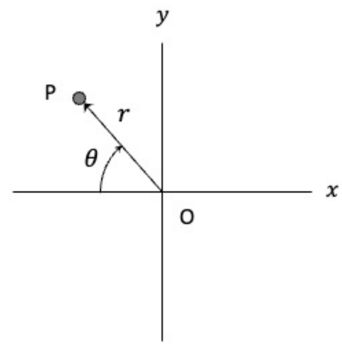
ME 274: Basic Mechanics II – Spring 2024 ***SOLUTION***

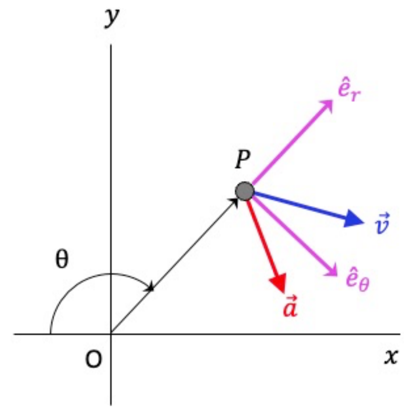
Exam 1 – February 14

Problem 1 *(20 points)*

Given: Particle P moves within a plane on a path given by: $r^2 \sin\theta = c$, where *c* is a constant, *r* is the radial distance from O to P, and θ is measured clockwise from the negative *x*-axis. It is known that the angle θ is increasing at a *constant* rate of $\dot{\theta}$.



Find: In this problem, you are asked to determine the acceleration of point P. To this end, you need to respond to the following four parts A-D of the question. Use the following parameter values in your work: $\sin\theta = 4/5$, $\cos\theta = -3/5$, $c = (16/5)\, m^2$ and $\dot{\theta} = 8\, rad/s$.



Solution

<u>Part A</u>: On the axes to the right, draw P in the position specified by the angle provided above in "Find". Show the polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$ for that position in the drawing.

<u>Part B</u>: Determine the polar components of the velocity and acceleration vectors for P at the position indicated. Write the velocity and acceleration of P as vectors using the polar unit vectors.

$$r^2 \sin\theta = c \quad \Rightarrow \quad r = \sqrt{\frac{c}{sin\theta}} = \sqrt{\frac{16/5}{4/5}} = 2m$$

$$\frac{d}{dt}(r^2 \sin\theta) = 2r\dot{r}sin\theta + r^2\dot{\theta}cos\theta = 0 \quad \Rightarrow$$

$$\dot{r} = -\frac{1}{2}\frac{rcos\theta}{sin\theta}\dot{\theta} = -\frac{1}{2}\frac{(2)(-3/5)}{(4/5)}(8) = 6m/s \longleftarrow$$

$$0 = \frac{d}{dt}\left(2r\dot{r}sin\theta + r^2\dot{\theta}cos\theta\right) = (2\dot{r}^2 + 2r\ddot{r})sin\theta + 4r\dot{r}\dot{\theta}cos\theta - r^2\dot{\theta}^2 sin\theta + r^2\ddot{\theta}cos\theta \quad \Rightarrow$$

$$\ddot{r} = \frac{\left[-2\dot{r}^2 + r^2\dot{\theta}^2\right]sin\theta - 4r\dot{r}\dot{\theta}cos\theta}{2rsin\theta} = \frac{[-2(6)^2 + (2)^2(8)^2](4/5) - 4(2)(6)(8)(-3/5)}{2(2)(4/5)} = 118\, m/s^2 \longleftarrow$$

Therefore:

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = (6\hat{e}_r + 16\hat{e}_\theta)m/s \longleftarrow$$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta = [118 - (2)(8)^2]\hat{e}_r + 2(6)(8)\hat{e}_\theta = (-10\hat{e}_r + 96\hat{e}_\theta)m/s^2 \longleftarrow$$

Problem 1 (continued)

Part C: Show the *velocity* and *acceleration vectors* for P on the axes provided in Part A above.

Part D: Determine the *rate of change of speed* of P. Is the speed of P increasing, decreasing or constant?

