Exam Final - Main Time

Particle Kinetics and Kinetics;
Planar Rigid Body Kinematics and Kinetics; Vibrations

Last Name: KEY________________ **First Name:** ________________________

Instructor (please circle your lecture time): (8:30am) Krousgrill (10:30am) Krousgrill

(11:30am) Gibert (1:30pm) Davies (2:30pm) Sotelo (4:30pm) Rojas

Instructions:

- You have 120 minutes for this CLOSED-BOOK, CLOSED-NOTES, NO CRIB SHEETS exam.
- You are allowed to use a calculator but only for simple calculations during the exam.
- The only person you should communicate with during the exam is the exam supervisor.
- ALL THE WORK YOU SUBMIT MUST BE YOUR OWN WORK, see page (ii).

- If you are using the BLANK exam, there is no need to write out the problem statements in full. Only redraw the Figure, if you are asked to draw something on it.

- Each problem should start on a new page, and write on one side of each page only.
- Draw FBDs for each kinetics problem. Clearly show the coordinate system(s) being used.
- Use PROPER VECTOR NOTATION in your solution and include UNITS in your answers.
- For most problems, work the problem using variables, substituting in numbers at the end.

- You must SHOW ALL WORK to receive full credit.
- If your solution does not reflect a logical thought process, it will be assumed to be in error.
- If it is not clear how you produced your answers, you will not get credit for your solution.
- When errors greatly simplify the subsequent steps in a problem, you will lose additional points.

- Put your NAME, PROBLEM NUMBER on EACH PAGE.
- SCAN SHEETS IN ORDER: Cover page (filled in), honor statement (signed), Problem 1, Problem 2, Problem 3, and Problem 4 sheets, and sheets in the correct order for each problem.
- Identify which sheets belong to which problem when submitting via Gradescope.

Problem	Score
1 (20 pts)	
2 (20 pts)	
3 (20 pts)	
4 (20 pts)	
Total (80 max)	

Code of Honor for Students taking ME 274 Examinations

I understand that this is a CLOSED BOOK EXAM and I may only use my calculator to do simple arithmetic and trigonometric calculations.

During the examination, I will NOT USE crib sheets, course notes, text books, nor any other similar materials. I WILL NOT communicate with anyone else during the exam, nor will I accept help from anyone else. I WILL NOT use information on the internet or elsewhere to help me solve the exam problems.

All the work that I submit will be my own work.

I have read Code of Honor for Students taking ME274 and I agree to abide by this Code.

Signature: ______________________________

PRINT Last Name: ______________________________

PRINT First Name: ______________________________

Date: ______________________________

The Purdue Honor Code: *As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together. We are Purdue.*

Equation Sheet

$$\vec{v}_P = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} = v_P\hat{e}_t = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k}$$

$$\vec{a}_P = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k} = \dot{v}_P\hat{e}_t + \frac{v_P^2}{\rho}\hat{e}_n + 0\ \hat{e}_b = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{k}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

$$\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

$$\sum \vec{F} = m\vec{a}_P$$

$$T_1 + V_1 + U_{1\to 2}^{nc} = T_2 + V_2$$

where:

$T = \sum_i^N \frac{1}{2}m_i v_i^2$ is the kinetic energy of each particle and N is the number of particles,

$V = V_g + V_{sp}$ where $V_g = mgh,\ h$ is the signed distance above or below the datum,

$V_{sp} = \frac{1}{2}k(l - l_o)^2,$ and l, l_o are the deformed and original length of the spring, respectively, and

$U_{1\to 2}^{nc}$ is the work done by nonconservative forces.

$$m\vec{v}_{P_1} + \int_{t_1}^{t_2} \sum \vec{F}dt = m\vec{v}_{P_2} \qquad e = -\frac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$$

$$\vec{H}_{O_1} + \int_{t_1}^{t_2} \sum \vec{M}_O dt = \vec{H}_{O_2} \qquad \vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P$$

$$\sum \vec{F} = m\vec{a}_G \qquad \sum \vec{M}_A = I_A\vec{\alpha} + \vec{r}_{G/A} \times m\vec{a}_A$$

$$T_1 + V_1 + U_{1\to 2}^{nc} = T_2 + V_2 \qquad T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A \bullet (\vec{\omega} \times \vec{r}_{G/A})$$

$$m\vec{v}_{G_1} + \int_{t_1}^{t_2} \sum \vec{F}dt = m\vec{v}_{G_2}$$

$$\vec{H}_{A_1} + \int_{t_1}^{t_2} \sum \vec{M}_A dt = \vec{H}_{A_2} \qquad \vec{H}_A = I_A\ \vec{\omega},\ \text{for special cases of point A.}$$

