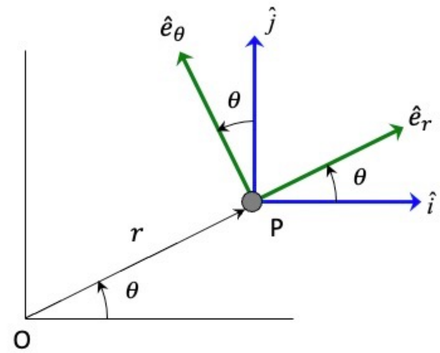
ME 274 – Spring 2024 **SOLUTION**
Quiz 2 – 11:30/1:30 sections
Worksheet

Consider the radial distance r and the angle θ used to describe the position of point P. When $\theta = 36.87°$, the velocity and acceleration of P in terms of their polar coordinates are, respectively: $\vec{v} = (4\hat{e}_r - 3\hat{e}_\theta)\, m/s$ and $\vec{a} = (10\hat{e}_\theta)\, m/s^2$.

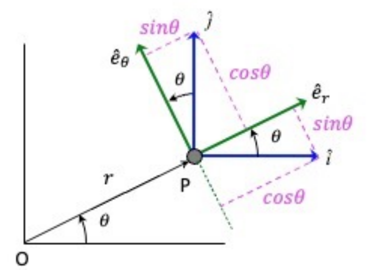
In your calculations, please use $cos 36.87° = 0.8$ and $sin 36.87° = 0.6$.



PART a) Write down the Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$ in terms of the polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$:

$$\hat{\imath} = cos\theta\hat{e}_r - sin\theta\hat{e}_\theta = 0.8\hat{e}_r - 0.6\hat{e}_\theta$$

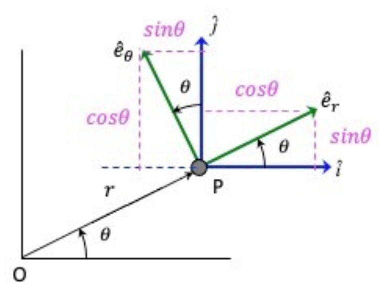
$$\hat{\jmath} = sin\theta\, \hat{e}_r + cos\theta\hat{e}_\theta = 0.6\hat{e}_r + 0.8\hat{e}_\theta$$



PART b) Write down the polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$ in terms of the Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$:

$$\hat{e}_r = cos\theta\hat{\imath} + sin\theta\hat{\jmath} = 0.8\hat{\imath} + 0.6\hat{\jmath}$$

$$\hat{e}_\theta = -sin\theta\hat{\imath} + cos\theta\hat{\jmath} = -0.6\hat{\imath} + 0.8\hat{\jmath}$$



PART c) What are the numerical values for $\dot{x}$ and $\dot{y}$? What are the numerical values for $\ddot{x}$ and $\ddot{y}$?

$$\dot{x} = \vec{v} \bullet \hat{\imath} = (4\hat{e}_r - 3\hat{e}_\theta)\bullet(0.8\theta\hat{e}_r - 0.6\hat{e}_\theta) = 5.0 \text{ m/s}$$

$$\dot{y} = \vec{v} \bullet \hat{\jmath} = (4\hat{e}_r - 3\hat{e}_\theta)\bullet(0.6\hat{e}_r + 0.8\hat{e}_\theta) = 0$$

$$\ddot{x} = \vec{a} \bullet \hat{\imath} = (10\hat{e}_\theta)\bullet(0.8\theta\hat{e}_r - 0.6\hat{e}_\theta) = -6.0 \text{ m}/s^2$$

$$\ddot{y} = \vec{a} \bullet \hat{\jmath} = (10\hat{e}_\theta) \bullet (0.6\hat{e}_r + 0.8\hat{e}_\theta) = 8.0 \text{ m}/s^2$$

BONUS part Is the speed of P increasing, decreasing or is it constant? Explain.

$$\dot{v} = \vec{a} \bullet \hat{e}_t = \vec{a} \bullet \frac{\vec{v}}{|\vec{v}|} = (10\hat{e}_\theta) \bullet \left(\frac{4\hat{e}_r - 3\hat{e}_\theta}{5}\right) = -6\frac{m}{s^2} < 0 \quad \Longrightarrow \quad \text{DECREASING in speed}$$