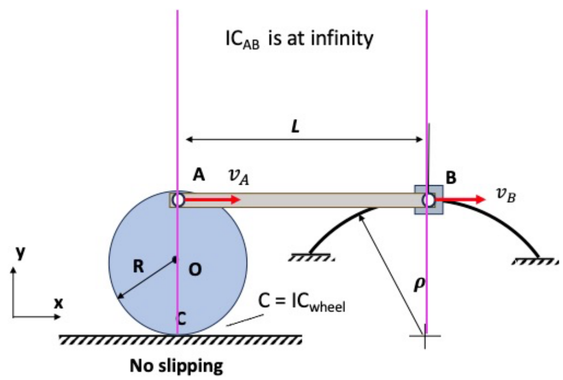
$$\dot{v} = \vec{a} \bullet \hat{e}_t = \vec{a} \bullet \frac{\vec{v}}{|\vec{v}|} = (-10\hat{e}_r + 96\hat{e}_\theta) \bullet \left(\frac{6\hat{e}_r + 16\hat{e}_\theta}{\sqrt{6^2 + 16^2}}\right)$$

$$= \frac{(-10)(6) + (96)(16)}{\sqrt{292}} = 86.4\, m/s^2 > 0 \quad \Longrightarrow \quad INCREASING \text{ in speed}$$

***Problem 2** (20 points)*

***Given**:* A rigid link AB is connected to a wheel and a sliding block B with pin joints. The length of the link is L, and the size of the sliding block B is negligible. Block B is sliding on a circular track with a radius of curvature of ρ at **a constant speed $\boldsymbol{v_B}$**. The wheel has a center point O and a radius of R, it is rolling without slipping on a horizontal surface. At this instant, the wheel is contacting the horizontal surface at point C, B is at the top point of the circular track, and A and B have the same vertical height (i.e., the link AB is in a horizontal position).

Please use the following values for calculations: $v_B = 2$ m/s, $R = 1$ m, $\rho = 2$ m, and $L = 4$ m.



Find: For this problem, please do the following:

(a) Find the instantaneous centers of rotation of the wheel (IC_O) and of link AB (IC_{AB}). Please mark IC_O and IC_{AB} on the figure above.

(b) Find the angular velocity of the wheel, $\vec{\omega}_o$, and the angular velocity of the link AB, $\vec{\omega}_{AB}$. Express your answers in vector form using the *xyz*-coordinates in the figure above.

(c) Find the angular acceleration of the wheel, $\vec{\alpha}_o$, and the angular acceleration of the link AB, $\vec{\alpha}_{AB}$. Express your answers in vector form using the given *xyz*-coordinates in the figure above.

Part (a)

Draw perpendiculars to $\vec{v}_A$ and $\vec{v}_B$. These perpendiculars cross at the IC for AB at infinity. From that, we conclude that $\vec{\omega}_{AB} = \vec{0}$. Since AB has zero angular velocity, all points on AB have the same velocity. Therefore: $v_A = v_B = 2m/s$. ←

Also, since C is a no-slip point on the wheel, C is the IC for the wheel.

Part (b)

Since C is the IC for the wheel, we can write $v_A = 2R\omega_O$. From this, we have:

$$\vec{\omega}_O = -\left(\frac{v_A}{2R}\right)\hat{k} = -\left[\frac{2}{(2)(1)}\right]\hat{k} = -1\hat{k}\ rad/s \quad \longleftarrow$$

Problem 2 (continued)

Part (c)

$$\vec{a}_O = \vec{a}_C + \vec{\alpha}_O \times \vec{r}_{O/C} - \omega_O^2\, \vec{r}_{O/C}$$
$$a_O\hat{\imath} = a_C\hat{\jmath} + \left(\alpha_O\hat{k}\right) \times (R\hat{\jmath}) - \omega_O^2(R\hat{\jmath})$$
$$= -R\alpha_O\hat{\imath} + (a_C - R\omega_O^2)\hat{\jmath}$$

Therefore:

$\hat{\jmath}$: $0 = a_C - R\omega_O^2 \quad \Rightarrow \quad a_C = R\omega_O^2$

$$\vec{a}_A = \vec{a}_C + \vec{\alpha}_O \times \vec{r}_{O/C} - \omega_O^2\vec{r}_{O/C}$$
$$= a_C\hat{\jmath} + \left(\alpha_O\hat{k}\right) \times (2R\hat{\jmath}) - \omega_O^2(2R\hat{\jmath})$$
$$= (-2R\alpha_O)\hat{\imath} + (a_C - 2R\omega_O^2)\hat{\jmath} = (-2R\alpha_O)\hat{\imath} + (-R\omega_O^2)\hat{\jmath}$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times \vec{r}_{A/B} - \omega_{AB}^2\vec{r}_{A/B}$$
$$= (-v_B^2\,/\,\rho)\hat{\jmath} + \left(\alpha_{AB}\hat{k}\right) \times (L\hat{\imath})$$
$$= (-L\alpha_{AB} - v_B^2\,/\,\rho)\hat{\jmath}$$

