\hat{\imath}$: $a_O = L\alpha sin\theta$; $\hat{\jmath}$: $0 = a_A - L\alpha cos\theta$

Also,

$\vec{a}_G = \vec{a}_O + \vec{\alpha} \times \vec{r}_{G/O} - \omega^2 \vec{r}_{G/O}$

$= L\alpha sin\theta\hat{\imath} + (\alpha\hat{k}) \times (Lcos\theta\hat{\imath} + Lsin\theta\hat{\jmath})/2$

$= (L/2)\alpha sin\theta\hat{\imath} + (L/2)\alpha cos\theta\hat{\jmath}$ $\Rightarrow$

(6) $a_{Gx} = (L/2)\alpha sin\theta$

(7) $a_{Gy} = (L/2)\alpha cos\theta$

Step 4: Solve

Using the equations from Steps 2 and 3 above, solve for the angular acceleration of the bar. Write your answer as a vector. Your answer should be in terms of, at most: *m*, *L*, *g* and θ.

(4) $O_x = -2mL\alpha sin\theta$

(1) and (6): $N_A = -2mL\alpha sin\theta - m(L/2)\alpha sin\theta = -m(5\,L/2)\alpha sin\theta$

(2) and (7): $O_y = mg + m(L/2)\alpha cos\theta$

Combining with (3):

$$-[mg + m(L/2)\alpha cos\theta](L/2)cos\theta + [-2mL\alpha sin\theta](L/2)sin\theta + [-m(5\,L/2)\alpha sin\theta](L/2)sin\theta = (1/12)mL^2\alpha \quad \Rightarrow$$

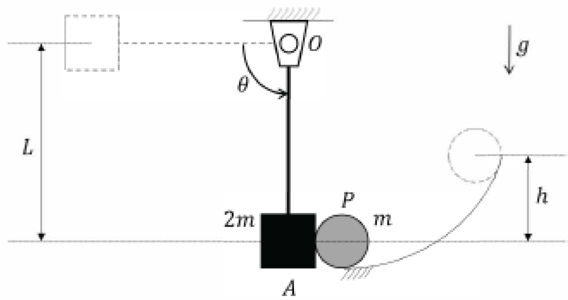
$$\vec{\alpha} = -\left[\frac{2gcos\theta/L}{1/3 + \cos^2\theta + 9\sin^2\theta}\right]\hat{k}$$ ←

Name SOLUTION

Problem 2 (20 points)

Given: The first stage in a Rube Goldberg machine requires releasing block A (mass $2m$) from rest at $\theta = 0$ rad. Block A is suspended by a cord (length L) from fixed point O. When $\theta = \frac{\pi}{2}$ rad, block A must strike particle P (mass m). The coefficient of restitution for this impact event is e. After impact, particle P must travel on the smooth semicircular segment up to a maximum height h to activate the next stage in the machine. All motion occurs in the vertical plane.

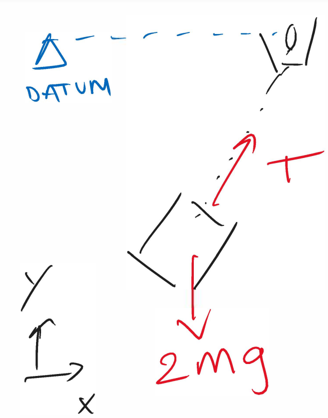
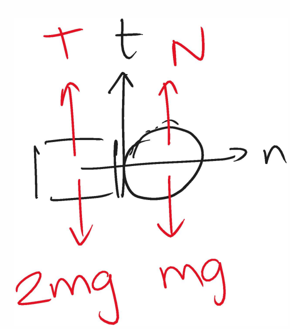
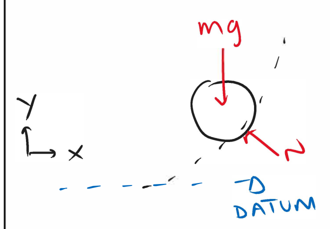
The relevant motion states are:
State 1: Upon release of block A
State 2: Right before impact
State 3: Right after impact
State 4: P reaches height of h



Find: Follow the solution steps to write an expression for L in terms of, at most, the known variables h, m, e, and g, to ensure the particle P is able to activate the next stage of the machine.

Solution

Part A: Use the spaces below to draw the pertinent FBDs for the motion at state 1-2, state 2-3, and state 3-4. Your FBDs must indicate the relevant coordinates, any conserved quantities, and denote gravitational datums as necessary.

State 1-2	State 2-3	State 3-4
		 ENERGY CONSERVED

ENERGY CONSERVED

LINEAR MOMENTUM IN n CONSERVED

Name SOLUTION

Problem 2 (continued)

Part B: Use the work-energy equation to find an expression for the velocity of block A at state 2: $\vec{v}_{A2}$. Write the correct expressions in the appropriate spaces below.