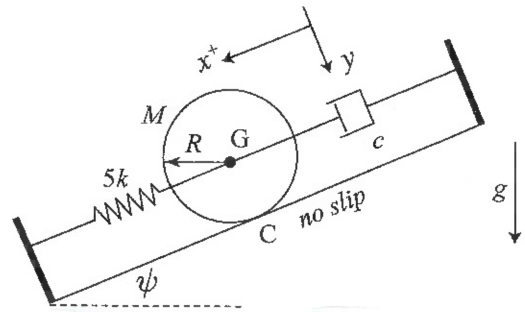
$$I_{G,disk} = \frac{1}{2}mr^2 \qquad I_{G,bar} = \frac{1}{12}mL^2 \qquad I_{G,plate} = \frac{1}{12}m\left(a^2 + b^2\right) \qquad I_{G,sphere} = \frac{2}{5}mr^2 \qquad I_{G,ring} = mr^2$$

$$M\ddot{x} + C\dot{x} + Kx = F(t) \qquad \omega_n = \sqrt{\frac{K}{M}} \qquad 2\zeta\omega_n = \frac{C}{M} \qquad \omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

$$x_P(t) = A_p \cos\omega t + B_p \sin\omega t \qquad x_C(t) = e^{-\zeta\omega_n t}\Big[A_c \cos\omega_d t + B_c \sin\omega_d t\Big]$$

Problem 1 (20 points):

Given:
A disk of radius R meters and mass M Kg is released from rest on an inclined plane when the spring (of stiffness $5k$ N/m) is unstretched (x=0). Note k is the spring constant.
The center of the disk is also attached to a viscous damper (damping coefficient, c Kg/s). Note c is the damping constant. The disk rolls without slipping on the rough inclined plane.



Let $\vec{\alpha}$ denote the angular acceleration and $\vec{\omega}$ denote the angular velocity of the disk.
Known variables in this problem are: M, R, k, c, ψ and g.

Find:

b(b)

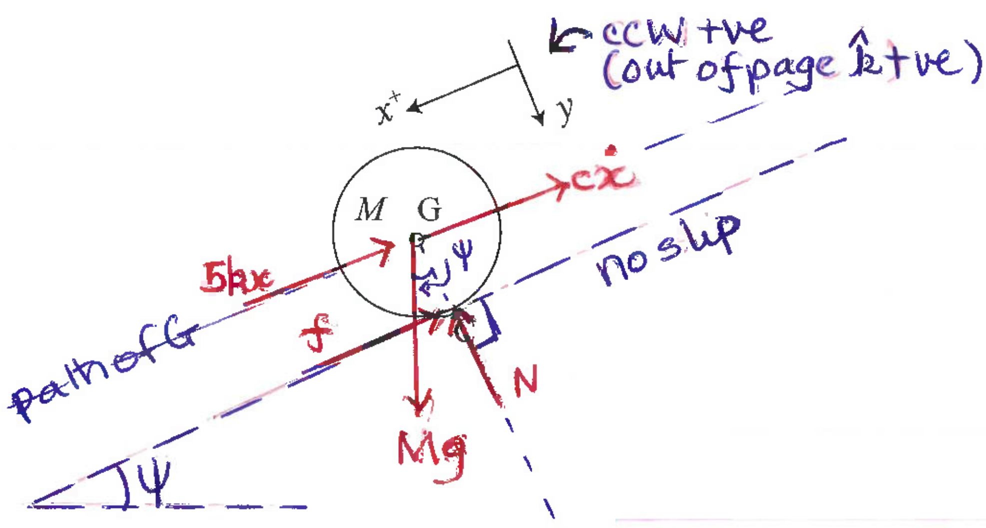
(a) Draw the free body diagram of the disk at a position x meters down the plane. Write down Euler's equation for the disk taking moments about G, and also write down Newton's equations for the disk in both the x and y directions.

(c) Starting from the rigid body kinematics equations, DERIVE the relationships between α and $\ddot{x}$. This expression should be true at any location x along the path of G.

(d) Use some or all of the equations in parts (b) and (c) to derive the equation of motion of the center of the disk (G). Put this equation in ***standard form*** with the coefficient of $\ddot{x}$ equal to one. [There is no need to transform the equation of motion to describe the motion about the static equilibrium position.]

(e) Use your equation of motion in part (d), to determine the undamped natural frequency (ω_n) and the damping ratio (ζ) for this system.

Solution:
(a) Draw the free body diagram of the disk at a position x meters down the plane.



(b) Write down Euler's equation for the disk taking moments about G, and also write down Newton's equations for the disk in both the x and y directions.