Equating the two expressions above for $\vec{a}_A$ and balancing coefficients gives:

$\hat{\imath}$: $0 = -2R\alpha_O \quad \Rightarrow \quad \vec{\alpha}_O = \vec{0}$ ⟵

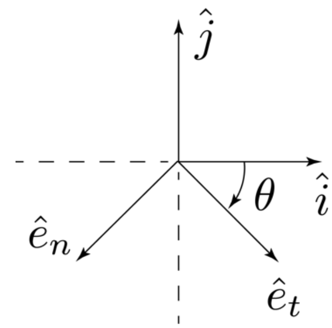
$\hat{\jmath}$: $-R\omega_O^2 = -L\alpha_{AB} - v_B^2/\rho \quad \Rightarrow$

$$\vec{\alpha}_{AB} = \frac{1}{L}[R\omega_O^2 - v_B^2/\rho]\hat{k} = \frac{1}{4}[(1)(1) - (2)^2/2]\hat{k} = -\left(\frac{1}{4}\right)\hat{k}\ rad\,/\,s^2$$ ⟵

Problem 3 *(20 points)*

Part A (2 points)

Given: To the right two coordinate systems are illustrated: a *Cartesian* coordinate system and a *Path* coordinate system. The Path coordinate system is oriented at an angle θ from the Cartesian system, as indicated.



Find: To identify the unit normal vector in terms of the Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$, circle the best answer from the options provided.

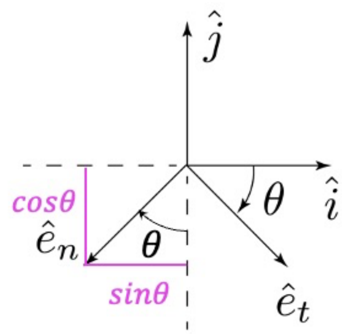
a) $\hat{e}_n = -\cos\theta\hat{i} + \sin\theta\hat{j}$

b) $\hat{e}_n = -\cos\theta\hat{i} - \sin\theta\hat{j}$

c) $\hat{e}_n = -\sin\theta\hat{i} - \cos\theta\hat{j}$

d) $\hat{e}_n = -\sin\theta\hat{i} + \cos\theta\hat{j}$

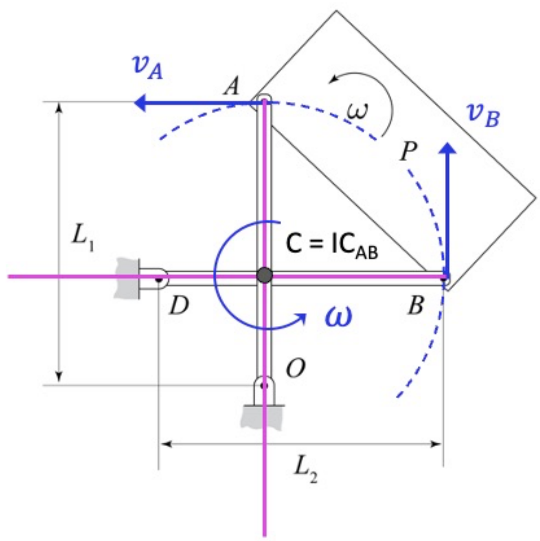
e) The correct answer is not given.



$\hat{e}_n = -\sin\theta\ \hat{\imath} - \cos\theta\ \hat{\jmath}$

Part B (3points)

Given: The mechanism consists of two links, AO and BD, with lengths L_1 and L_2 respectively, and plate P. Link AO is attached to a fixed pivot at point O, and link BD is attached to a fixed pivot at point D. The motion of the rectangular plate P is guided by these two links, which are arranged to cross each other without making contact. At the specific moment depicted, when the links are perpendicular to each other, the plate is moving with a counterclockwise angular velocity, denoted by ω.



