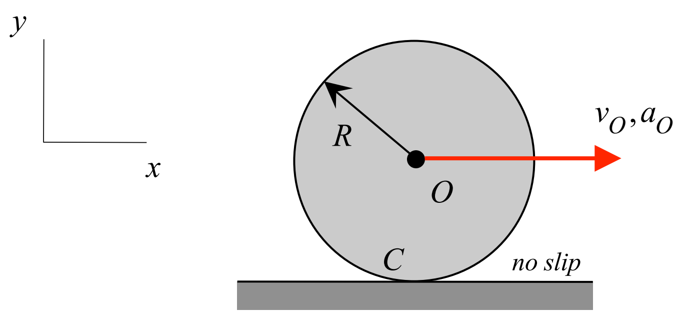
Rolling Without Slipping

Consider the wheel shown below that rolls along a rough, stationary horizontal surface with the center of the wheel O having a velocity and acceleration of v_O and a_O, respectively. As the wheel moves, it is assumed that sufficient friction acts between the wheel and the fixed horizontal surface that the contact point C does not slip. The consequences of the "no slip" condition at C are:

$$v_{cx} = 0$$

$$a_{cx} = 0$$

(If either of the above is not true, then point C "slips" as the wheel moves.)



We will encounter many problems throughout the course that involve the rolling without slipping of a body on a stationary surface. Although this concept is defined through a simple set of equations, the consequences of rolling without slipping on the velocity and acceleration of other points on the body can become quite complicated. We will see this through a number of examples.

CHALLENGE QUESTION: If C is a no-slip point, what are the y-components for the velocity and acceleration of C?

ANSWER: Since O moves on a straight, horizontal path, the y-components of the velocity and acceleration of O are zero. From this and the no-slip condition above for C, we can write:

$$\vec{v}_C = \vec{v}_O + \vec{\omega} \times \vec{r}_{C/O}$$
$$v_{Cy}\hat{j} = v_O\hat{i} + (\omega\hat{k}) \times (-R\hat{j}) = (v_O + R\omega)\hat{i} \qquad \Rightarrow \qquad v_{Cy} = 0$$

and

$$\vec{a}_C = \vec{a}_O + \vec{\alpha} \times \vec{r}_{C/O} - \omega^2\vec{r}_{C/O}$$
$$a_{Cy}\hat{j} = a_O\hat{i} + (\alpha\hat{k}) \times (-R\hat{j}) - \omega^2(-R\hat{j})$$
$$= (a_O + R\alpha)\hat{i} + R\omega^2\hat{j} \qquad \Rightarrow \qquad a_{Cy} = R\omega^2$$

From this we see that: $\vec{v}_c = \vec{0}$ and $\vec{a}_C = R\omega^2\hat{j} \neq \vec{0}$.