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$\bar{a}_G = a_{G_x}\hat{i} = \ddot{x}\hat{i}$

$\Sigma M_G = I_G\alpha$: $fR = I_G\alpha = \frac{1}{2}MR^2\alpha$ — (1) ANS

$\Sigma F_x = M\ddot{x}_G$: $-5kx - c\dot{x} - f + Mg\sin\psi = M\ddot{x}$ — (2) ANS

$\Sigma F_y = M\ddot{y}_G$: $Mg\cos\theta - N = M\ddot{y} = 0$ — (3) ANS

(c) Starting from the rigid body kinematics equations, DERIVE the relationships between α and $\ddot{x}$. This expression should be true at any location x along the path of G.

$\bar{a}_G = \bar{a}_C + \bar{\alpha}\times\bar{r}_{G/C} - \omega^2\bar{r}_{G/C} \Rightarrow a_G\hat{i} = a_C\hat{j} + \alpha\hat{k}\times(-R\hat{j}) - \omega^2(-R\hat{j})$

no slip

$\therefore a_G\hat{i} = a_C\hat{j} + \alpha R\hat{i} + \omega^2 R\hat{j}$

$\hat{i}$ components: $a_G = \alpha R$, i.e., $\ddot{x} = \alpha R$ — Ans (4)

(d) Use some or all of the equations in parts (b) and (c) to derive the equation of motion of the center of the disk (G). Put this equation in ***standard form*** with the coefficient of $\ddot{x}$ equal to one. [There is no need to transform the equation of motion to describe the motion about the static equilibrium position.]

From (1) & (4): $f = \frac{1}{2}M\cancel{R}\ddot{x}/\cancel{R} = \frac{1}{2}M\ddot{x}$

Sub. into (2): $-5kx - c\dot{x} - \frac{1}{2}M\ddot{x} + Mg\sin\theta = M\ddot{x}$

Rearrange: $\frac{3}{2}M\ddot{x} + c\dot{x} + 5kx = Mg\sin\theta$

Divide by $3/2\,M$:

$$\ddot{x} + \frac{2c}{3M}\dot{x} + \frac{10k}{3M}x = Mg\sin\theta \quad (5) \text{ Ans}$$

(e) Use your equation of motion in part (d), to determine the undamped natural frequency (ω_n) and the damping ratio (ζ) for this system.

Standard form of the 2nd order differential equation:

$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t)$

comparing with (5) gives

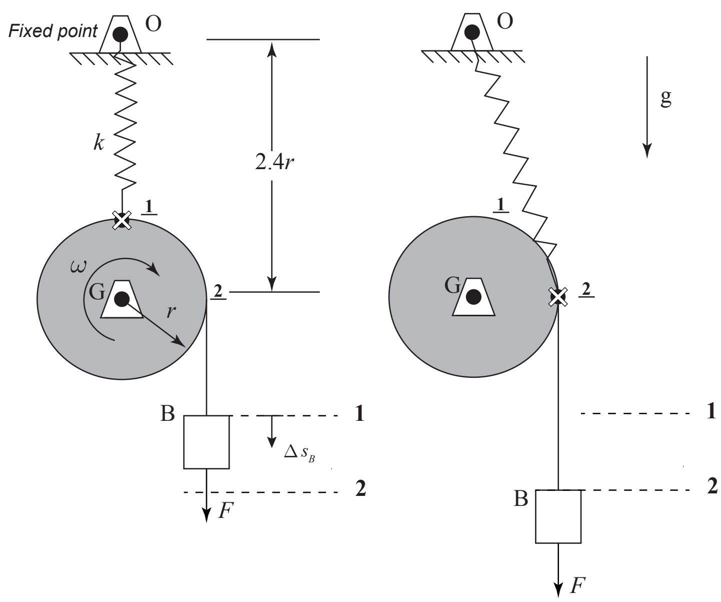
$\cancel{2}\zeta\omega_n = \frac{\cancel{2}c}{3M}$ and $\omega_n^2 = \frac{10k}{3M}$

$\therefore \omega_n = \sqrt{\frac{10k}{3M}}$ ANS, and $\zeta = \frac{c}{3M}\sqrt{\frac{3M}{10k}} = \frac{c}{\sqrt{30kM}}$ ANS

This page is for extra work related to Problem 1 ______________________________

Problem 2 (20 points):

Given: A homogeneous disk with radius, r (m), and mass m (kg), rotates about its center and fixed point, **G**. A spring with stiffness k (N/m) is fixed at one end at point **O** and is attached initially unstretched at point **1**. A block **B**, with mass M (kg), is attached to a cable that is wrapped around the disk and is pulled straight down by an externally applied force F (N) a distance Δs_B (m) (**unknown**, determine this distance in terms of known variables based on the positions the disk rotates to get from **1** to **2** corresponding to the rotation amount (arc length) of the disk at each point (**1** (no rotation) to **2** (quarter rotation)). The system is released from rest.



System at Position I **System at Position 2**

Find:

(a) Draw the free body diagram of the **total system** (disk, cable, block) at an arbitrary position so that the spring force can be displayed. Clearly indicate your **coordinate system** and define your gravitational potential energy **datum(s)**.

