Homework H.6.G

Given: A stepped drum (of mass $2m$) is pinned to ground at its center, with the inner and outer radii of the drum given by R and $2R$, respectively. The radius of gyration of the drum about its center of mass is given by k_O. A cable is wrapped around the inner radius of the drum with the other end of the cable connected to particle A that has a mass of m. A second cable is wrapped around the outer radius of the drum with the other end of the cable being attached to connector B, with B, in turn, connected to a grounded spring of stiffness k and a grounded dashpot having a damping coefficient c, as shown in the figure. As the system moves, the cables are known to not slip on the drum nor do the cables do slack. Let θ represent the rotation of the drum, with θ being defined positive in the clockwise direction. The mass of B is to be considered to be negligible.

Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate θ;
(b) From the EOM, determine the static rotation of the drum, θ_{st};
(c) Rewrite the EOM of the system in terms of the variable $z = \theta - \theta_{st}$, where z represents the rotation of the drum relative to its static equilibrium rotation;
(d) Determine the natural frequency of the system in terms of, at most, the given parameters of the problem: m, R, k and k_O; and,
(e) Determine the ratio of the parameters $c/\sqrt{km}$ that is required for critical damping to exist in the response of the system. Use $R/k_O = 1$.