$T_1 =$	0
$V_1 =$	0
$U_{1\to 2}^{NC} =$	0
$T_2 =$	$\frac{1}{2}(2m)v_{A2}^2$
$V_2 =$	$-2mgL$
$\vec{v}_{A2} =$	$\sqrt{2gL}\ \hat{i}$

$$0 = \frac{1}{2}(2m)v_{A2}^2 - 2mgL$$

$$v_{A2} = \sqrt{2gL}$$

Part C: Use linear impulse-momentum equations to find the velocity of particle P at state 3: $\vec{v}_{P3}$.

LIM – n :

$$2\cancel{m}v_{A2} + \cancel{m}\cancel{v_{P2}}^{\,0} = 2\cancel{m}v_{A3} + \cancel{m}v_{P3}$$

$$2v_{A2} = 2v_{A3} + v_{P3} \qquad (1)$$

CENTRAL IMPACT :

$$e = \frac{v_{P3} - v_{A3}}{v_{A2} - \cancel{v_{P2}}^{\,0}}$$

$$e\,v_{A2} = v_{P3} - v_{A3} \qquad (2)$$

(1) & (2) $\rightarrow$ $v_{P3} = \frac{2}{3}(e+1)\sqrt{2gL}$

Name SOLUTION

Problem 2 (continued)

__Part D__: Use the previous results and the work-energy equation from state 3-4 to find an expression for L in terms of the known variables. Write the correct expressions in the appropriate spaces below.

$T_3 =$	$\frac{1}{2} m v_{P_3}^2$
$V_3 =$	0
$U_{3\to4}^{NC} =$	0
$T_4 =$	0
$V_4 =$	mgh
$L =$	$9h/4(1+e)^2$

$$\frac{1}{2} m \left[\frac{2}{3}(1+e)\sqrt{2gL} \right]^2 = mgh$$

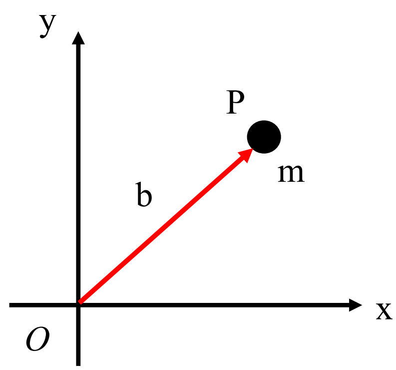
$$L = \frac{9h}{4(1+e)^2}$$

***Problem 3** (20 points)*

<u>*Part A*</u> *(2 points)*

Given: A particle of mass m is moving in a circular path where its position as a function of time is given by the expression $\vec{r} = bcos(\omega t)\,\hat{\imath} + bsin\,(\omega t)\hat{\jmath}$, where b and ω are constants.

Find: Calculate the total angular momentum of the particle about point O and mathematically show it is constant.



Solution:

$$\vec{H}_O = \vec{r}_{P/O} \times (m\vec{v}_P) = [bcos(\omega t)\,\hat{\imath} + bsin(\omega t)\,\hat{\jmath}] \times [-b\omega sin(\omega t)\,\hat{\imath} + bcos(\omega t)\hat{\jmath}]$$

$$= bcos(\omega t)\,\hat{\imath} \times (-b\omega sin(\omega t)\,\hat{\imath}) + bcos(\omega t)\,\hat{\imath} \times bcos(\omega t)\hat{\jmath} + bsin(\omega t)\hat{\jmath} \times (-b\omega sin(\omega t)\,\hat{\imath}) + bsin(\omega t)\hat{\jmath} \times bcos(\omega t)\hat{\jmath}$$

$$= b^2\omega(cos(\omega t))^2\hat{k} + b^2\omega(sin(\omega t))^2\hat{k}$$

$$= b^2\omega\hat{k} \quad since \quad (cos(\omega t))^2 + (sin(\omega t))^2 = 1$$

The angular momentum is constant that does not change with time.

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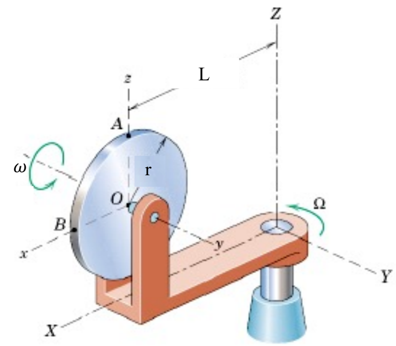
Part B *(8 points)*

Given: An arm rotates around the fixed $Z-axis$ with a rate of Ω. A circular disk rotates about its own axis with a constant rate of ω relative to the arm. At the instant shown, Ω increases at a rate of $\dot{\Omega}$. It is desired to know the acceleration of point A on the circumstance of the disk when A is directly above the center O and using the following equation:



$$\vec{a}_A = \vec{a}_O + (\vec{a}_{A/O})_{rel} + \vec{\alpha} \times \vec{r}_{A/O} + 2\vec{\omega} \times (\vec{v}_{A/O})_{rel} + \vec{\omega} \times \vec{\omega} \times \vec{r}_{A/O}$$

Find: Calculate the following vector quantities using the coordinate system incorporating the parameters: L, r, ω, Ω, and $\dot{\Omega}$. Note that not all parameters may be needed for each calculation.