(b) Write the potential energy in terms of all known variables due to gravity for the block **B**, $V_{gravity}$, as defined by your datum at positions **1** and **2**.

(c) Write the potential energy in terms of all known variables due to the spring, V_{spring}, at positions **1** and **2**.

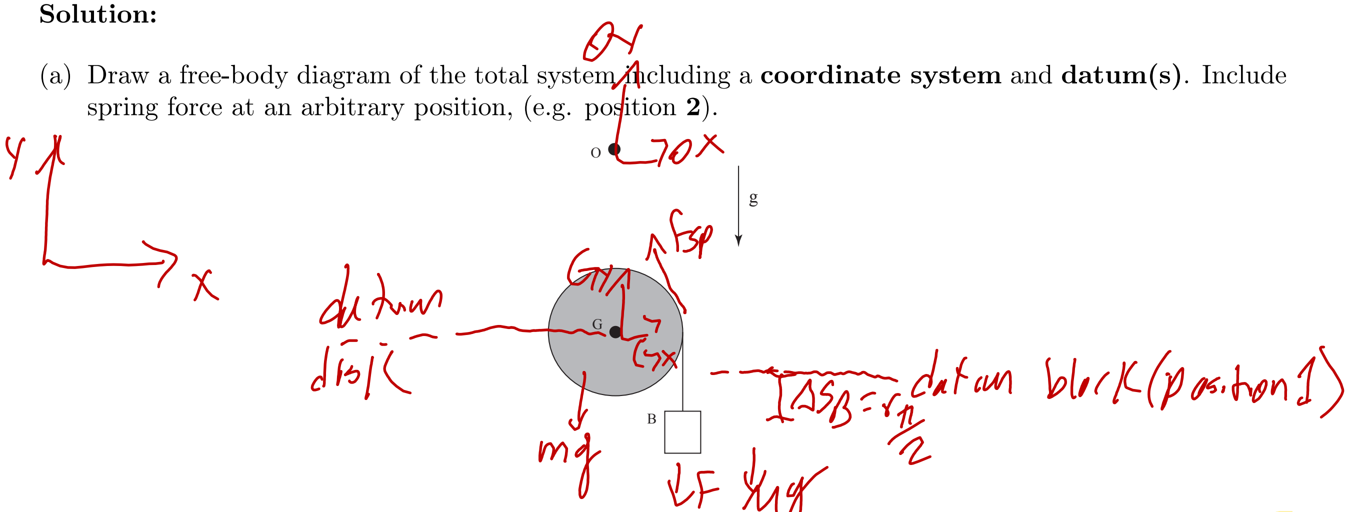
(d) Clearly write out the Work-Energy equation and the individual terms associated including the kinetic energies (T_i), potential energies V_i, and non-conservative work ($U^{nc}_{1\rightarrow 2}$) from position **1** to **2**.

(e) From position **1** to **2**, find the angular speed ω_2 of the disk in terms of all known variables. You may have to use kinematics to relate linear velocity of the block **B** and angular velocity of the disk.

Units must be clearly stated as part of the answer (only at the very end) you do not have to carry units throughout the analysis. Note that your final answers should be in terms of (at most) the known variables: m, M, r, k, F, π, and gravity, g.

Solution:

(a) Draw a free-body diagram of the total system including a **coordinate system** and **datum(s)**. Include spring force at an arbitrary position, (e.g. position **2**).

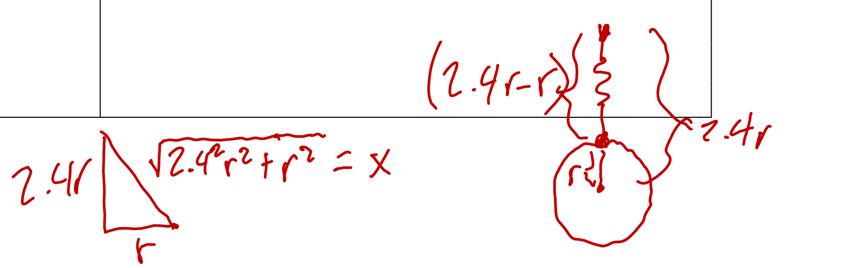


(b) Using the datum you have drawn on the FBD, write the potential energy in terms of all known variables due to gravity for the block **B** at positions **1** and **2**. Note: Δs_B is unknown, this variable must be in terms of known constants.

