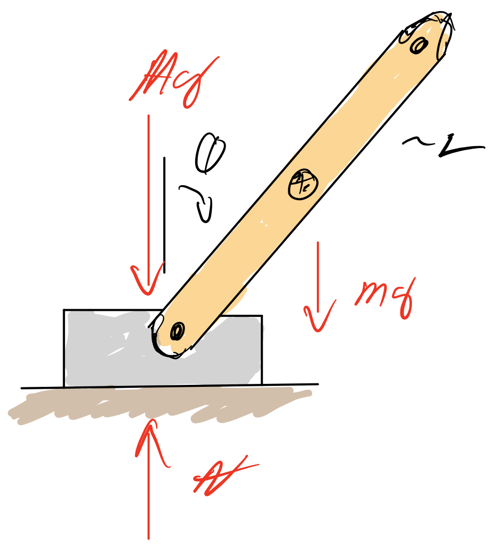
Example 5.C.9 Answer Key

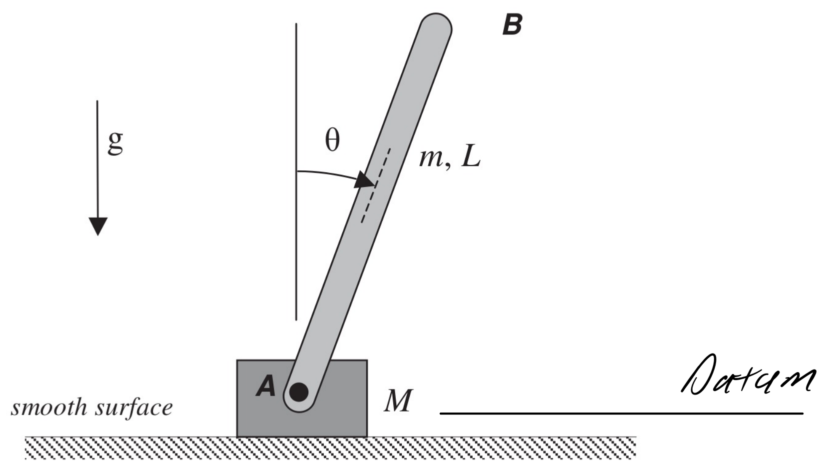
Given: A rod of mass m and length L is pinned to a block of mass M. The block can slide on a frictionless horizontal surface. The rod is released from rest when $\theta = 30°$.

Find: Determine:
(a) The angular velocity of the rod when $\theta = 90°$; and
(b) The velocity of the block when $\theta = 90°$.

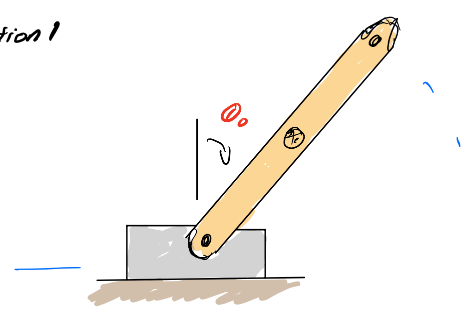
Consider using both the work-energy and linear impulse-momentum equations in your solution. Use the following parameters in your analysis: $m = 10$ kg, $M = 30$ kg and $L = 2$ m.

① FBD





Position 1



$\theta_0 = 30$ deg.

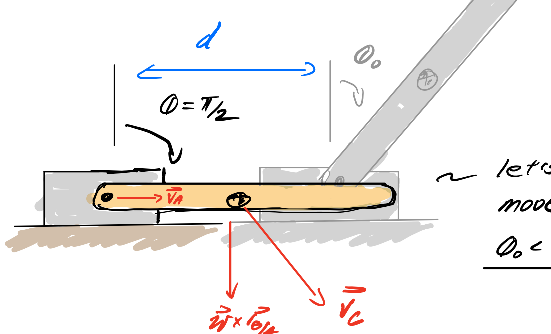
$T_1 + V_1 + \Sigma U_{1-2}^{NC} = T_2 + V_2$

$T_1 = 0 \qquad V_1 = mg\,L/2 \cos\theta_0$

$T_2 = 1/2\, m V_G^2 + 1/2\, I_G \omega^2 + 1/2\, M V_A^2$

$mg\,L/2 \cos\theta_0 = 1/2\, m V_G^2 + 1/2\, I_G \omega^2 + 1/2\, M V_A^2$

Position 2



I don't need 'd' since the system is conservative.

~ lets say it move to $\theta = \theta_2$

$\theta_0 < \theta_2 < \pi/2$

Equations 1, unknowns: V_G, ω, V_A

Now momentum is conserved in "x" direction

$\xrightarrow{+}$ $\quad 0 = M V_A + m V_{Gx}$

~ correct drawing but I am going to assume positive in my solution.

ame
ve is
$\theta = 90°$.

③ Kinematics

$$\bar{V}_G = \bar{V}_A + \vec{\omega} \times \vec{r}_{G/A}$$

$$V_{Gx}\hat{i} + V_{Gy}\hat{j} = V_A\hat{i} + \omega\hat{k} \times L/2\,\hat{i}$$

$$V_{Gx}\hat{i} + V_{Gy}\hat{j} = \bar{V}_A\hat{i} + \omega L/2\,\hat{j}$$

$\hat{i}$ $\quad V_{Gx} = V_A$

$\hat{j}$ $\quad V_{Gy} = \omega L/2$

④ <u>Solve</u>

ok, we have a contradiction

$V_A = -\frac{\cancel{M}}{m} V_{Gx}$ - and $V_{Gx} = V_{Ax}$

The only solution that satisfies is

$V_{Gx} = V_{Ax} = 0$. This means that it falls and stops.

$$mgL/2\cos\theta_0 = 1/2\, m V_G^2 + 1/2\, I_G \omega^2 + 1/2\, \cancel{M} V_A^2$$

$$mgL/2\cos\theta_0 = 1/2\, m V_{Gy}^2 + 1/2\, I_G\omega^2 + 0$$

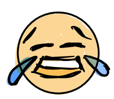
$$mgL/2\cos\theta = 1/2\, m L^2/4\, \omega^2 + 1/2\,(1/12\, mL^2)\omega$$

$$= 1/2\,(1/3\, mL^2)\,\omega^2$$

$$3\,mgL\cos\theta = mL^2\omega^2$$

$$\boxed{\omega = \sqrt{\frac{3g}{L}\cos\theta}}$$

$$\boxed{V_A = 0}$$



This was nasty
you have to realize
it stops moving
in "x" direction when
$\theta = 90°$. You will have
a HW problem that
does the same.