Find: Please answer the following:

a) (1 points) On the figure above please draw and label the instantaneous center of rotation, *IC*, of rectangular plate P.

b) (2 points) Let $\vec{\omega}_{AO}$ represent the angular velocity of link AO. Choose the correct response below and provide an explanation for your response.

i. $\vec{\omega}_{AO}$ is counterclockwise

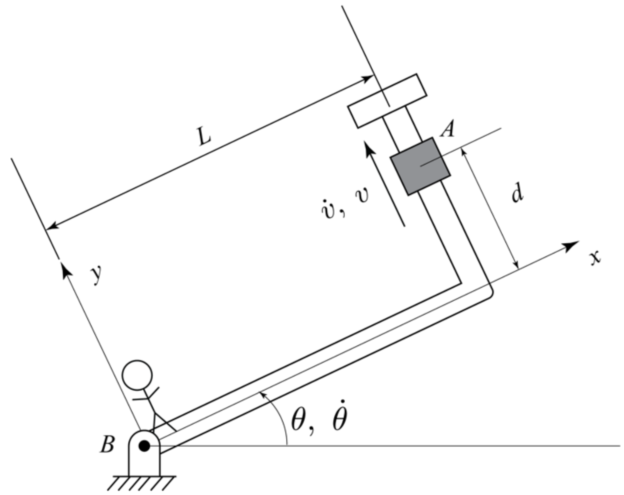
ii. $\vec{\omega}_{AO} = \vec{0}$

iii. $\vec{\omega}_{AO}$ is clockwise

- Draw perpendiculars to the directions of motion for A and B.
- The intersection of the perpendiculars is the location of the IC for AB (here, point C).
- Since AB is rotating CCW around point C, then the velocity of A is to the left.
- Since v_A is to the left and OA is rotating about O, then the angular velocity of OA is CCW.

Part C (4 points)

Given: The mechanism features a bent bar with the dimensions L and d as shown and is equipped with a small collar labeled A. This collar moves relative to the bar at a speed v with a constant change in speed $\dot{v}$. At the same time, the bar rotates around a fixed pivot point B at a constant counterclockwise angular velocity $\dot{\theta}$. An observer and an xyz coordinate system are affixed to the bent bar, as depicted.



The following equation is to be used to calculate the acceleration of the collar A:

$$\vec{a}_A = \vec{a}_B + \left(\vec{a}_{A/B}\right)_{rel} + \vec{\alpha} \times \vec{r}_{A/B} + 2\vec{\omega} \times \left(\vec{v}_{A/B}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{A/B}\right)$$

Find: Calculate the following vector quantities using the xyz coordinate system incorporating the parameters: L, d, v, $\dot{v}$, and $\dot{\theta}$. Note that not all parameters may be needed for each calculation. Answers should include the correct unit vectors.

a) $\vec{\omega} = angular\ velocity\ of\ the\ observer = \dot{\theta}\hat{k}$

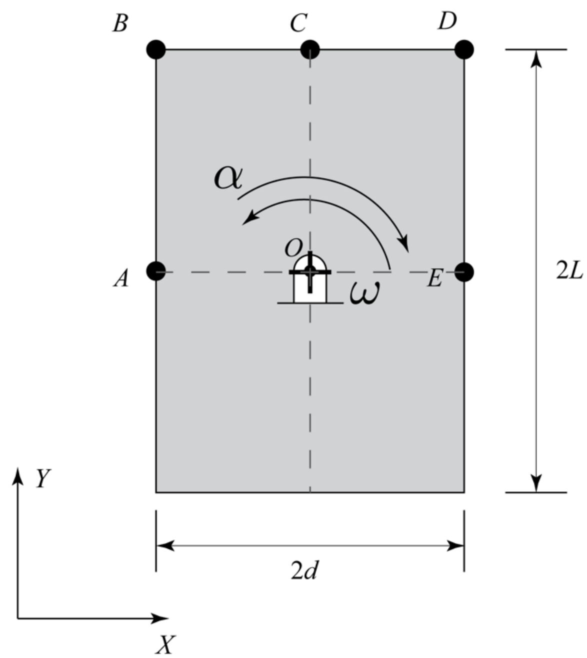
b) $\vec{\alpha} = angular\ acceleration\ of\ the\ observer = \ddot{\theta}\hat{k} = \vec{0}$

c) $\left(\vec{v}_{A/B}\right)_{rel} = velocity\ of\ A\ as\ seen\ by\ observer = v\hat{j}$

d) $\left(\vec{a}_{A/B}\right)_{rel} = acceleration\ of\ A\ as\ seen\ by\ observer = \dot{v}\hat{j}$

Part D (8 points)

Given: A rectangular plate with dimensions $2L$ (length) and $2d$ (width) rotates about a fixed pivot located at its center, point O. The plate's angular velocity ω is in a counterclockwise direction, while its angular acceleration α is in a clockwise direction, as illustrated in the figure.