$V_{gravity}$ at **1**	$V_{gravity}$ at **2**
$Mgh = 0$ J @ datum shown for block	$Mg\Delta S_B = -Mgr\frac{\pi}{2}$

(c) Write the potential energy in terms of all known variables due to the spring at positions **1** and **2**

V_{spring} at **1**	V_{spring} at **2**
$\frac{1}{2}k\Delta^2 = 0$ initially unstretched	$\frac{1}{2}k\Delta^2 = \frac{1}{2}k\left(x - (2.4r - r)\right)^2$ $= \frac{1}{2}k(1.2r)^2 = 0.72kr^2$



(d) Write out the Work-Energy equation in its full form (T,V,U) followed by filling out the contributions of energies associated with the system for each state (1,2). Note: Δs_B is unknown, this variable must be in terms of known constants. You may have unknown angular and linear velocities in your expressions for kinetic energy.

$$\text{W-E:}\quad T_1 + V_1 + U^{nc}_{1\to2} = T_2 + V_2$$

(i) $T_1 = 0$

(ii) $V_1 = V_{1,spring} + V_{1,gravity} = 0 + 0 + 0$

(iii) $U^{nc}_{1\to 2} = F\Delta s_B = F r \frac{\pi}{2}$

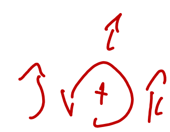
(iv) $T_2 = \frac{1}{2} I_G \omega_2^2 + \frac{1}{2} M v_{B2}^2$

(v) $V_2 = V_{2,spring} + V_{2,gravity} = -Mgr\frac{\pi}{2} + \frac{1}{2} k \Delta^2 = -Mgr\frac{\pi}{2} + 0.72 k r^2$

(e) Is energy for the total system conserved? Provide a very brief explanation.

NO, $\vec{F}$ applied

(f) Determine the angular speed of the disk, ω_2. You may have to use kinematics to relate linear velocity of the block **B** and angular velocity of the disk.



$$Fr\frac{\pi}{2} = \frac{1}{2} I_G \omega_2^2 + \frac{1}{2} M v_{B2}^2 - Mgr\frac{\pi}{2} + 0.72 k r^2$$

$$Fr\frac{\pi}{2} = \frac{1}{2} I_G \omega_2^2 + \frac{1}{2} M (r\omega_2)^2$$

$$\omega_2 = \sqrt{\frac{2\left(Fr\frac{\pi}{2} + Mgr\frac{\pi}{2} - 0.72 k r^2\right)}{I_G + M r^2}}$$

$v_{B'} = v_B = \vec{v}_G + \vec{\omega} \times \vec{r}_{B'/G}$

$= \vec{0} + -\omega \hat{k} \times (r\hat{i})$

$\vec{v}_{B'} = \vec{v}_B = -r\omega \hat{j}$

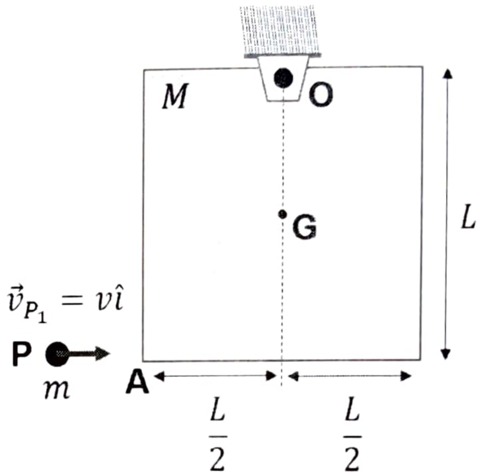
This page is for extra work related to Problem 2

Problem 3(20 points):

Given: A particle P of mass m (kg) travels with a horizontal velocity of $\vec{v}_{P1} = v\hat{i}$ (m/s) when it impacts the corner A of a homogeneous **square** plate of mass M (kg) as depicted. Immediately after impact the particle P becomes embedded to the plate at point A. The plate is pinned to the ground at O, and has a center of mass G. The sides of the plate are length L (m). All motion occurs in the HORIZONTAL PLANE.

Known variables in this problem are m, M, v, and L.

Find:



(a) Draw the free body diagram of the system made up of particle P and the square plate during impact, clearly indicating the coordinate system.

(b) Why is it beneficial to consider angular momentum about point O instead of G for this system? Justify briefly.

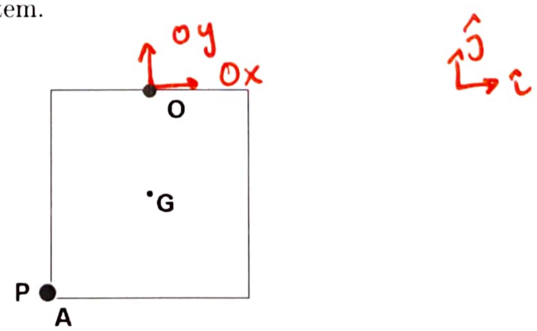
(c) Write the angular momentum expressions for the particle P and the square plate about point O right before impact. Answers should be in terms of, at most, the parameters m, M, L, and v.