1) For the case of ***an observer and a set of XYZ-axes attached to arm***, determine the following terms:

a) $\vec{\omega}$ = ________ $\Omega\hat{K}$ ______________________

b) $\vec{\alpha}$ = _________ $\dot{\Omega}\hat{K}$ ____________________

c) $(\vec{v}_{A/O})_{rel}$= ____ $\omega r\hat{I}$ ____________________

d) $(\vec{a}_{A/O})_{rel}$= ____ $-\omega^2 r\hat{K}$ ________________

Note: you can use either $\hat{I}, \hat{J}, \hat{K}$ or $\hat{\imath}, \hat{\jmath}, \hat{k}$

2) For the case of ***an observer and a set of xyz-axes attached to disk***, determine the following terms:

a) $\vec{\omega}$ = ________ $\Omega\hat{k} + \omega\,\hat{\jmath}$ __________________

b) $\vec{\alpha}$ = _________ $\dot{\Omega}\hat{k} - \Omega\omega\hat{\jmath}$ _________________

c) $(\vec{v}_{A/O})_{rel}$= ____ $\vec{0}$ _________________________

d) $(\vec{a}_{A/O})_{rel}$= ____ $\vec{0}$ _________________________

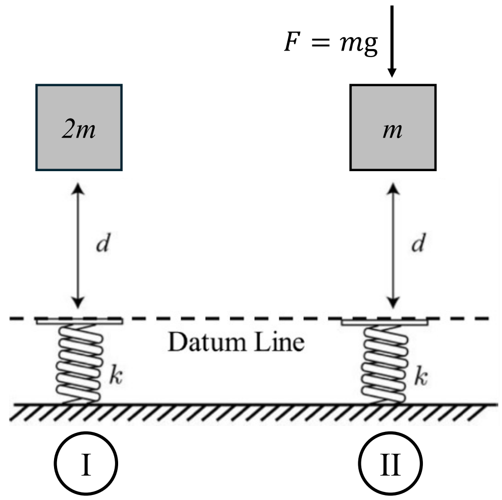
Note: you can use either $\hat{I}, \hat{J}, \hat{K}$ or $\hat{\imath}, \hat{\jmath}, \hat{k}$

Part C *(6 points)*

Given: Two systems as shown to the right are initially at rest, and the blocks are able to move in the vertical plane. The springs in both systems are massless with a stiffness of k, initially unstretched.

In system I), a single block having mass of $2m$ falls through a distance d before compressing the spring.

In system II), a block of mass m is pushed downward by a constant force $F = mg$ as it falls through the same distance d and during the subsequent compression of the spring.



Considering the following 3 instants:

Instant 1: initially at rest;
Instant 2: the block touches the spring;
Instant 3: the spring reaches a maximum compression;

Find:

1) Circle the correct answer for each of the statement:

a) TRUE or FALSE:
Using the provided datum line, the potential energy for system I is larger in magnitude than system II at Instant 1.

Tips: 2mgd > mgd

b) TRUE or FALSE:
From Instant 1 to Instant 2, mechanical energy is conserved for both systems from Instant 1 to 2.

Tips: In system 2, F does non-conservative work

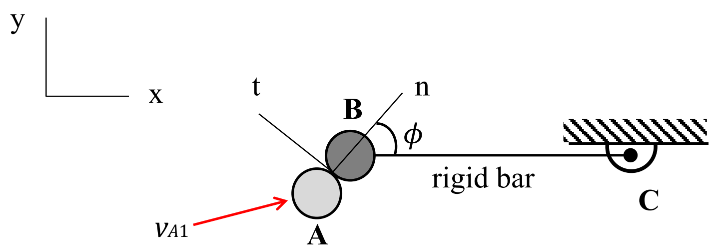
2) Circle the answer that most accurately describes the speed of the block in the two systems when it initially touches the spring (Instant 2):

a) $v_I < v_{II}$
b) $v_I > v_{II}$
c) $v_I = v_{II}$
d) More information is needed about the problem in order to answer this question.

Tips: Both systems have the same kinetic energy using work energy equation, but mass for system 2 is smaller, thus the speed has to be higher

Part D (4 points)

Given: Particle A strikes a stationary particle B with a speed of v_{A1} with $0° < \phi < 90°$. Both particles are smooth. Particle B is attached to a rigid bar of negligible mass that is pinned to the ground at point O as shown in the figure. The whole system is on a horizontal and smooth surface.



Find: Circle all the correct responses below. Some problems might have more than one correct response.

1) For ***system made up of A alone*** during impact:

   a) linear momentum in the x-direction is conserved
   b) linear momentum in the y-direction is conserved
   c) linear momentum in the n-direction is conserved
   d) linear momentum in the t-direction is conserved
   e) angular momentum about C is conserved

   Tips: Draw an FBD for A alone, only t-direction has no force

2) For ***system made of A and B*** during impact:

   a) linear momentum in the x-direction is conserved
   b) linear momentum in the y-direction is conserved
   c) linear momentum in the n-direction is conserved
   d) linear momentum in the t-direction is conserved
   e) angular momentum about C is conserved

   Tips: Draw an FBD for A and B together, no force in y-direction, and no moment about point C.