Find: Calculate the following kinematic quantities. Write vector answers in terms of L, d, ω, and α. Not all parameters may be used in each answer.

a) $\vec{v}_C = -L\omega\hat{\imath}$

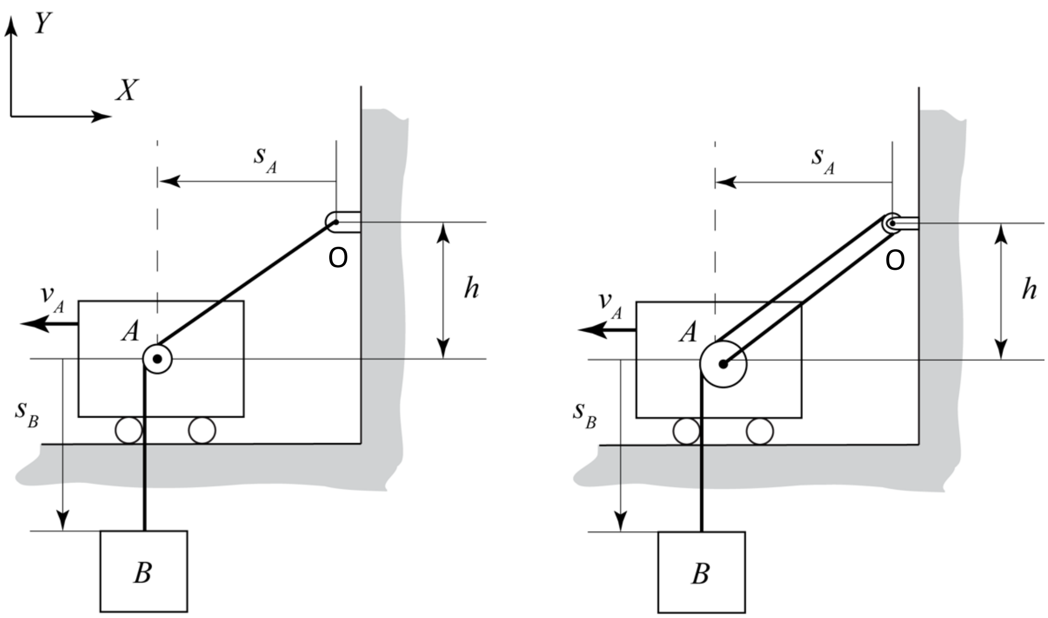
b) $\vec{v}_A = -d\omega\hat{\jmath} \;\Rightarrow\; \vec{v}_{C/A} = \vec{v}_C - \vec{v}_A = -L\omega\hat{\imath} + d\omega\hat{\jmath}$

c) $\vec{a}_A = d\alpha\hat{\jmath} + d\omega^2\hat{\imath}$

d) $\vec{a}_C = L\alpha\hat{\imath} - L\omega^2\hat{\jmath} \;\Rightarrow\; \vec{a}_{C/A} = \vec{a}_C - \vec{a}_A = (L\alpha - d\omega^2)\hat{\imath} - (d\alpha + L\omega^2)\hat{\jmath}$

Part E (3 points)

Given: Consider the two systems described below, each featuring a cart labeled A and a block labeled B. Cart A is equipped with a small pulley. In System I, a rope extends from block B, wraps around the pulley on cart A, and is anchored to the wall at point O. In System II, a rope extends from block B, wraps around the pulley on cart A, then passes around another pulley mounted on the wall at point O, and finally connects back to cart A. In both scenarios, cart A moves to the left at a speed v_A. In both Systems I and II, the pulleys' radii are considered negligible.



System I System II

Find: (3 points) Let $|\dot{s}_B|_I$ and $|\dot{s}_B|_{II}$ be the magnitudes of the y component of velocity of B for Systems I and II, respectively. Choose the correct response below:

a) $|\dot{s}_B|_I > |\dot{s}_B|_{II}$

b) $|\dot{s}_B|_I = |\dot{s}_B|_{II}$

c) $|\dot{s}_B|_I < |\dot{s}_B|_{II}$

$$L_I = length\ of\ cable\ for\ System\ I = s_{BI} + \sqrt{s_A^2 + h^2} + constant \quad \Rightarrow$$

$$dL_I/dt = 0 \quad \Rightarrow \quad |\dot{s}_{BI}| = \frac{s_A v_A}{\sqrt{s_A^2 + h^2}}$$

$$L_{II} = length\ of\ cable\ for\ System\ II = s_{BII} + 2\sqrt{s_A^2 + h^2} + constant \quad \Rightarrow$$

$$dL_{II}/dt = 0 \quad \Rightarrow \quad |\dot{s}_{BII}| = 2\frac{s_A v_A}{\sqrt{s_A^2 + h^2}} = 2|s_{BI}|$$