(d) Write the angular momentum expressions for the particle P and the square plate about point O immediately after impact. Answers should be in terms of, at most, the parameters m, M, L, v, and the unknown ω.

(e) Find the angular velocity $\vec{\omega}$ of the square plate immediately after impact. The answer should be in terms of, at most, the parameters m, M, L, and v.

Solution:

(a) Draw the free body diagram of the system made up of particle P and the square plate during impact, clearly indicating the coordinate system.



(b) Why is it beneficial to consider angular momentum about point O instead of G for this system? Justify briefly.

$\Sigma M_G \neq 0$ but $\Sigma M_O = 0$, Thus angular momentum is conserved about point O.

(c) Write the angular momentum expressions for the particle P, $\vec{H}_{O_{P1}}$, and the square plate, $\vec{H}_{O_{S1}}$, about point O right before impact Answers should be in terms of, at most, the parameters m, M, L, and v.

$\vec{H}_{O_{P1}} = \vec{r}_{P/O} \times m\vec{v}_{P_1} = (-\frac{L}{2}\hat{i} - L\hat{j}) \times mv\hat{i} = mLv\hat{k}$

$\vec{H}_{O_{S1}} = 0$

(d) Write the angular momentum expressions for the particle P , $\vec{H}_{O_{P2}}$, and the square plate, $\vec{H}_{O_{S2}}$, about point O immediately after impact. Answers should be in terms of, at most, the parameters m, M, L, v and the unknown ω.

$\vec{H}_{O_{P2}} = \vec{r}_{P/O} \times m\vec{v}_{P_2} = (-\frac{L}{2}\hat{i} - L\hat{j}) \times m[\cancel{\vec{v}_O}^{\,0} + \vec{\omega} \times \vec{r}_{P/O}]$

$= (-\frac{L}{2}\hat{i} - L\hat{j}) \times m[\omega\hat{k} \times (-\frac{L}{2}\hat{i} - L\hat{j})]$

$= m\omega[(\frac{L}{2})^2 + (L)^2] = \frac{5}{4}mL^2\omega\,\hat{k}$

$\vec{H}_{O_{S2}} = I_O\vec{\omega} = (I_G + M|\vec{r}_{G/O}|^2)\,\omega\hat{k} = (\frac{1}{12}M(L^2+L^2) + M(\frac{L}{2})^2)\,\omega\hat{k}$

$= \frac{5}{12}ML^2\omega\,\hat{k}$

(e) Find the angular velocity $\vec{\omega}$ of the square plate immediately after impact. The answer should be in terms of, at most, the parameters m, M, L, and v.

$\vec{H}_{O_1} = \vec{H}_{O_2}$

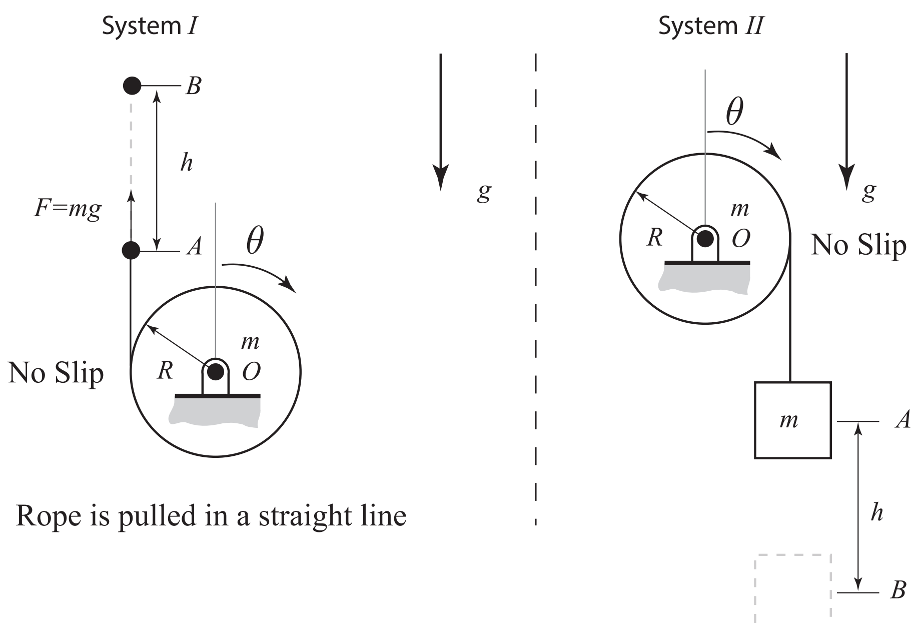
$m\cancel{L}v\hat{k} = [\frac{5}{4}mL^{\cancel{2}}\omega + \frac{5}{12}ML^{\cancel{2}}\omega]\hat{k}$

$\vec{\omega} = \frac{m}{(\frac{5}{4}m + \frac{5}{12}M)}\frac{v}{L} = \frac{m}{(3m+M)}\frac{12v}{5L}\ \frac{\text{rad}}{\text{s}}\,\hat{k}$

This page is for extra work related to Problem 3 ____________________

Problem 4, Part 1 (6 points): ______________________________

Given: Consider Systems I and II shown below. System I consists of a homogenous disk pinned at O of mass m (kg) attached to a rope where force F is applied. System II consists of a homogenous disk pinned at O of mass m (kg) attached to a rope that passes over a frictionless pulley. The rope is connected to a block of mass m. In system I the magnitude of force F equals mg where g is the gravitational constant and the rope is pulled from position A to position B by a distance h. In system II the mass drops from position A to position B by a distance h. In both systems the rope does not slip with respect to the disk.



Find:

A: *Fill in the circle with the correct answer.* Compare the angular speeds of the disks of system I to system II.

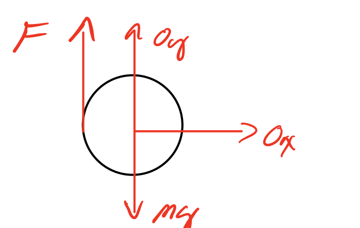
(a) ● $(\omega)_I > (\omega)_{II}$

(b) ○ $(\omega)_I = (\omega)_{II}$

(c) ○ $(\omega)_I < (\omega)_{II}$

(d) ○ It is NOT possible to answer.

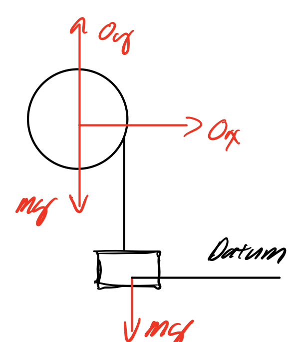
B: Justify your answer.



$$\cancel{T_1}^{0} + \cancel{V_1}^{0} + \Sigma U_{1-2}^{NC} = T_2 + \cancel{V_2}^{0}$$

$$\cancel{F}^{mg} h = \tfrac{1}{2}\left(\tfrac{1}{2} m r^2\right)\omega^2$$

$$\omega = \sqrt{4gh/r^2} = 2\sqrt{gh/r^2}$$



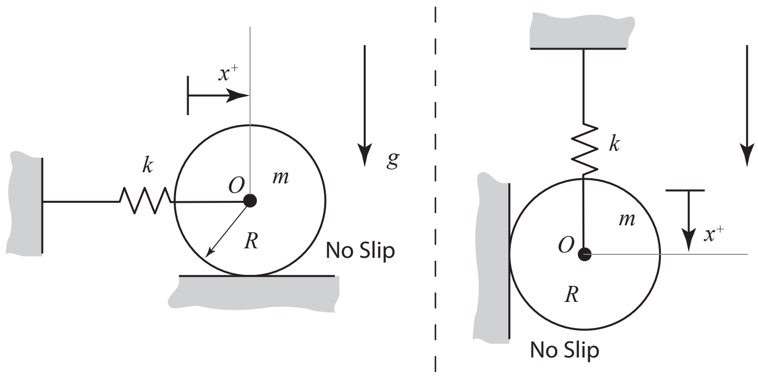
$$\cancel{T_1}^{0} + \cancel{V_1}^{0} + \Sigma \cancel{U_{1-2}^{NC}}^{0} = T_2 + V_2$$

$$0 = \tfrac{1}{2}\left(\tfrac{1}{2} m r^2\right)\omega^2 + \tfrac{1}{2} m \underbrace{\cancel{v_B}^2}_{(\omega r)^2} - mgh$$

$$\omega = \sqrt{\frac{mgh}{\frac{1}{4} m r^2 + \frac{1}{2} m r^2}} = \sqrt{\tfrac{4}{3}\, gh/r^2} = 2\sqrt{\frac{gh}{3r^2}}$$

Problem 4, Part 2 (4 points):

Given: The position x of each mass is measured from the unstretched position. Which of the systems will oscillate around a non-zero static equilibrium state?



System A — Yes or No

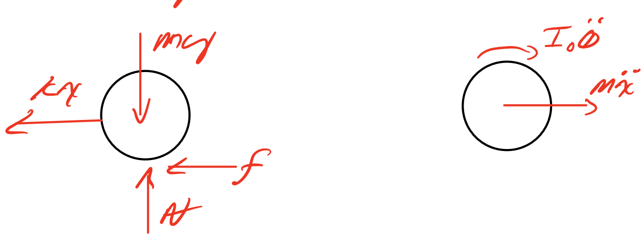
System B — Yes or No

Find:

A: *Circle Yes or No underneath the figure.*

B: Justify your answer for system A.

Gravity not a factor $\Rightarrow x_{ST} = 0$ or



$\xrightarrow{+} \Sigma F_x: -kx - f = m\ddot{x}$

$\circlearrowleft + \Sigma M_O: fr = \underbrace{I_O}_{\frac{1}{2}mr^2}\ddot{\theta}$

$\frac{3}{2}m\ddot{x} + kx = 0$

C: Justify your answer for system B.

Gravity not a factor $\Rightarrow x_{ST} > 0$ or

$+\downarrow \Sigma F_x: -kx - f + mg = m\ddot{x}$

$\circlearrowleft + \Sigma M_O: fr = \underbrace{I_O}_{\frac{1}{2}mr^2}\ddot{\theta}$

$\frac{3}{2}m\ddot{x} + kx = mg$

Problem 4, Part 3 (4 points): ______

Given: A system consisting of a mass m (kg) and a stiffness $2k$ (N/m) rolls along a smooth surface. The system is excited by a sinusoidal force $F_o \sin \Omega t$ (N), where $\Omega = \sqrt{4k/m}$ (rad/s). The equation of motion of the system can be written as

$$m\ddot{x} + 2kx = F_o \sin \Omega t \tag{1}$$

Recall the steady state oscillations are given as $x(t) = X \sin \Omega t$ (m), where X is a function of the frequency of excitation, amplitude of force, and mass and stiffness of the system and for this problem is given by $X = \dfrac{F_o}{2k - m\Omega^2}$.

Find:

A: *Fill in the circle corresponding to the correct answer.* The natural frequency is ω_n is

(a) ◯ $\omega_n = \sqrt{\frac{k}{m}}$,

(b) ◯ $\omega_n = \sqrt{\frac{k}{2m}}$,

(c) ● $\omega_n = \sqrt{\frac{2k}{m}}$,

(d) ◯ $\omega_n = \sqrt{\frac{2k}{3m}}$,

(e) ◯ Not enough information to answer/ Correct response not available.

$m\ddot{x} + 2kx = F_o \sin \Omega t$ divide by m put into standard form

$\ddot{x} + \frac{2k}{m}x = \frac{F_o}{m} \sin \Omega t$

$\Rightarrow \omega_n^2 = 2k/m \rightarrow \underline{\omega_n = \sqrt{2k/m}}$

B: *Fill in the circle corresponding to the correct answer.* The absolute value of the ratio of amplitude X at $\Omega = 0$ or $X(0)$ to X at $\Omega = \sqrt{4k/m}$ or $X(\Omega)$ can be written as $\left|\dfrac{X(0)}{X(\Omega)}\right|$ and is equal to

(a) ◯ $\left|\dfrac{X(0)}{X(\Omega)}\right| = 2$,

(b) ◯ $\left|\dfrac{X(0)}{X(\Omega)}\right| = \frac{1}{2}$,

(c) ● $\left|\dfrac{X(0)}{X(\Omega)}\right| = 1$,

(d) ◯ $\left|\dfrac{X(0)}{X(\Omega)}\right| = 6$,

(e) ◯ $\left|\dfrac{X(0)}{X(\Omega)}\right| = \frac{1}{6}$,

(f) ◯ Not enough information to answer/ Correct response not available.

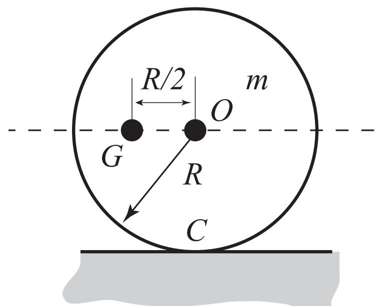
$X = \dfrac{F_o}{2k - m\Omega^2}$

$X(0) = \dfrac{F_o}{2k}$ $\quad X(\Omega) = \dfrac{F_o}{2k - m\left(\sqrt{4k/m}\right)^2} = \dfrac{F_o}{-2k}$

C: Justify your answer for B.

Problem 4, Part 4 (6 points): ______

Given: A non-homogeneous disk of radius R shown has a center of mass G, a geometric center located at O, and contacts the ground at point C. The center of mass is a distance $R/2$ from the geometric center. At the position shown G is directly to the left of the geometric center O. The disk has a mass of m and a centroidal radius of gyration k_G.



A: *Fill in the circle* with the correct answer that describes the moment of inertia of the disk about point O, I_O, compared to the moment of inertia about point C, I_C.

(a) ○ $I_O = I_C$
(b) ● $I_O < I_C$
(c) ○ $I_O > I_C$

B: Justify your answer for A.

$$I_O = I_G + m(R/2)^2 = mk_G^2 + mR^2/4$$

$$I_C = I_G + m((R/2)^2 + R^2) = mk_G^2 + m5R^2/4$$

This page is for extra work related to Problem ______

This page is for extra work related to Problem ____________________

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